SIDDHARTH INSTITUTE OF ENGINEERING AND TECHNOLOGY
AUTONOMOUS

RANDOM SIGNAL AND STOCHASTIC PROCESS

UNIT-1

1. **(a)** Define the following with examples. i.
   Sample space
   ii. Event
   iii. Mutually exclusive events. iv.
   Independent events.
   (b) Two cards are drawn from a 52 –card deck (the first is not replaced).
       i) Given that first card is a queen, what is the probability that the second is also a
         Queen?
       ii)   Repeat part (i) but replace the first card with a queen and the second card with a 7.
2. **(a)** Explain about Joint and Conditional probability and also state the properties of Joint & Conditional probability?
   (b) In a box there are 100 resistors having resistance and tolerance values given in table below. Let a resistor be selected from the box and assume that each resistor has the same likelyhood of being chosen. Define 3 events:
   Event A: Draw a 47Ω resistor
   Event B: Draw a resistor with 5% tolerance
   Event C: Draw a 100Ω resistor
   Find the individual, joint and conditional probabilities.

   Table: Details of resistances

Resistance(Ω)	Tolerance		Total
	5%	10%	
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

3. **(a)** Discuss about Total Probability Theorem?
   (b) Two boxes are selected randomly. The first box contains 2 white balls and 3 black balls. The second box contains 3 white and 4 black balls. What is the probability of drawing a white ball?
   (c) When two dice are thrown, find the probability of getting sum of 10 or 11?

4. **(a)** Explain about Baye's theorem?
   (b) In a bolt factory, machines A,B,C manufacture 30%,30%,40% of the total output respectively. From their outputs 4,5,3 percents are defective bolt. A bolts is drawn at random and found to be defective. What are the probabilities that it was manufacturing by machines A,B and C?
5. (a).Define axioms of probability.
   (b).Define probability as a relative frequency.
   (c) When two dice are thrown, determine the probabilities from axiom3 for the following three events
       i) A= {sum=7}   ii) B= { 8< sum<11}   iii) C={10<sum}

6. **(a)** Define Random variable? Explain about probability distribution function with properties?
   (b) let X be a continuous random variable with density function

   fx(x)=   (x/9)+k $0 \leq x \leq 6$

                   0   Otherwise

   i) Find the value of 'k'   ii) find $P(2 \leq x \leq 5)$

7. **(a)** Explain about probability density function? And State its properties?
   (b) The random variable X has the discrete variable in the set {-1,-0.5,0.7,1.5,3}. The corresponding probabilities are assumed to be a {0.1, 0.2, 0.1, 0.4, 0.2}. Plot the distribution function?

8. **(a)** Discuss about Gaussian distribution function? Plot the distribution and density function?
   (b) Discuss about uniform distribution function? Plot the distribution and density function?

9. **(a)** Explain about Exponential distribution function? Plot the distribution and density function? **(b)** Explain about Rayleigh distribution function? Plot the distribution and density function?

10. **(a)** Explain about binomial distribution function? Plot the distribution and density function?
    (b) Explain about Poisson distribution function? Plot the distribution and density function?

UNIT-2

1. **(a)** Explain about Joint distribution & density function? And discuss its properties?
   (b) If the joint Pdf of two dimensional random variable (x, y) is given by:

$$f_{X,Y}(x,y) \quad = \quad kxy \quad ; \quad 0<x<y<1$$
$$= \quad 0 \quad ; \quad \text{Otherwise}$$

   Find the 'k' value and marginal density function of X and Y.

2. **(a).** Define statistical independence of random variables? And explain about Point Conditioning in distribution and density functions?
   (b) Random variable X and Y have the density:

$$f_{X,Y}(x,y) \quad = \quad \begin{cases} \frac{1}{18}e^{-(x/6+y/3)} & \text{for } x \geq 0 \text{ and } y \geq 0 ; \\ 0 & \text{Elsewhere} \end{cases}$$

   Show that X and Y are independent random variables?

