RANDOMSIGNAL ANDSTOCHASTICPROCESS

UNIT-1

1. **(a)**Definethefollowingwith examples. i.Sample space
   ii.Event
   iii.Mutuallyexclusive events.
   iv.Independentevents.
   (b) Two cards are drawn froma 52–card deck (thefirstisnotreplaced).
   i)Giventhatfirstcardisaqueen,whatistheprobabilitythatthesecondisalsoa Queen?
   ii) Repeatpart(i)but replacethe firstcard witha queen and thesecond card witha 7.
2. **(a)**Explain aboutJointandConditionalprobabilityand alsostatethe properties ofJoint& Conditional probability?
   (b) Inaboxthereare100resistorshavingresistanceandtolerancevaluesgivenintablebelow.Leta resistorbeselectedfromtheboxandassumethateachresistorhasthesamelikelyhoodofbeing chosen. Define 3events:
   EventA:Draw a47Ωresistor
   EventB:Draw aresistor with 5%tolerance
   EventC:Draw a100Ωresistor
   Find theindividual,jointand conditionalprobabilities.

   Table: Details of resistances

Resistance(Ω)	Tolerance		Total
	5%	10%	
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

3. **(a)**Discuss about TotalProbabilityTheorem?
   (b) Two boxesare selected randomly.The firstboxcontains2whiteballsand3 black balls.The secondboxcontains3whiteand4blackballs. Whatistheprobabilityofdrawingawhite ball?
   (c) Whentwo dicearethrown,findtheprobabilityofgettingsumof10 or11?

4. **(a)**ExplainaboutBaye’stheorem?

(b) Inaboltfactory, machinesA,B,Cmanufacture30%,30%,40%ofthetotaloutputrespectively. Fromtheiroutputs4,5,3percentsaredefectivebolt.Abolts isdrawnatrandomandfoundtobe defective. Whatare the probabilities that it was manufacturingbymachinesA,Band C?

5. (a).Define axioms of probability.
   (b).Define probability as a relative frequency.

(c) Whentwodicearethrown,determinetheprobabilitiesfromaxiom3forthefollowingthree events
i)A= {sum=7} ii)B= { 8<sum<11} iii)C={10<sum}

6. **(a)**Define Randomvariable?Explain aboutprobabilitydistribution function withproperties?

(b) let X be a continuous random variable with density function
fx(x)= (x/9)+k $0 \le x \le 6$
0 Otherwise
i) Find the value of 'k' ii) find $P(2 \le x \le 5)$

7. **(a)** Explain about probabilitydensityfunction? And Stateits properties?
(b) The random variable X has the discrete variable in the set {-1,-0.5,0.7,1.5,3}. The correspondingprobabilitiesareassumedtobea{0.1, 0.2,0.1,0.4,0.2}.Plotthe distribution function?
8. **(a)** Discuss about Gaussian distribution function? Plotthe distribution anddensityfunction?
(b) Discuss about uniformdistribution function? Plotthedistribution anddensityfunction?
9. **(a)** ExplainaboutExponentialdistribution function? Plot the distributionand densityfunction? **(b)** Explain aboutRayleigh distribution function? Plotthedistributionand densityfunction?
10. **(a)** Explainaboutbinomialdistributionfunction? Plot the distribution and densityfunction?
(b) Explain aboutPoissondistributionfunction? Plotthe distribution and densityfunction?

UNIT-2

1.**(a)**Explain aboutJoint distribution & densityfunction? And discuss its properties?

(b) If thejointPdfof two dimensionalrandomvariable (x, y) is given by:

$f_{X,Y}(x,y) =$ kxy; 0<x<y<1

= 0 ; Otherwise

Find the 'k' value and marginaldensity function ofXand Y.

2.**(a).** Define statistical independence of random variables? And explain about Point Conditioning in distribution and density functions?

(b) RandomvariableXandYhavethedensity:

$f_{X,Y}(x,y) = \begin{cases} \frac{1}{18} e^{-(x/6+y/3)} & \text{for } x \geq 0 \text{ and } y \geq 0; \\ 0 & \text{Elsewhere} \end{cases}$

Show that X and Y are independent random variables?

