SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY::PUTTUR
(AUTONOMOUS)
(Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu)
(Accredited by NBA for Civil, EEE, Mech., ECE & CSE)
(Accredited by NAAC with 'A' Grade)
Puttur -517583, Chittoor District, A.P. (India)

CONTENTS OF THE COURSE FILE

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6. Question Bank
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8. Previous end exam and mid question papers
9. PPTs (soft copy)
10. Links to the video lectures
11. Assignment and tutorial questions

INSTITUTE VISION AND MISSION

Vision

To emerge as one of the premier institutions through excellence in education and research, producing globally competent and ethically strong professionals and entrepreneurs.

Mission

Imparting high-quality technical and management education through the state-of-the- art resources.

Creating an eco-system to conduct independent and collaborative research for the betterment of the society.

Promoting entrepreneurial skills and inculcating ethics for the socio-economic development of the nation.

VISION AND MISSION OF THE DEPARTMENT (EEE)

Vision

To produce competent Electrical & Electronics Engineering professionals through quality technical education and research and taking responsible positions to develop a value-based society.

Mission

M1: Providing strong technical knowledge to formulate, solve, and analyze Electrical and Electronics Engineering problems.

M2: Collaborating with industry, research organizations, and academia to research for the betterment of society.

M3: Preparing graduates with a positive attitude and ethical values for socio-economic growth and encouraging entrepreneurial ideas.

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY: PUTTUR (AUTONOMOUS)

(19EE0207) ELECTROMAGNETIC FIELDS

L	T	P	C
3	-	-	3

COURSE OBJECTIVES

The objectives of this course:

1, To learn the laws concerning static electric fields: Coulomb's law, Gauss law; the laws concerning static magnetic fields: Biot, savart law, Ampere circuital law
2. To learn the equations concerned with static electric fields
3. To learn the equations concerned with static magnetic fields
4. To find the difference between the behaviors of conductors and dielectrics in electric fields
5. To determine the energy stored and energy density in (i) static electric field (ii) magnetic field, Electric dipole and dipole moment, magnetic dipole and dipole moment

COURSE OUTCOMES (COs)

On successful completion of this course, the student will be able to

1. Acquires mathematical foundation on vector calculus
2. Analyse and estimate Electric field quantities with charge distribution
3. Study the behaviour of electric fields in conductor and dielectric materials
4. Estimate the magnetic field strengths due to different current carrying elements
5. Evaluate the magnetic forces generated due to interaction of electric and magnetic fields 6. Understand the electromagnetic wave propagation in free space

UNIT-I INTRODUCTION TO VECTOR CALCULUS

Three orthogonal coordinate systems (rectangular, cylindrical and spherical)- Representation of a point and a vector in three coordinates, Conversion of point and vector from one coordinate system to another. Vector algebra- Vector addition, subtraction and multiplications; vector operators gradient, divergence and curl; integral theorems of vectors. Representation of differential length, surface and volume.

UNIT-II STATIC ELECTRIC FIELD

Coulomb"s law, Electric field intensity, Electrical field due to point charges. Line, Surface and Volume charge distributions. Gauss law and its applications. Divergence theorem and Maxwell"s First equation, Absolute Electric potential, Potential difference, Calculation of potential differences for different configurations. Electric dipole, Electrostatic Energy and Energy density,

UNIT-III CONDUCTORS, DIELECTRICS AND CAPACITANCE

Current and current density, Ohms Law in Point form, Continuity of current, Boundary conditions of perfect dielectric materials. Permittivity of dielectric materials, Capacitance, Capacitance of a two wire line, Poisson"s equation, Laplace"s equation, Solution of Laplace and Poisson"s equation, Application of Laplace"s and Poisson"s equations.

UNIT-IV STATIC MAGNETIC FIELDS

Biot-Savart Law, Amperes Law, Stokes theorem and Maxwell"s second equation. Magnetic flux and magnetic flux density, Maxwell"s third equation. Scalar and Vector Magnetic potentials. Steady magnetic fields produced by current carrying conductors.

MAGNETIC FORCES, MATERIALS AND INDUCTANCE

Force on a moving charge in a magnetic field, Force on a differential current element and straight current carrying conductor in a magnetic field, Force between differential current elements, Force between two parallel current carrying conductors. Torque on a rectangular loop carrying current in a magnetic field. S e l f inductance of solenoid, toroid and coaxial cable. Definition of Mutual Inductance, Mutual inductance between a loop and straight conductor.

UNIT-V TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

Faraday"s law of Electromagnetic induction, Maxwell"s fourth equation Displacement current, Modification of Maxwell"s third equation for time varying fields. Point and integral form of Maxwell"s equations for time varying fields.

TEXT BOOKS:

1. William.H.Hayt, "Engineering Electromagnetics" Mc.Graw,Hill, 2010.
2. Gangadhar,"Field Theory", Khanna Publications, 2003.

REFERENCE BOOKS:

1. Griffith,"Electrodynamics" PHI, 3rd Edition, 1999.
2. Sadiku,"Electromagnetic Fields" Oxford University Press, 5th Edition, 2010.
3. Joseph Edminister, "Electromagnetics" Tata Mc Graw Hill, 2006.
4. J.D.Kraus, "Electromagnetics" Mc.Graw,Hill Inc,5th edition,1999.

