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QUESTION BANK (DESCRIPTIVE)

Subject with Code : Mathematics-I **(18HS0830)** **Course & Branch**: B.Tech – ALL

Year & Sem: I-I **Regulation:** R18

UNIT –I

1. a) Find the Rank of A= $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$. [2M]

b) Reduce the matrix A= $\begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$ into Echelon form and find its rank? [2M]

c) Define Symmetric & Skew-symmetric matrices. [2M]

d) If A= $\begin{bmatrix} 3 & a & b \\ -2 & 2 & 4 \\ 7 & 4 & 5 \end{bmatrix}$ $is\ symmetric$, then find a, b values? [2M]

e) Find the Eigen values of A= $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. [2M]

2. a) Define the rank of the Matrix. [2M]

b) Find whether the following equations are consistent if so solve them $x + y + 2z = 4\,; 2x - y + 3z = 9\,; 3x - y - z = 2.$ [8M]

3. a) Show that the matrix $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ is a orthogonal matrix. [5M]

b) Express the matrix $A = \begin{bmatrix} 3 & -2 & -6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix. [5M]

4. a) Find the rank of the matrix A = $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ [5M]

b) Determine the Eigen values of $A^{-1}\ where\ A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. [5M]

5. a) Show that A= $\frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ is orthogonal. [5M]

b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ [5M]

6. a) State Cayley-Hamilton theorem [2M]

b) Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation. [8M]

7. Find the Eigen values and corresponding Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. [10M]

8. Diagonalise the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ and hence find A^4. [10M]

9. Reduce the Quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into sum of squares form by Orthogonal transformation. [10M]

10. Reduce the Quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy + 2xz - 2yz$ into the canonical form by Orthogonal transformation. [10M]

UNIT –II

1. a) Evaluate the improper integral $\int_1^\infty \frac{1}{x^4} dx$. [2M]

b) Define gamma and beta function. [2M]

c) Prove that $\Gamma(1)=1$. [2M]

d) State Rolle's theorem. [2M]

e) State Lagrange's mean value theorem. [2M]

2. a) Find the surface area generated by the revolution of an arc of (catenary) curve $y = c.\cosh\frac{x}{c}$ from x=0 to x=c about the x-axis. [5M]

b) Find the volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis. [5M]

3. a) Find the surface area of the sphere of radius 'a'. [5M]

b) Find the volume of the reel-shaped solid formed by the revolution about the y- axis, of the part of the parabola $y^2 = 4ax$ cut off by the latus- rectum. [5M]

4. a) State and verify the Roller's theorem then $f(x) = \log[\frac{x^2 + ab}{x(a+b)}]$ in [a, b]. [5M]

b) Verify lagrange's mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in [0, 4]. [5M]

5. a) verify Cauchy's mean value theorem for the function $\sin x$ and $\cos x$ in the interval $\left[0, \frac{\pi}{2}\right]$. [5M]

b) Express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$ assigning Taylor's series. [5M]

6.a) Calculate the approximate value of $\sqrt{10}$ correct to 4 decimal places using Taylor's theorem. [5M]

b) Expand $\log_e x$ in power of $(x-1)$ and hence evaluate log 1.1 correct to 4 decimal places using Taylor's theorem. [5M]

7. a) Using Maclaurin's sexries expand $tanx$ up to the fifth power of x and hence find the series for $\log(secx)$. [5M]

b) Evaluate $\int_0^1 x^2 \left(\log\frac{1}{x}\right)^3$. [5M]

8. a) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}}\, dx = \frac{1}{5} B\left(\frac{2}{5},\frac{1}{2}\right)$. [5M]

b) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^2}}\, dx = \frac{1}{2}\, \beta\left(1,\frac{1}{2}\right)$. [5M]

9. a) Prove that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. [5M]

b) Evaluate $\int_0^\infty \sqrt{x}\, e^{-x^2} dx$ [5M]

10. a) Prove that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta . cos^{2n-1}\theta\, d\theta$. [5M]

b) Prove that $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx = \tau(n)$. [5M]