3.**(a).**consider that the joint pdf of random variables, X and Y is

$f_{X,Y}(x,y) = \begin{cases} \frac{1}{8}(x+y) & \text{for } 0 \leq x \leq 2, 0 \leq y \leq 2 \\ = 0 & \text{otherwise} \end{cases}$

Find conditional density functions?

b) The joint cdf of a random variable X and Y is given by

$F_{X,Y}(x,y) = \begin{cases} (1-e^{-ax})(1-e^{-by}) & x \geq 0, y \geq 0, a, b > 0 \\ 0 & \text{otherwise} \end{cases}$

Find the marginal cdfs of X and Y

4.**(a).** Discuss about the Sum of Two Random Variables?

(b). Statistically independent random variables X and Y have densities

$f_X(x) = 5u(x)e^{-5x}$

$f_Y(y) = 2u(x)e^{-2y}$

find the density of the sum W= X+Y

5.**(a).**State central limit theorem?

(b). Find the distribution function $F_{X,Y}(x,y)$ and the marginal distribution functions?

(X,Y)	(0,0)	(1,2)	(2,3)	(3,2)
P(x,y)	0.2	0.3	0.4	0.1

6.**(a)**State andprove the propertiesofcorrelationfunction.

(b) Consider two random variables X and Y such that Y= -4X+20. The mean value and the variance of X are 4 and 2 respectively. Find the correlation?

7.**(a)** Define Covariance and correlation coefficient?

(b) Discuss about Joint characteristic function and its properties?

8.(a). Define Expected value of a function of two random variables?

(b). Random variables X and Y have the joint density $f_{X,Y}(x,y) = \frac{1}{24}, 0 \leq x \leq 6, \ 0 \leq y \leq 4$. what is the expected value of the function $g(x,y) = (XY)^2$?

9.**(a)** Explain about Jointly Gaussian Function for Two random variables? And its properties?

(b) given the function $f_{x,y}(x,y) = \begin{cases} b(x+y)^2 & -2 \leq x \leq 2, -3 \leq y \leq 3 \\ 0 & \text{elsewhere} \end{cases}$

i)Find the a and b values?

ii)Determine the marginal density functions?

10.**(a).** The joint density function of X and Y is

$$f_{X,Y}(x,y)=100 \begin{cases} \frac{1}{100} & 0 \le x \le 5,\ \ 0 \le y \le 20 \\ 0 & \text{otherwise} \end{cases}$$

Find the expected value of the functions (a) XY (b) X^2Y(c) $(XY)^2$

(b) Explain about Joint moment generating functions?And its properties?

UNIT-3

1. What is ACF? State and explain any four properties of ACF?

2. Explain about first order,second,wide-sense and strict sense stationary process.

3. (a)Show that the autocorrelation function of a stationary random process is an even function of τ
   (b)Give the classification of random processes.

4. A random process is defined by x(t) =At where A is a continuous random variable uniformly
5. Distributed on (0,1) and t represents time. Find (a) E [x (t)] (b) Rxx [t, t + τ] (c) Is the process stationary?

5. (a) A random process is defined as X(t)=A sin(ωt+Θ),where A is a constant and Θ is a random

   Variable uniformly distributed over (π,-π),check X(t) is stationary.

   (b). Prove the following 1. $|R_{XX}(\tau)| \leq R_{XX}(0)$ 2. $R_{XX}(-\tau) = R_{XX}(\tau)$ 3. $R_{XX}(0) = E[X^2(t)]$

6. (a) State the conditions for wide sense stationary random process.
   (b) Write short notes on ergodic random processes.

7. What is cross correlation function of a random process? state and explain any four properties of cross correlation function of a random process?

8. (a) Explain about mean-ergodic process.

   (b).If x (t) is a stationary random process having auto correlation function:
   $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean and variance of the random variable.