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ELETRICAL AND ELECTRONICS ENGINEERING

COURSE INFORMATION SHEET

PROGRAMME: UG	**DEGREE:** B.TECH
COURSE: Electromagnetic Fields	**SEMESTER:** 2-2 **CREDITS:** 3
COURSE CODE: 19EE0212 **REGULATION :** R19	**COURSE TYPE:** CORE
COURSE AREA/DOMAIN:	**CONTACT HOURS:** 3+1 (Tutorial) hours/Week.
CORRESPONDING LAB COURSE CODE (IF ANY): NO	**LAB COURSE NAME:**

SYLLABUS:

UNIT	DETAILS	HOURS
I	**INTRODUCTION TO VECTOR CALCULUS** Three orthogonal coordinate systems (rectangular, cylindrical and spherical)- Representation of a point and a vector in three coordinates, Conversion of point and vector from one coordinate system to another. Vector algebra- Vector addition, subtraction and multiplications; vector operators gradient, divergence and curl; integral theorems of vectors. Representation of differential length, surface and volume.	18
II	**STATIC ELECTRIC FIELD** Coulomb"s law, Electric field intensity, Electrical field due to point charges. Line, Surface and Volume charge distributions. Gauss law and its applications. Divergence theorem and Maxwell"s First equation, Absolute Electric potential, Potential difference, Calculation of potential differences for different configurations. Electric dipole, Electrostatic Energy and Energy density,	14
III	**CONDUCTORS, DIELECTRICS AND CAPACITANCE** Current and current density, Ohms Law in Point form, Continuity of current, Boundary conditions of perfect dielectric materials. Permittivity of dielectric materials, Capacitance, Capacitance of a two wire line, Poisson"s equation, Laplace"s equation, Solution of Laplace and Poisson"s equation, Application of Laplace"s and Poisson"s equations.	11
IV	**MAGNETIC FORCES, MATERIALS AND INDUCTANCE** Force on a moving charge in a magnetic field, Force on a differential current element and straight current carrying conductor in a magnetic field, Force between differential current elements, Force between two parallel current carrying conductors. Torque on a rectangular loop carrying current in a magnetic field. Self inductance of solenoid, toroid and coaxial cable. Definition of Mutual Inductance, Mutual inductance between a loop and straight conductor.	18
V	**TIME VARYING FIELDS AND MAXWELL'S EQUATIONS** Faraday"s law of Electromagnetic induction, Maxwell"s fourth equation Displacement current, Modification of Maxwell"s third equation for time varying fields. Point and integral form of Maxwell"s equations for time varying fields.	14
	TOTAL HOURS	75

TEXT/REFERENCE BOOKS:

T/R	BOOK TITLE/AUTHORS/PUBLICATION
T1	William.H.Hayt, "Engineering Electromagnetics" Mc.Graw,Hill, 2010.
T2	Gangadhar,"Field Theory", Khanna Publications, 2003.
R1	Griffith,"Electrodynamics" PHI, 3rd Edition, 1999.
R2	Sadiku,"Electromagnetic Fields" Oxford University Press, 5th Edition, 2010.
R3	Joseph Edminister, "Electromagnetics" Tata Mc Graw Hill, 2006

WEB SOURCE REFERENCES:

1	*www.wikipedia.com*
2	*https://www.britannica.com/science/electromagnetic-field*
3	*https://nptel.ac.in/courses/117/103/117103065/*

COURSE PRE-REQUISITES:

C.CODE	COURSE NAME	DESCRIPTION	SEM
19EE0203	ELECTRICAL MACHINES-I	Dc Generators And Dc Motors	II
19EE0208	ELECTRICAL MACHINES-II	Induction & Synchronous Motors	II

COURSE OBJECTIVES:

1	To learn the laws concerning static electric fields: Coulomb's law, Gauss law; the laws concerning static magnetic fields: Biot, savart law, Ampere circuital law
2	To learn the equations concerned with static electric fields
3	To learn the equations concerned with static magnetic fields
4	To find the difference between the behaviors of conductors and dielectrics in electric fields
5	To determine the energy stored and energy density in (i) static electric field (ii) magnetic field, Electric dipole and dipole moment, magnetic dipole and dipole moment

COURSE OUTCOMES:

SNO	DESCRIPTION
1	Acquires mathematical foundation on vector calculus
2	Analyse and estimate Electric field quantities with charge distribution
3	Study the behaviour of electric fields in conductor and dielectric materials
4	Estimate the magnetic field strengths due to different current carrying elements
5	Evaluate the magnetic forces generated due to interaction of electric and magnetic fields
6	Understand the electromagnetic wave propagation in free space

CO PO MAPPING:

CO Mapping with PO's and PSO's:

1-low 2-medium 3-high															
CO	**PO 1**	**PO 2**	**PO 3**	**PO 4**	**PO 5**	**PO 6**	**PO 7**	**PO 8**	**PO 9**	**PO 10**	**PO 11**	**PO 12**	**PS O1**	**PS O2**	**PS O3**
1	2	2	2	1	1						1				
2	1	2	1	2			1								
3	1	1	2	2				1							
4	2	1	2	2		2									

TOPICS BEYOND SYLLABUS/ADVANCED TOPICS/DESIGN:

1	WAVE EQUATION
2	WAVE LENGTH

DELIVERY/INSTRUCTIONAL METHODOLOGIES:

☑ CHALK & TALK	☑ STUD. ASSIGNMENT	☑ POWER POINT PRESENTATION	☑ STUD. SEMINARS
☑ ICT			

ASSESSMENT METHODOLOGIES-DIRECT/INDIRECT

☑MID EXAMS	☑ASSIGNMENTS	☑UNIVERSITY EXAM	☑CLASS ROOM DISCUSSION
☑STUDENT FEEDBACK			

Prepared by
Dr. J. Gowrishankar
Professor

Approved by
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Professor & HEAD

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING
TEACHING PLAN

ACADEMIC YEAR: 2020-21 **YEAR/SEM: II BTECH/II SEM**

SUBJECT NAME (CODE):19EE0207

NAME OF THE FACULTY: Dr. J. GOWRISHANKAR

S.NO	NAME OF THE TOPIC	NO OF HOURS		TEXT BOOK REFERED (BOOK NO-)	PAGE NO	MODE OF LECTURE		
		LECTURE (L)	TUTORIAL (T)			BLACK BOARD (BB)	PPT	URL
UNIT-I INRODUCTION TO VECTOR CALCULUS								
1	Three orthogonal coordinate systems (rectangular, cylindrical and spherical)-	3		B1	3-10		√	
2	Representation of a point and a vector in three coordinates,	1		B1	11-13	√		
3	Conversion of point and vector from one coordinate system to another.	2		B1	14-16	√		
4	Vector algebra- Vector addition, subtraction and multiplications		1	B1	17-18	√		
5	vector operator's gradient	2		B1	19-20	√		
6	divergence and curl	2		B1	22-25	√		
7	Integral theorems of vectors	1		B1	26-27	√		
8	Representation of differential length, surface and volume	2		B1	28-30	√		
UNIT-II STATIC ELECTRIC FIELD								
9	Coulomb"s law,	1		B1	35-36	√		
10	Electric field intensity, Electrical field due to point charges. Line, Surface and Volume charge distributions.	2	1	B1	37-42	√		
11	Gauss law and its applications.	1		B1	45-46	√		https://byjus.com/jee/gauss-law/
12	Divergence theorem and Maxwell"s First equation,	2		B1	47-48	√		
13	Absolute Electric potential	1		B1	52-53	√		
14	Potential difference	1		B1	55-57	√		

