RTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR

(AUTONOMOUS)

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QUESTION BANK

Subject with Code **ENGINEERING MATHEMATICS-II** (16HS611)

Course & Branch: B.Tech (All Branches) **Year &Sem:** I-B.Tech& II-Sem**Regulation:** R16

UNIT –I

1. a) Find the rank of the matrix $\begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$ by using Echelon form. [6 M]

   b) Reduce the matrix $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ into normal form. Find its rank. [6 M]

2. .a) Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ by using Echelon form [6M]

   b) Reduce the matrix $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 5 & 6 \end{bmatrix}$ into normal form. Find its rank [6M]

3.(a)Test for consistency and if consistent solve them [6M]

$5x + 3y + 7t = 4; 3x + 26y + 2t = 9; 7x + 2y + 10t = 5;$

(b)Solve the system of equations $2x - y + 3z = 0; 3x + 2y + z = 0; x - 4y + 5z = 0$ [6M]

4.(a) Discuss for what values of λ and μ, the simultaneous equations $x + y + z = 6$

$x + 2y + 3z = 10; x + 2y + \lambda = \mu$have $i)$ no solution $ii)$ a unique solution

$iii)$An infinitely many solutions. [6 M]

(b)Solve the system of equations $2x + y + 2z = 0; x + y + 3z = 0; 4x + 3y + 8z = 0$ [6M]

5. Find Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ [12M]

6. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I.$ [12 M]

7. Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ find A^{-1} *and* A^4 [12M]

8. Diagonalize the matrix A= $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ also find A^2 [12M]

9. Reduce the quadratic form to the sum of squares form by orthogonal reduction. Find index, Nature and Signature of the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy.$ [12 M]

10.Reduce the quadratic form by orthogonalreduction and obtain the corresponding transformation. Find the index, signature and nature of the quadratic form $q = 2xy + 2yz + 2zx$

[12M]

UNIT – II

1. a) find the directional derivative of the function $xy^2 + yz^2 + zx^2$along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point $(1,1,1)$. [6M]
   b) Find a unit normal vector to the given surface $x^2y + 2xz = 4$ at the point (2,-2, 3). [6M]
2. a) find the directional derivative of the function $\emptyset = xyz$ along the direction of the normal to the surface $xy^2 + yz^2 + zx^2 = 3$ at the point $(1,1,1)$. [6M]
   b) Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the Point$(2, -1,2)$. [6M]
3. a) find $div\ f$ where $\bar{F} = grad(x^3 + y^3 + y^3 - 3xyz)$. [6M]
   b) Find $div\ f$ where $\bar{F} = r^n\bar{r}$. Find n if it is solenoidal. [6M]
4. a) compute the line integral $\int (y^2\, dx - x^2 dy)$ round the triangle whose vertices are $(1,0), (0,1) (-1,0)$ in the $xy - plane$ [6M]
   b) Find the work done in moving a particle in the force field $\bar{F} = 3x^2\bar{\imath} - (2xz - y)\bar{\jmath} + z\bar{k}$Along the straight line from$(0,0,0)$ to $(2,1,3)$. [6M]
5. a) Verify stokes theorem for the function $\bar{F} = x^2\bar{\imath} + xy\bar{\jmath}$ integrated round the square in the Plane $z = 0$ whose sides are along the lines $x = 0, y = 0, x = a, y = a.$ [8M]
   b) Evaluate by stokes theorem $\bar{F} = (2x - y)\bar{\imath} - yz^2\bar{\jmath} + y^2z\bar{k}$ over the upper half surface of the surface $x^2 + y^2 + z^2 = 1$ bounded by the projection of the xy-plane. [4M]

6. a) verify Gauss divergence theorem for $\bar{F} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ over the cube formed by the Planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$ [10M]

b) Define the statement of greens theorem [2M]

7. Verify Gauss Divergence theorem for $\bar{F} = (x^3 - yz)\bar{i} - 2x^2y\bar{j} + z\bar{k}$ taken over the surface of the cube bounded by the planes $x = y = z = a$and coordinate planes. [10M]
b) Define the statement of stokes theorem [2M]

8. Verify Greens theorem for $\int[(3x^2 - 8y^2)dx - (4y - 6xy)dy]$ where c is the region Bounded by $x = 0, y = 0$ and $x + y = 1$. [10M]
b) Define the statement of Gauss Divergence theorem [2M]