3. **(a).** consider that the joint pdf of random variables, X and Y is

$$f_{X,Y}(x,y)= \begin{cases} \frac{1}{8}(x+y) & \text{for } 0\leq x\leq 2, 0\leq y\leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   Find conditional density functions?

   b) The joint cdf of a random variable X and Y is given by

$$F_{X,Y}(x,y)= \begin{cases} (1-e^{-ax})(1-e^{-by}) & x \geq 0, y \geq 0, a, b > 0 \\ 0 & \text{otherwise} \end{cases}$$

   Find the marginal cdfs of X and Y

4. **(a).** Discuss about the Sum of Two Random Variables?
   (b). Statistically independent random variables X and Y have densities
   $f_X(x)=5u(x)e^{-5x}$
   $f_Y(y)=2u(x)e^{-2y}$

   find the density of the sum W= X+Y

5. **(a).** State central limit theorem?
   (b). Find the distribution function $F_{X,Y}(x,y)$ and the marginal distribution functions?

(X,Y)	(0,0)	(1,2)	(2,3)	(3,2)
P(x,y)	0.2	0.3	0.4	0.1

6. **(a)** State and prove the properties of correlation function.
   (b) Consider two random variables X and Y such that Y= -4X+20. The mean value and the variance of X are 4 and 2 respectively. Find the correlation?

7. **(a)** Define Covariance and correlation coefficient?
   (b) Discuss about Joint characteristic function and its properties?

8. (a). Define Expected value of a function of two random variables?

   (b). Random variables X and Y have the joint density $f_{X,Y}(x,y)=\frac{1}{24}, 0 \leq x \leq 6, \ 0 \leq y \leq 4$. what is the expected value of the function $g(x,y)=(X Y)^2$?

9. **(a)** Explain about Jointly Gaussian Function for Two random variables? And its properties?

(b) given the function $f_{x,y}(x,y) = \begin{cases} b(x+y)^2 & -2 \leq x \leq 2, -3 \leq y \leq 3 \\ 0 & \text{elsewhere} \end{cases}$

  i)Find the a and b values?
  ii)Determine the marginal density functions?

10.**(a).** The joint density function of X and Y is

$$f_{x,y}(x,y)= \begin{cases} \dfrac{1}{100} & 0 \leq x \leq 5, \ 0 \leq y \leq 20 \\ \\ 0 & \text{otherwise} \end{cases}$$

  Find the expected value of the functions (a) XY (b) X^2Y (c) $(XY)^2$

  (b) Explain about Joint moment generating functions? And its properties?

UNIT-3

1. What is ACF? State and explain any four properties of ACF?

2. Explain about first order, second, wide-sense and strict sense stationary process.

3. (a)Show that the autocorrelation function of a stationary random process is an even function of τ
   (b)Give the classification of random processes.

4. A random process is defined by x(t) =At where A is a continuous random variable uniformly
5. Distributed on (0,1) and t represents time. Find (a) E [x (t)] (b) Rxx [t, t + τ]   (c) Is the process stationary?

5. (a) A random process is defined as $X(t)=A \sin(\omega t+\Theta)$,where A is a constant and Θ is a random

   Variable uniformly distributed over $(\pi,-\pi)$,check X(t) is stationary.

   (b). Prove the following 1. $|RXX (\tau)| \le RXX (0)$ 2. $RXX (-\tau)= RXX (\tau)$ 3. $RXX (0) = E[X^2(t)]$

6. (a) State the conditions for wide sense stationary random process.
   (b) Write short notes on ergodic random processes.

7.What is cross correlation function of a random process? state and explain any four properties of cross correlation function of a random process?

8. (a) Explain about mean-ergodic process.

   (b).If x (t) is a stationary random process having auto correlation function:
   $RXX (\tau) = 9 + 2e^{-|\tau|}$. Find the mean and variance of the random variable.
9.(a) Explain the significance of auto correlation.
   (b) Find auto correlation function of a random process whose power spectral density is given by
   $4/(1+(\omega^2/4))$

10. Determine whether the random process $X(t)= A \cos(wot+\theta)$ is wide sense stationary or Not where A, wo are constants and θ is a uniformly distributed random variable on the Interval $(0,2\pi)$

UNIT-4

1.(a) Briefly explain the concept of cross power density spectrum.
   (b) Find the cross correlation of functions sin ωt and cos ωt.