15	Calculation of potential differences for different configurations.	2		B1	58-60	√		
16	Electric dipole, Electrostatic Energy and Energy density,	2		B1	65-70	√		
UNIT-III CONDUCTORS, DIELECTRIC AND CAPACITANCE								
17	Current and current density,	2		B1	85-86	√		
18	Ohms Law in Point form,	1		B1	87-89	√		
19	Continuity of current,	2		B1	92-93	√		
20	Boundary conditions of perfect dielectric materials.	2		B1	95-98	√		
21	Permittivity of dielectric materials,	2		B1	99-101	√		
22	Capacitance, Capacitance of a two wire line,	1		B1	102-103	√		
23	Poisson"s equation, Laplace's equation,	1		B1	105-108	√		http://www.niser.ac.in/~sbasak/p201/02_02_09.pdf
24	Solution of Laplace and Poisson"s equation		1	B1	109-110	√		
25	Application of Laplace's and Poisson"s equations.	1		B1	112-114	√		https://eng.libretexts.org/Bookshelves/Electrical_Engineering/Electro-Optics/Book%3A_Electromagnetics_I_(Ellingson)/05%3A_Electrostatics/5.15%3A_Poisson%E2%80%99s_and_Laplace%E2%80%99s_Equations
UNIT -IV STATIC MAGNETIC FIELDS								
26	Biot-Savart Law,	2		B1	116-118	√		
27	Amperes Law,	2		B1	121-125	√		
28	Stokes theorem and Maxwell"s second equation.	2		B1	128-130	√		https://people.math.osu.edu/tanveer.1/m263.02/maxwell
29	Magnetic flux and magnetic flux density,		1	B1	132-134	√		
30	Maxwell"s third equation.	1		B1	136-138	√		

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31	Scalar and Vector Magnetic potentials.		2	B1	140-141	√		
32	Steady magnetic fields produced by current carrying conductors.	1		B1	141143	√		
MAGNETIC FORCE, MATERIALS AND INDUCTANCE								
33	Force on a moving charge in a magnetic field,	2		B1	144-146	√		
34	Force on a differential current element and straight current carrying conductor in a magnetic field,	1		B1	147-148	√		
35	Force between differential current elements,	2		B1	152-155	√		
36	Force between two parallel current carrying conductors.	2		B1	156-158	√		
37	Torque on a rectangular loop carrying current in a magnetic field.	2		B1	160-165	√		
3	S e l f inductance of solenoid, toroid and coaxial cable.	2		B1	165-168	√		
39	Definition of Mutual Inductance, Mutual inductance between a loop and straight conductor.	3		B1	170-172	√		
UNIT-V TIME VARYING FIELDS AND MAXWELLS EQUATIONS								
40	Faraday"s law of Electromagnetic induction,	2		B1	172-173	√		https://www.electrical4u.com/faraday-law-of-electromagnetic-induction/
41	Maxwell"s fourth equation Displacement current,	3		B1	174-175	√		
42	Modification of Maxwell"s third equation for time varying fields.	3		B1	175-76	√		
43	Point and integral form of Maxwell"s equations for time varying fields	3		B1	178-182	√		

FACULTY

HOD

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BOOK NO	S.NO	TEXT BOOK NAME & AUTHOR
B1	1	Engineering Electromagnetics" William.H.Hayt
B2	2	Gangadhar Field Theory
B3	3	Sadiku Electromagnetic Fields

UNIT-1

VECTOR ANALYSIS

INTRODUCTION:

Electromagnetics is a branch of Physics (or) electrical engineering which is used to study the electric and magnetic Phenomena.

what is a field ?

Consider a magnet. It has its own effect in a region surrounding it. The effect can be Placed by Placing another magnet near the first magnet. such an effect can be defined by a Particular Physical function.

In the region surrounding the magnet, there exists a Particular value ~~for~~ that Physical function, at every Point, describing the effect of magnet.

So field can be defined as the region in which, at each point there exists a corresponding value of some Physical function.

If the field ~~is~~ Produced is due to magnetic effects, it is called MAGNETIC FIELD.

There are two types of electric charges, Positive and negative. such an electric charge Produces a field around it which is called an ELECTRIC FIELD.

Moving charges produces current and current carrying conductor produces a magnetic field. In such case electric and magnetic fields are related to each other. such a field is called ELECTROMAGNETIC FIELD. such fields may be time varying or time independent.

2 It is seen that distribution of a quantity in a space is defined by a field. Hence to quantify the field, three dimensional representation plays an important ~~field~~ role. Such three dimensional representation can be made easy by the use of vector analysis.

SCALARS & VECTORS:

The various quantities involved in the study of engineering electromagnetics can be classified as

1. Scalars &
2. Vectors.

SCALAR:

The scalar is a quantity whose value may be represented by a single real number, which may be +ve (or) -ve. The direction is not at all required in describing a scalar. Thus

A scalar is a quantity which is wholly characterized by its magnitude.

eg:- temperature, mass, volume, density, speed electric charge etc.

VECTOR:-

A quantity which has both, a magnitude and a specific direction in space is called a vector.

In electromagnetics vectors defined in two and three dimensional spaces are required but vectors may be defined in n-dimensional space.

A vector is a quantity which is characterized by both, a magnitude and a direction.

eg:- Force, velocity, displacement, electric field intensity, magnetic field intensity, acceleration etc.