9. (a) Explain the significance of auto correlation.
   (b) Find auto correlation function of a random process whose power spectral density is given by $4/(1+(\omega^2/4))$

10. Determine whether the random process X(t)= A cos(wot+ɵ) is wide sense stationary or Not where A, wo are constants and ɵ is a uniformly distributed random variable on the Interval (0,2π)

UNIT-4

1.(a) Briefly explain the concept of cross power density spectrum.
(b) Find the cross correlation of functions sin ωt and cosωt.

2.(a) The power spectral density of a stationary random process is given by

$$S_{xx}(\omega) = \begin{cases} A; & -k < \omega < k \\ 0; & \text{otherwise} \end{cases}$$

Find the auto correlation function.
(b)Discuss the properties of power spectral density.

3. (a)Discuss the properties of cross power density spectrum.
(b)Discuss the relation between cross power spectrum and cross correlation function.

4. State and prove properties of PDS

5. (a) If the PSD of x(t) is SXX(ω).Find the PSD of dx(t)/dt.
Find the PSD of a stationary random process for which autocorrelation is $R_{XX}(\tau)=6e^{-\alpha|\tau|}$

6. (a) State and prove wiener –khintchins relations
(b)Prove that 1. SXX(-ω)= SXX(ω) 2. SXY(ω)= SYX(-ω)

7. The psd of X(t) is given by

$$S_{xx}(\omega) = \begin{cases} 1+\omega^2 & \text{for } |\omega|<1 \\ 0 & \text{otherwise} \end{cases}$$

Find the autocorellation function.

8.(a)Discussthe properties of CPSD?.
(b) The autocorrelation of a WSS random process X(t) is given by $R_{xx}(\tau)= A\cos(w_0 \tau)$ where A and W_0 are constants. Find psd?.

9.(a) A stationary random process X(t) has autocorellation $R_{XX}(\tau)=10+5\cos(2\tau)+10e^{-2|\tau|}$.Find the dc and ac powers of X(t).
(b) Prove that the psd of the derivatieX(t) is equal to ω^2 times the psd of $S_{xx}(\omega)$?

10.(a)Assume that the ergodic random process X(t) has an autocorellation function$R_{XX}(\tau)=18+(2/(6+\tau^2))[1+4\cos(12\tau)]$
What is the average power of X(t).
(b).Prove that SXX(ω)= SXX(-ω)

UNIT-5

1.(a). Derive the relation between PSDs of input and output random process of an LTI system.
(b)Discuss about cross correlation between the input X (t) and output Y (t).

2. (a) Explain about LTI system
(b) Find the power density spectrum of response of a linear system

3.X(t) is a stationary random process with zero mean and auto correlation Rxx(t)=$e^{-2|\tau|}$ is applied to a system of function H(ω)=1/jω+2.Find mean and PSD of its output.

4.Write notes on:
(a) Band Pass random process.
(b) Band limited random process
(c) Narrow band random process.

5.(a) Derive the relation between PSD of input and output random process of an LTI system.
(b) Discuss about cross correlation between the input X (t) and output Y (t)

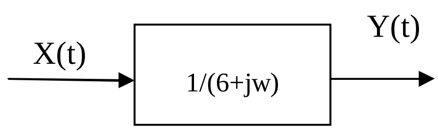
6.Derive the expressions for mean. Autocorrelation cross correlation and PSD of response of a linear system response is 5te$^{-2t.}$

7.(a)How mean of the system response Y(t) is calculated?
(b)Write different types of band pass processes with band limited processes.

8.(a)Define mean value of system response.
(b)Find mean square value of Y(t).

9.(a) A WSS random process x(t) is applied to the input of an LTI system whose impulse The mean of x(t) is 3.Find the mean output of the system
(b) Give any two spectral characteristics of the system response.

10.(a) consider a linear system as shown in fig



Linear system

The autocorrelation of x(t)=5$\delta(\tau)$. Find the psd, autocorrelation and mean square value of the output y(t).

(b). A random process X(t) is applied to a network with impulse response h(t)= e^{-bt}u(t), where b>0 is constant. The cross correlation X(t) with the output Y(t) is known to have the form Rxx(τ) = $u(\tau)\tau e^{-bt}$. Find the autocorrelation of Y(t).