9. a) Find $curl\, f$ where$\bar{F} = 2xz^2\bar{i} - yz\bar{j} + 3xz^3\bar{k}$. [6M]
b) Prove that if $\bar{r}$ is position vector of any point in space, then$r^n\bar{r}$ is irrational. [6M]
10. a) if $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ evaluate $\iint \bar{F}.\bar{n}ds$ where s is the surface of the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a$ [6M]

b) Find the work done where $\bar{F} = (x - 3y)\bar{i} + (y - 2x)\bar{j}$and c is the closed curve in the

$xy - plane, x = 2cost, y = 3sint$From $t = 0\ to\ t = 2\pi$. [6M]

UNIT –III

1. A) Expand the function $f(x) = x^2$ as a fourier series in $[-\pi,\pi]$ and hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - - - - - - = \frac{\pi^2}{12}$ [6M]
b)Obtain Fourier Series in $(-\pi,\pi)$ for $f(x) = \begin{cases} 0, -\pi < x < 0 \\ sinx,\ 0 < x < \pi \end{cases}$ [6M]

2.a)Find the Fourier Series to represent the $f(x) = |cosx|, -\pi < x < \pi$ [6M]
b) Obtain the Fourier Series for $f(x) = e^{ax}$ in $(0,2\pi)$ [6M]

3. a) Obtain the Fourier Series expansion f(x) given that $f(x) = \left(\frac{\pi-x}{2}\right)^2, 0 < x < 2\pi$ .and deduce that value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + - - - - = \frac{\pi^2}{6}$ [10M]
b)What is the formula for half range sine series? [2M]

4. a) Expand the function $f(x) = xsinx$ as a Fourier series in the interval$-\pi \leq x \leq \pi$. Hence deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - - - - - = \frac{\pi-2}{4}$ [8M]
b)Expand the function $f(x) = x^3\ in - \pi < x < \pi$ [4M]

5 a) Write Dirichlet conditions and Eulers coefficients of Fourier Series. [4M]
b) Find Fourier Series of $f(x)$,If$f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$. Hence deduce that

$1 + \frac{1}{3^2} + \frac{1}{5^2} + - - - - - = \frac{\pi^2}{8}$ [8M]

6 a)Find the half range cosine series for $f(x) = x, 0 < x < \pi$ and hence deduce

that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+-----=\frac{\pi^2}{8}$ [6M]

b)Obtain Fourier Series $f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$ [6M]

7 a) Find the half range sine series for $f(x)=x(\pi-x)$,$0<x<\pi$ and hence deduce that

$\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}------=\frac{\pi^3}{32}$ [6M]

b) Express $f(x)=x^2-2$ as a Fourier series in $-2\le x\le 2$ [6M]

8) a) Find Fourier Series $f(x)=x^2$ in $[-l,l]$ [6M]

b) Find the half range cosine series for $f(x)=x$ in (0,2) [6M]

9) a) Expand $f(x)=e^{-x}$ as a Fourier series in the interval (-1,1) [6M]

b) Find the half range sine series for $f(x)=ax+b$,$0<x<1$ [6M]

10) a) Find half-range cosine series for $f(x)=(x-1)^2\ in\ 0<x<1$.

Hence S.T $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+-----=\frac{\pi^2}{6}$ [6M]

b) Find the Fourier Series for $f(x)=2lx-x^2\ in\ 0<x<2l$ [6M]

UNIT – IV

FOURIER TRANSFORMS

1.a) Express f(x) = $\begin{cases}1, 0\le x\le\pi\\ 0, x>\pi\end{cases}$ as a fourier sine integral and hence evaluate

$\int_0^\infty \frac{1-\cos(\pi\lambda)}{\lambda}\sin(x\lambda)d\lambda$ [6M]

b) Prove that (i) F_s { a f(x) + b g(x) }= a F_s(p) + bG_s(p)

(ii) F_c { a f(x) + b g(x) }= a F_c(p) + bG_c(p) [6M]

2. a) Prove that F[x^n f(x)] = $(-i)^n\frac{d^n}{dp^n}[F(p)]$ [6M]

b) Prove that F_s { x f(x) } = $-\frac{d}{dp}[F_c(p)]$ [6M]

3. Find the Fourier transform of f(x) = $\begin{cases}a^2-x^2, |x|<a\\ 0, |x|>a>0\end{cases}$ Hence show that

$\int_0^\infty \frac{\sin x-x\cos x}{x^3}dx=\frac{\pi}{4}$. [12M]

4. a)Find the Fourier transform of f(x) =. $e^{-\frac{x^2}{2}}, -\infty < x < \infty$