VECTOR ~~FIELD~~

SCALAR FIELD:

The distribution of a scalar quantity with a definite position in a space is called SCALAR FIELD.

eg: 1. Temperature of atmosphere.
(It has a definite value in the atmosphere but no need of direction to specify).

2. Height of surface of earth above sea level
3. Sound intensity in an auditorium.
4. Light intensity in a room.
5. Atmospheric pressure in a given region etc.

VECTOR FIELD:

If a quantity which is specified in a region to define a field is a vector then the corresponding field is called a vector field.

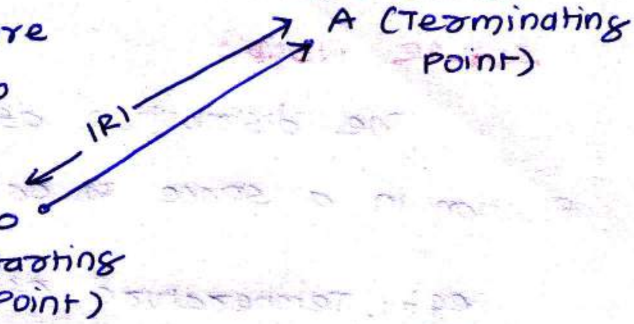
eg: 1. Gravitational force on a mass in a space is a vector field. [This force has a value at various points in a space and always has a specific direction].

2. Velocity of particles in a moving fluid
3. wind velocity of atmosphere
4. Voltage gradient in a cable
5. Displacement of a flying bird in a space.
6. Magnetic field existing from north to south poles.

REPRESENTATION OF A VECTOR:

In two dimensional, a vector can be represented by a straight line with an arrow in a plane. The length of the ~~vector~~ segment is the magnitude of a vector while the arrow indicates the direction of the vector.

The vector shown in figure is symbolically denoted as $\overrightarrow{OA}$. It's length is called as magnitude, which is R for the vector OA.

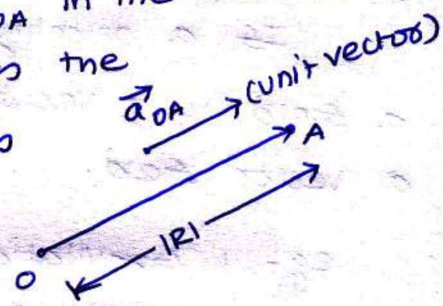


It is represented as $|\overrightarrow{OA}| = R$

UNIT VECTOR:

A unit vector has a function to indicate the direction. It's magnitude is always unity, irrespective of its direction. Thus for any vector, to indicate its direction a unit vector can be used.

consider a unit vector $\vec{a}_{OA}$ in the direction of $\overrightarrow{OA}$ as shown in fig. This indicates the direction of $\overrightarrow{OA}$ but its magnitude is unity.



so vector $\overrightarrow{OA}$ can be represented completely as its magnitude R and the direction as indicated by the unit vector along its direction.

$$\overrightarrow{OA} = |\overrightarrow{OA}|\,\vec{a}_{OA} = R\underbrace{\vec{a}_{OA}}$$

↳ unit vector along the direction OA and $|\vec{a}_{OA}| = 1$

(X)

letter $\vec{a}$ is used to indicate the unit vector

2 Mark Question:

1. Mention the purpose of unit vector in vector algebra.

In case if a vector is known then the unit vector along that vector can be obtained by dividing the vector by its magnitude. Thus unit vector can be expressed as,

$$\text{Unit vector } \vec{a}_{OA} = \frac{\overrightarrow{OA}}{|OA|}$$

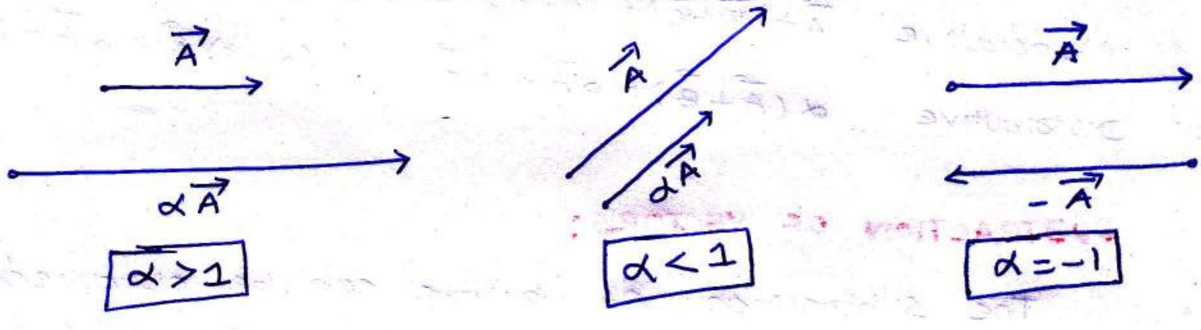
2 mark question.

1. Express unit vector in terms of a vector and its magnitude.

VECTOR ALGEBRA: [Scaling, Addition, subtraction].

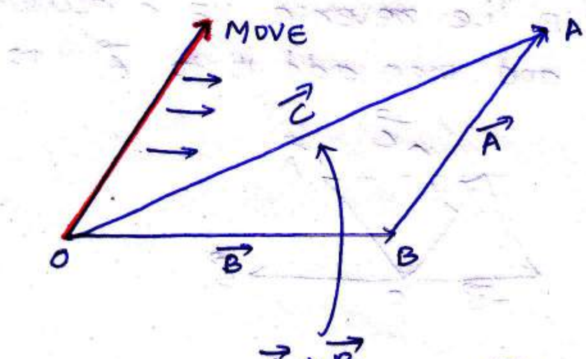
SCALING OF VECTOR

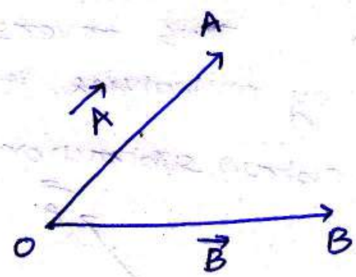
- → This is multiplication by a scalar to a vector
- → This changes the magnitude (length) of a vector but not its direction, when scalar is POSITIVE
- → When scalar = −1, the magnitude remains same but direction of the vector reverses.