UNIT –III

1) a) Evaluate $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$. [2M]

b) Define Total differential Coefficient. [2M]

c) Find the stationary points of $f(x,y) = x^3+y^3-3axy$. [2M]

d) Define CURL of a Vector. [2M]

e) If $\vec{f} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$ find $div.\vec{f}$ at $(1,-1,1)$. [2M]

2) a) Discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & at\ (0,0) \end{cases}$ [5M]

b) If $U = \frac{1}{\sqrt{x^2+y^2+z^2}}$; $x^2+y^2+z^2 \neq 0$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$. [5M]

3) a) If $u = tan^{-1}\left[\frac{2xy}{x^2-y^2}\right]$,prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. [5M]

b) If $U = log(x^3+y^3+z^3-3xyz)\ prove\ that \left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 U = \frac{-9}{(X+Y+Z)^2}$. [5M]

4) a) Find $\frac{du}{dt}$ $as\ a\ total\ derivative; if\ u = x^2y^3$ where $x = logt\ \ and\ y = e^t$ [5M]

b) If $z = xy^2 + x^2y\ ; where\ x = at^2, y = 2at,\ find\ \frac{dz}{dt}$ as a total derivative. [5M]

5) a) If $u = sin^{-1}(x-y), where\ x = 3t, y = 4t^3\ , then\ show\ that\ \frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$. [5M]

b) If $u = x^2+y^2+z^2$ and $x = e^{2t}$, $y = e^{2t}cos3t$, $z = e^{2t}sin3t$, find $\frac{du}{dt}$ =? [5M]

6) a) Examine the function for extreme values $f(x,y) = x^4+y^4-2x^2+4xy-2y^2$; (x>0,y>0) [5M]

b) Find the stationary points of $u(x,y) = sinx.siny.sin(x+y)$ where $0 < x < \pi, 0 < y < \pi$ and find the maximum of u. [5M]

7) a) Find the shortest distance from origin to the surface $xyz^2 = 2$. [5M]

b) Find the minimum value of $x^2+y^2+z^2$ given $x+y+z = 3a$. [5M]

8) a) Find a point on the plane $3x+2y+z-12 = 0$, which is nearest to the origin. [5M]

b) Find the shortest and longest distance from the point $(3,1,-1)$ to the sphere $x^2+y^2+z^2 = 4$. [5M]

9) a) Find the directional derivative of the fuction $f = x^2-y^2+2z^2$ at the point $P = (1,2,3)$ in the direction of the line PQ where $Q = (5,0,4)$. [5M]

b) Find the directional derivative of $f(x,y,z) = 2xy+z^2$ at $(1,-1,3)$ in the direction of $\vec{i}+2\vec{j}+3\vec{k}$. [5M]

10) a) Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point $(2,-1,2)$. [5M]

b) Find $curl.\vec{f}$ where $\vec{f} = grad(x^3+y^3+z^3-3xyz)$. [5M]

UNIT –IV

1. a) Define Convergent and Divergent of a sequence [2M]

b) Examine the sequence $a_n = 2^n$ for convergence. [2M]

c) Define Sequence and Series. [2M]

d) Test for convergence the series $\sum \frac{n^3}{3^n}$. [2M]

e) Define Power Series. [2M]

2. Examine the following sequences for convergence:

$i)\ a_n = \frac{n^2-2n}{3n^2+n}$ $\quad ii)\ a_n = 3+(-1)^n$. [10M]

3. Show that the series $1+r+r^2+r^3+$-----∞

$i)$ Convergence if $|r| < 1$ $\quad ii)$ Divergence if $r \geq 1$ and $\quad iii)$ Oscillates if $r \leq -1$. [10M]

4. Show that the $p-series$ $\sum_{n=1}^{\infty}\frac{1}{n^p} = \frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+$------∞

$i)$ Converges for $p > 1$ $\quad ii)$ Diverges for $p \leq 1$ [10M]