[6M]

b) If F(p) is the complex Fourier transform of f(x) , then prove that the complex Fourier transform of f(x) cos ax is $\frac{1}{2}[F(p+a)+F(p-a)]$

[6M]

5 . a) Find the Fourier cosine transform of $e^{-ax}\cos ax, a>0$

[6M]

b) Find the Fourier cosine transform of f(x) = $\begin{cases} x, \ for \ \ 0<x<1 \\ 2-x, \ for \ \ 1<x<2 \\ 0, \ for \ \ x>2 \end{cases}$ [6M]

6.Find the Fourier sine and cosine transforms of f(x)=$\frac{e^{-ax}}{x}$ and deduce that

$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\sin sx\, dx = \tan^{-1}\left(\frac{s}{a}\right)-\tan^{-1}\left(\frac{s}{b}\right).$

[12M]

7. Find the Fourier sine and cosine transforms of f(x)= $e^{-ax}, a>0$ and hence deduce the integrals

(i) $\int_0^\infty \frac{p\sin px}{a^2+p^2}dp$ (ii) $\int_0^\infty \frac{\cos px}{a^2+p^2}dp$ [12M]

8. Find the inverse Fourier sine transformof f(x) of $F_s(\text{p}) = \frac{p}{1+p^2}$ [12M]

9.a) Find the finite Fourier sine transform of f(x),defined by $\begin{cases} x, \ 0 \le x \le \frac{\pi}{2} \\ \pi - x, \ \frac{\pi}{2} \le x \le \pi \end{cases}$ [6M]

b) Find the inverse finite Fourier sine transformof f(x),If $F_s(\text{n}) = \frac{16(-1)^{n-1}}{n^3}$, where n is a positive integer and 0 < x < 8.

[6M]

10.a) Using Parseval's Identity, show that $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2ab(a+b)}$

[6M]

b) Evaluate the following using Parseval's Identity $\int_0^\infty \frac{x^2}{(a^2+x^2)^2}dx$, (a > 0). [6M]

UNIT – V

1. a)From P.D.E by eliminating arbitrary constants a&b $4(1+a^2)z = (x+ay+b)^2$ [6M]
   b) Form the partial differential equation by eliminating the arbitrary functions from $f(x+y+z, x^2+y^2+z^2) = 0$ [6M]
2. a)Form the partial differential equation by eliminating a and b from $log(az-1) = x+ay+b$ [6M]
   b) Form the partial differential equation by eliminating the arbitraryfunction from $z = y^2 + 2f\left(\frac{1}{x} + logy\right)$ [6M]
3. a) Form the P.D.E by eliminating the arbitraryfunction from $z = xy + f(x^2+y^2)$[6M]
   b) Form the P.D.E by eliminating arbitrary functions $f(x)\ g(y)\ from\ \ z = yf(x) + xg(y)$ [6M]
4. a)Using the method of separation of variables solve$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$ [6M]
   b)Form the partial differential equation by eliminating the arbitrary constants a,b,c from$z = a(x+y) + b(x-y) + abt + c$ [6M]
5. a) Solve by Method of separation of variables $4u_x + u_y = 3u$and $u(0,y) = e^{-5y}$ [6M]
   b)Form the P.D.E by eliminating arbitrary function $\emptyset\left(\frac{y}{x}, x^2+y^2+z^2\right) = 0$ [6M]
6. a)Solve by Method of separation of variables$3u_x + 2u_y = 0$and $u(x,0) = 4e^{-x}$[6M]
   b)Form the P.D.E by eliminating arbitrary functions $z = f(y) + \emptyset(x+y)$ [6M]
7. A string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b\ sin^3\left(\frac{\pi x}{l}\right)$.Find displacement $y(x,t)$ [12M]
8. A tightly stretched string with fixed end points $x = 0, x = l$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $y = kx(l-x)$ Find the Displacement of the string at any distance x from one end at any time t. [6M]

9) A homogenous rod of conducting material of length 100cm has its ends kept at zero Temperature and the temperature initially is $u(x,0) = \begin{cases} x & ; \quad 0 \le x \le 50 \\ 100-x & ; 50 \le x \le 100 \end{cases}$

Find the temperature $u(x,t)$ at any time [12M]

10) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions $u(0,y) = 0, u(L,y) = 0,$
$u(x,0) = 0, u(x,a) = sin\left(\frac{n\pi x}{L}\right)$ [12M]