ADDITION OF VECTORS:

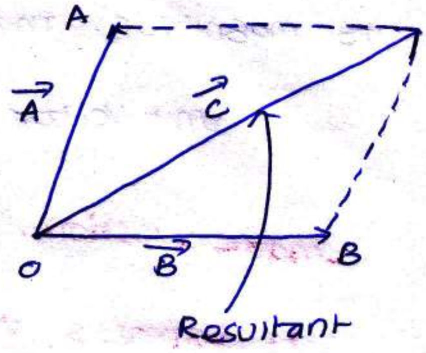
→ The vectors which lie on the same plane are called coplanar vectors.



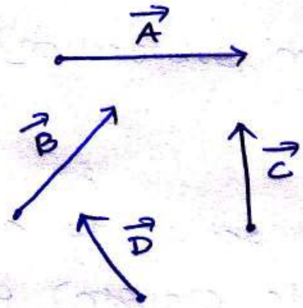


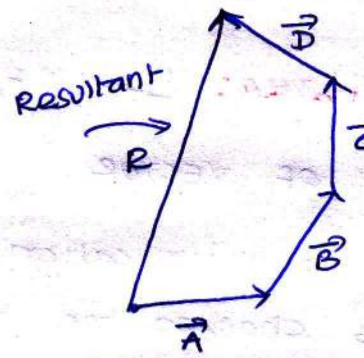
PARALLELOGRAM RULE:-

Complete the Parallelogram as shown in fig. Then the diagonal of the parallelogram represents the addition of the two vectors.



HEAD TO TRIAL RULE:-





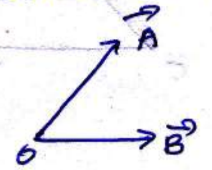
Law	Addition	Multiplication by scalar
commutative	$\vec{A}+\vec{B}=\vec{B}+\vec{A}$	$\alpha\vec{A}=\vec{A}\alpha$
Associative	$\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$	$\beta(\alpha\vec{A})=(\beta\alpha)\vec{A}$
Distributive	$\alpha(\vec{A}+\vec{B})=\alpha\vec{A}+\alpha\vec{B}$	$(\alpha+\beta)\vec{A}=\alpha\vec{A}+\beta\vec{A}$

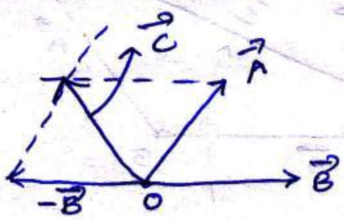
SUBTRACTION OF VECTORS:

The subtraction of vectors can be obtained from the rules of addition. If $\vec{B}$ is to be subtracted from $\vec{A}$ then based on addition it can be represented as

$$\vec{C}=\vec{A}+(-\vec{B}).$$

Thus reverse the sign $\vec{B}$ i.e reverse its direction by multiplying it with (-1). and then add it to $\vec{A}$ to obtain subtraction.





Identical vectors:

Two vectors are said to be identical if their difference is zero.

eg: $\vec{A} - \vec{B} = 0$ $\qquad$ $\vec{A}$ & $\vec{B}$ are identical.

$\Rightarrow \vec{A} = \vec{B}$

VECTOR MULTIPLICATION:

Consider two vectors $\vec{A}$ and $\vec{B}$. There are two types of products existing depending upon the result of the multiplication. These two types of products are

1. Scalar (or) DOT product
2. Vector (or) cross product

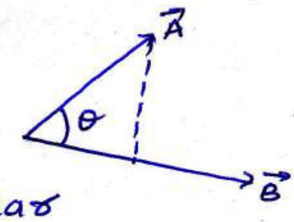
SCALAR (OR) DOT PRODUCT OF VECTORS:

→ It is denoted by $\vec{A} \cdot \vec{B}$

→ It is defined as the product of the magnitude of $\vec{A}$, the magnitude of $\vec{B}$ and the cosine of smaller angle b/w them.

→ It also can be defined as the product of magnitude of $\vec{B}$ and the projection of $\vec{A}$ onto $\vec{B}$ or vice versa

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta_{AB}}$$



The result of such a dot product is scalar hence it is also called as scalar product.

PROPERTIES OF DOT PRODUCT

1. If the two vectors are $\parallel^{el}$ to each other i.e $\theta = 0$ then $\cos\theta_{AB} = 1$ Thus

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \text{ for } \parallel^{el} \text{ vectors.}$$

2. If two vectors are $\perp^{r}$ to each other i.e $\theta = 90^\circ$ then $\cos\theta_{AB} = 0^\circ$ Thus

$$\vec{A} \cdot \vec{B} = 0 \text{ for } \perp^{r} \text{ vectors.}$$

3. If the dot product of vector with itself is performed, the result is square of the magnitude of that vector.

$$\vec{A}\cdot\vec{A} = |\vec{A}|\,|\vec{A}|\cos 0^\circ = |\vec{A}|^2$$

4. Any unit vector dotted with itself is unity.

$$\vec{a}_x\cdot\vec{a}_x = 1 = \vec{a}_y\cdot\vec{a}_y = \vec{a}_z\cdot\vec{a}_z$$

5. The dot product obeys commutative, & distributive law

(ie) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

2 mark question.