5. Test for convergence the series

$i)\ \frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+$------∞ $\quad ii)\ \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n+1}}$. [10M]

6. Discuss the convergence of the series i) $\sum\left(\frac{n!}{(n^n)^2}\right)$ ii) $1+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+$-------$\infty$ [10M]

7. a) Test for convergence the series $\sum(1+\frac{1}{\sqrt{n}})^{-n^{\frac{3}{2}}}$. [5M]

b) Discuss the nature of the series $\sum\frac{(n+1)^n x^n}{x^{n+1}}$. [5M]

8. State the value of x, for which the following series converge:

i) $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}--------\infty$. ii) $\frac{1}{1-x}+\frac{1}{2(1-x)^2}+\frac{1}{3(1-x)^3}+-------\infty$. [10M]

9. a) Show that the exponential series

$1+x+\frac{x^2}{2!}+---+\frac{x^n}{n!}----\infty$ is convergent for all values of x. [5M]

b) For what values of x the series $x+\frac{2!}{2^2}x^2+\frac{3!}{3^3}x^3+----+\frac{n!}{n^n}x^n+$---- is convergent. [5M]

10. a) Discuss the convergence of the series $\sum\frac{1}{\sqrt{n}}tan\frac{1}{n}$. [5M]

b) Test for convergence of the series $\sum log\left(1+\frac{1}{n}\right)$. [5M]

UNIT –V

1. a) Find Fourier coefficient b_n when $f(x)=e^x$ in $[-\pi,\pi]$. [2M]

b) If $f(x)=|\sin x|$ in $-\pi<x<\pi$ then find Fourier coefficient a_0. [2M]

c) Find the half-range sine series for $f(x)=1$ in $0,\pi$ [2M]

d) Obtain the Fourier series for $f(x)=\pi x$ in $0\le x\le 2$ [2M]

e) Find Fourier constant a_0 for $f(x)=1-x^2$ in [-1,1]. [2M]

2. Find a Fourier series to represent the function $f(x)=e^x$ for $-\pi<x<\pi$. and hence derive a series for $\frac{\pi}{sinh\pi}$. [10M]

3 . a) Obtain the Fourier series expansion of $f(x)=(\pi-x)^2$ in $0<x<2\pi$ and deduce the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}----=\frac{\pi^2}{6}$. [5M]

b) Find the Fourier series for the function $f(x)=x$; in $-\pi<x<\pi$. [5M]

4. Find the Fourier series to represent the function $f(x)=x^2$ for $-\pi<x<\pi$ and hence show that

(i) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}----=\frac{\pi^2}{12}$. (ii) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}----=\frac{\pi^2}{6}$.

(iii) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}----=\frac{\pi^2}{8}$ [10M]

5. a) If $f(x)=|\sin x|$, expand f(x) as a Fourier series in the interval $-\pi,\pi$ [5M]

b) Find the half range cosine series for $f(x)=x$ in the interval $0\le x\le\pi$. [5M]

6. Expand the function $f(x)=|x|$ in $-\pi<x<\pi$ as a Fourier series and Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}----=\frac{\pi^2}{8}$ [10M]

7. Find the half range sine series for $f(x) = x(\pi - x)$ in the interval $0 \leq x \leq \pi$ and

   Deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} - - - - = \frac{\pi^3}{32}$. [10M]

8. a) Expand $f(x) = e^{-x}$ as a fourier series in the interval $(-1,1)$. [5M]

   b) Expand $f(x) = |x|$ as a fourier series in the interval $(-2,2)$. [5M]

9. a) Find the half range sine series expansion of $f(x) = x^2$ when $0 < x < 4$. [5M]

   b) Find the half range cosine series expansion of $f(x) = x(2 - x)$ in $0 \leq x \leq 2$. [5M]

10. Find half range fourier cosine series of $f(x) = (x - 1)^2$ in $0 < x < 1$.

    Hence show that $i)\ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} - - - - = \frac{\pi^2}{6}$ $\quad ii)\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$. [10M]