2.(a) The power spectral density of a stationary random process is given by

   $S_{xx}(\omega) = \begin{cases} A; & -k < \omega < k \\ 0; & \text{otherwise} \end{cases}$

   Find the auto correlation function.
   (b)Discuss the properties of power spectral density.

3. (a)Discuss the properties of cross power density spectrum.
   (b)Discuss the relation between cross power spectrum and cross correlation function.

4. State and prove properties of PDS

5. (a) If the PSD of x(t) is $S_{XX}(\omega)$.Find the PSD of dx(t)/dt.
   Find the PSD of a stationary random process for which autocorrelation is $R_{XX}(\tau)=6e^{-\alpha|\tau|}$

6. (a) State and prove wiener –khintchins relations
   (b)Prove that 1. $S_{XX}(-\omega)= S_{XX}(\omega)$ 2. $S_{XY}(\omega)= S_{YX}(-\omega)$

7. The psd of X(t) is given by

   $S_{xx}(\omega) = 1+ \omega^2$ for IωI<1
                0   otherwise
   Find the autocorellation function.

8. (a)Discuss the properties of CPSD?.
   (b) The auto correlation of a WSS random process X(t) is given by $R_{xx}(\tau)= A \cos(w0\ \tau)$ where A and *W*0 are constants. Find psd?.

9.(a) A stationary random process X(t) has autocorellation $R_{XX}(\tau)=10+5\cos(2\tau)+10e^{-2|\tau|}$.Find the dc and ac powers of X(t).
   (b) Prove that the psd of the derivatie X(t) is equal to ω^2 times the psd of $S_{xx}(\omega)$?

10.(a)Assume that the ergodic random process X(t) has an autocorellation
    function $R_{XX}(\tau)=18+(2/(6+\tau^2))[1+4\cos(12\tau)]$
    What is the average power of X(t).
    (b).Prove that $S_{XX}(\omega)= S_{XX}(-\omega)$

UNIT-5

1.(a). Derive the relation between PSDs of input and output random process of an LTI system.
(b)Discuss about cross correlation between the input X (t) and output Y (t).

2. (a) Explain about LTI system
(b) Find the power density spectrum of response of a linear system

3. X(t) is a stationary random process with zero mean and auto correlation $R_{xx}(t)=e^{-2|\tau|}$ is applied to a system of function $H(\omega)=1/j\omega+2$.Find mean and PSD of its output.

4.Write notes on:
(a) Band Pass random process.
(b) Band limited random process
(c) Narrow band random process.

5.(a) Derive the relation between PSD of input and output random process of an LTI system.
(b) Discuss about cross correlation between the input X (t) and output Y (t)

6.Derive the expressions for mean. Autocorrelation cross correlation and PSD of response of a linear system response is $5te^{-2t.}$

7.(a)How mean of the system response Y(t) is calculated?
(b)Write different types of band pass processes with band limited processes.

8.(a)Define mean value of system response.
(b)Find mean square value of Y(t).

9.(a) A WSS random process x(t) is applied to the input of an LTI system whose impulse The mean of x(t) is 3.Find the mean output of the system
(b) Give any two spectral characteristics of the system response.

10.(a) consider a linear system as shown in fig

Linear system
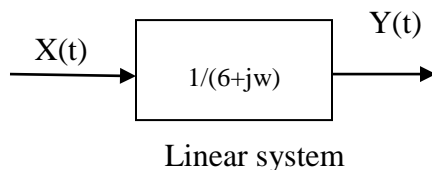
The autocorrelation of $x(t)=5\delta(\tau)$. Find the psd, autocorrelation and mean square value of the output y(t).

(b). A random process X(t) is applied to a network with impulse response h(t)= $e^{-bt}u(t)$, where b>0 is constant. The cross correlation X(t) with the output Y(t) is known to have the form $R_{xx}(\tau) = u(\tau)\tau e^{-bt}$. Find the autocorrelation of Y(t).