1. Given two vectors, How to identify whether they are $\perp^{re}$ or $\parallel^{el}$ to each other.

APPLICATION OF DOT PRODUCT:

1. To determine the angle b/w the two vectors.

$$\theta = \cos^{-1}\left\{\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}\right\}$$

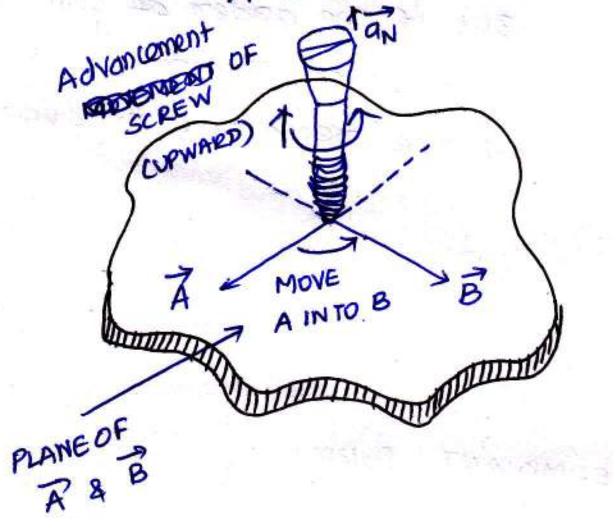
VECTOR (OR) CROSS PRODUCT OF VECTORS:

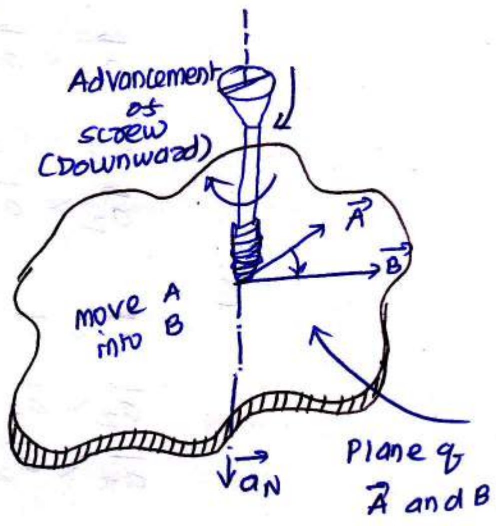
Consider two vectors $\vec{A}$ & $\vec{B}$ then the cross product is denoted as $\vec{A} \times \vec{B}$ and defined as the product of the magnitudes of $\vec{A}$ & $\vec{B}$ and the sine of the smaller angle between $\vec{A}$ and $\vec{B}$.

Cross product is a vector quantity and has a direction $\perp^r$ to the plane, containing the two vectors $\vec{A}$ and $\vec{B}$.

Mathematically cross product is expressed as

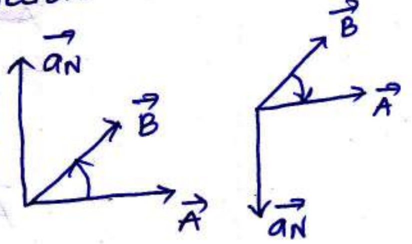
$$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta_{AB}\,\vec{a}_N$$





PROPERTIES OF CROSS PRODUCT:

1. The commutative law is not applicable to the cross product thus $\boxed{\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}}$



2. Reversing the order of the vectors $\vec{A}$ and $\vec{B}$, a unit vector $\vec{a}_N$ reverses its direction hence we can write

$$\vec{A} \times \vec{B} = -[\vec{B} \times \vec{A}] \quad \text{anticommutative}$$

3. The cross product is not associative, thus

$$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$$

4. With respect to addition cross product is distributive, thus

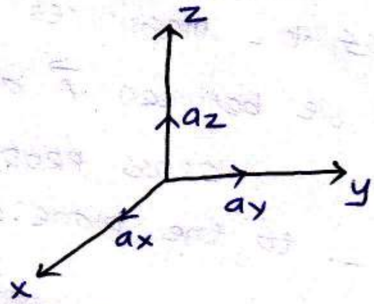
$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}.$$

5. If two vectors are $\parallel^{el}$ to each other, (ie) they are in same direction then $\theta = 0°$ & hence cross product of such two vectors is zero.

6. $\vec{A} \times \vec{A} = 0$ [cross product to itself].

7. cross product of unit vectors.

consider the unit vectors $\vec{a_x}$, $\vec{a_y}$ and $\vec{a_z}$ which are mutually $\perp^r$ to each other as shown in fig.



Then

$$\vec{a_x} \times \vec{a_y} = |\vec{a_x}||\vec{a_y}| \sin(90^\circ)\, \vec{a_N}$$

In this case $\vec{a_N} = \vec{a_z}$

and $|\vec{a_x}| = |\vec{a_y}| = \sin(90^\circ) = 1$

$$\therefore \quad \vec{a_x} \times \vec{a_y} = \vec{a_z}$$
$$\vec{a_y} \times \vec{a_z} = \vec{a_x}$$
$$\vec{a_z} \times \vec{a_x} = \vec{a_y}$$

But if the order of unit vectors is reversed, the result is -ve of the remaining third unit vector

Thus

$$\vec{a_y} \times \vec{a_x} = -\vec{a_z}$$
$$\vec{a_z} \times \vec{a_y} = -\vec{a_x}$$
$$\vec{a_x} \times \vec{a_z} = -\vec{a_y}$$

CROSS PRODUCT IN DETERMINANT FORM:

consider two vectors

$$\vec{A} = A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}$$
$$\vec{B} = B_x \vec{a_x} + B_y \vec{a_y} + B_z \vec{a_y}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\boxed{\vec{A} \times \vec{B} = [A_y B_z - B_y A_z]\vec{a_x} + [A_z B_x - A_x B_z]\vec{a_y} + [A_x B_y - A_y B_x]\vec{a_z}}$$

PRODUCTS OF THREE VECTORS :-

Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ are the three given vectors. Then the product of these three vectors is classified into two ways called,

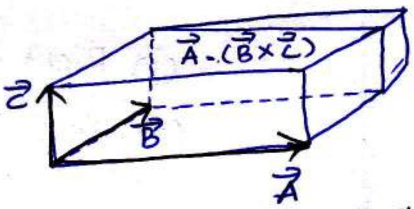
1. Scalar triple product
2. Vector triple product.

SCALAR TRIPLE PRODUCT: (scalar triple product of 3 vectors $\vec{A}, \vec{B}$ & $\vec{C}$).

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B}).$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}.$$

1. It represents the volume of parallelepiped.



2. If two (or) three vectors are equal then the result of the scalar triple product is zero.

3. Cyclic order a, b, c is to be followed. If the order is changed, the sign is reversed.

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = -\vec{B}\cdot(\vec{A}\times\vec{C}).$$

VECTOR TRIPLE PRODUCT:-

The vector triple product of the three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ is mathematically defined as

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$ $\boxed{\text{"bac-cab" RULE}}$

Problem

1. Three fields are given by $\vec{A} = 2\vec{a_x} - \vec{a_z}$, $\vec{B} = 2\vec{a_x} - \vec{a_y} + 2\vec{a_z}$. $\vec{C} = 2\vec{a_x} - 3\vec{a_y} + \vec{a_z}$

Find the scalar and vector triple product.

Scalar triple product. (ii)

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} 2 & 0 & 1 \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix}$$

$= 14.$

Vector triple product.

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$\vec{A}\cdot\vec{C} = (2)(2) + (0)(-3) + (-1)(1) = 3$

$\vec{A}\cdot\vec{B} = (2)(2) + (0)(-1) + (-1)(2) = 2$

$$\vec{A}\times(\vec{B}\times\vec{C}) = 3\vec{B} - 2\vec{C}$$
$$= 3[2\vec{a_x} - \vec{a_y} + 2\vec{a_z}] - 2[2\vec{a_x} - 3\vec{a_y} + \vec{a_z}]$$
$$= 2\vec{a_x} + 3\vec{a_y} + 4\vec{a_z}$$

CO-ORDINATE SYSTEM:

Three Types of co-ordinate systems are

(i) cartesian (or) Rectangular co-ordinate system

(ii) cylindrical co-ordinate system

(iii) spherical co-ordinate system.

CARTESIAN CO-ORDINATE SYSTEM:

- Also called Rectangular co-ordinate system
- Three co-ordinates x, y, z mutually $\perp^r$ to each other.
- Intersection of x, y, z is called origin.

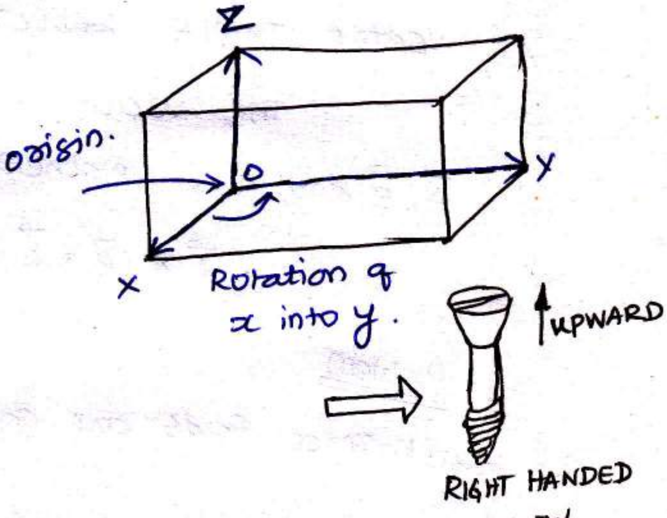
There are two types of such systems, they are

(i) Right handed system

(ii) Left handed system.

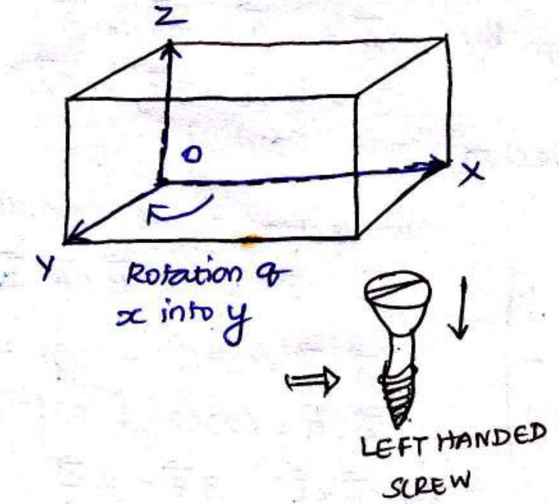
RIGHT HANDED SYSTEM:

If x axis is rotated towards y axis through a smaller angle, thus this rotation causes the upward movement of right handed screw in the 'z' axis direction.



LEFT HANDED SYSTEM:

Downward movement of screw.



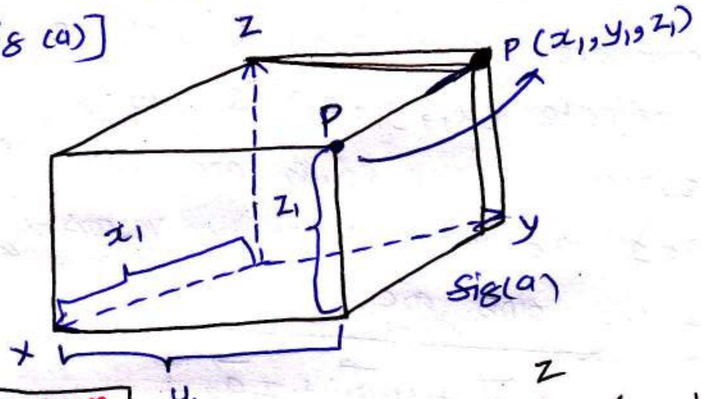
Note: RIGHT HANDED SYSTEM IS COMMONLY USED

REPRESENTING A POINT IN RECTANGULAR CO-ORDINATE SYSTEMS

→ A Point in rectangular co-ordinate system is located by three coordinates namely x, y and z co-ordinates.

→ The point can be reached by moving from origin, the distance x in x direction, then the distance y in y direction and finally distance z in z direction.

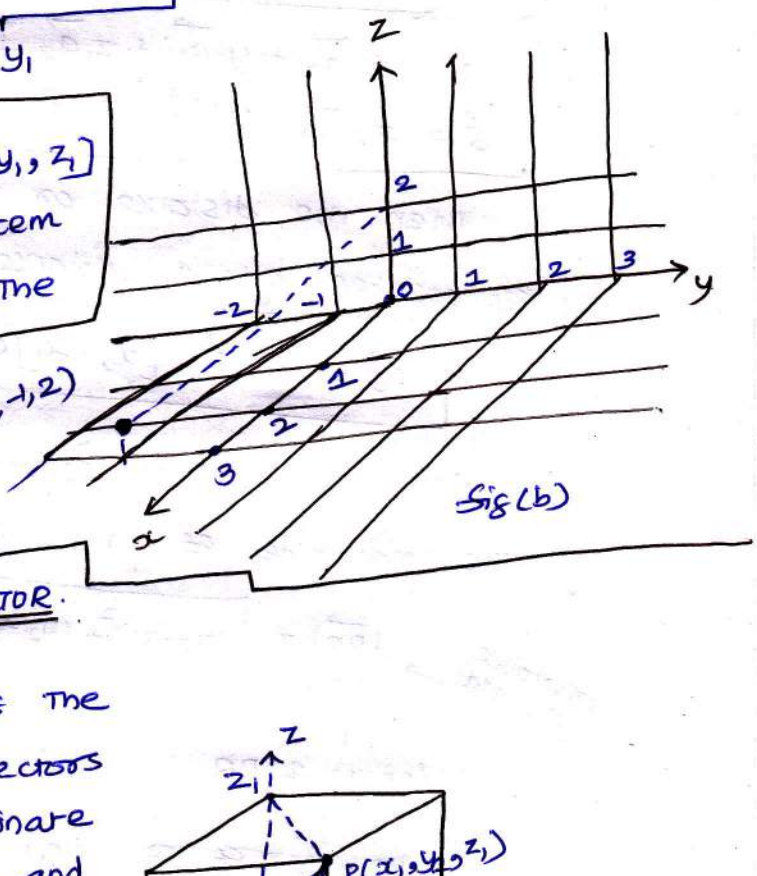
→ Consider a point P having co-ordinates x_1, y_1 and z_1. It is represented as $P(x_1, y_1, z_1)$. The co-ordinates x_1, y_1, z_1 may be +ve or -ve [fig (a)]



fig(a)

→ The Point Q(3,-1, 2) can be shown in this system as shown in fig(b)

FOLLOW 13A & 13B sheets & come here



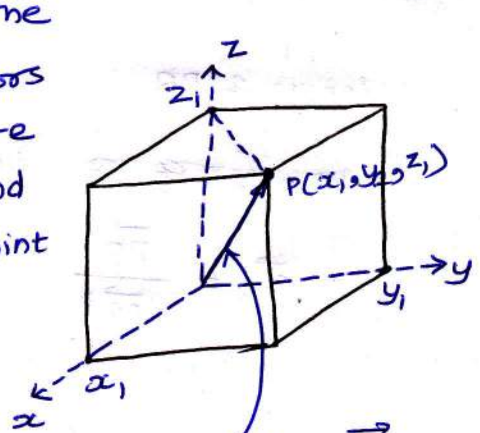
fig(b)

POSITION & DISTANCE VECTORS

Consider a point $P[x_1, y_1, z_1]$ in cartesian co-ordinate system as shown in fig (c). Then the Position vector of Point 'P' is represented by the distance of point P from the origin directed from origin to point P. This is also called RADIUS VECTOR.

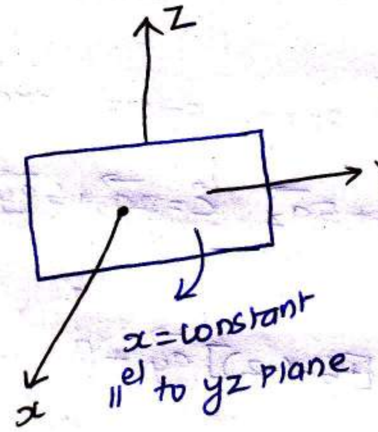
- The three components of the Position vector $\vec{r}_{OP}$ are three vectors oriented along the three co-ordinate axes with the magnitudes x_1, y_1 and z_1. Thus the position vector of point P can be represented as

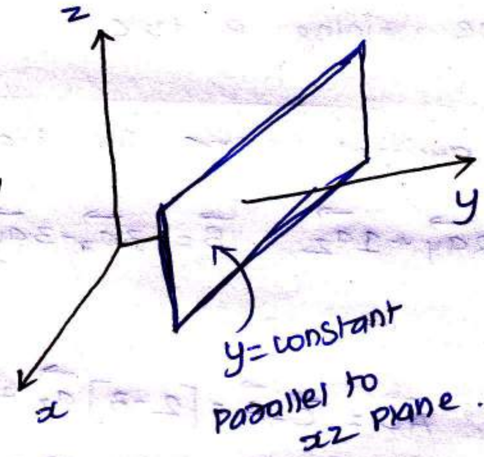
$$\boxed{\vec{r}_{OP} = x_1\vec{a}_x + y_1\vec{a}_y + z_1\vec{a}_z}$$

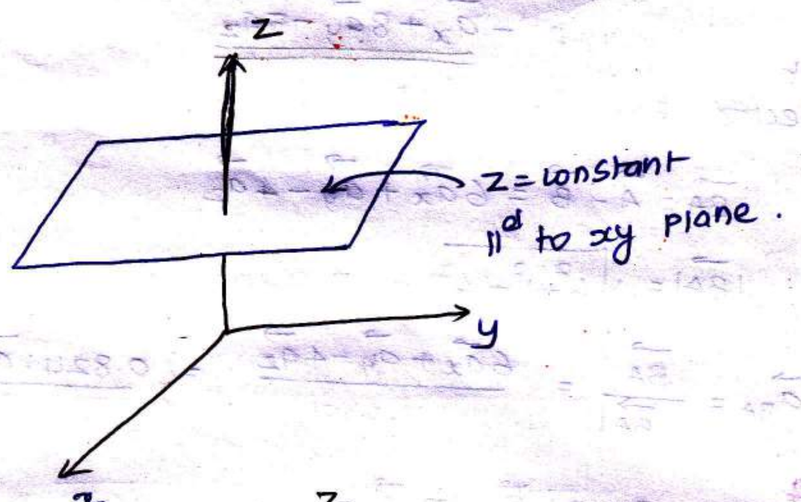


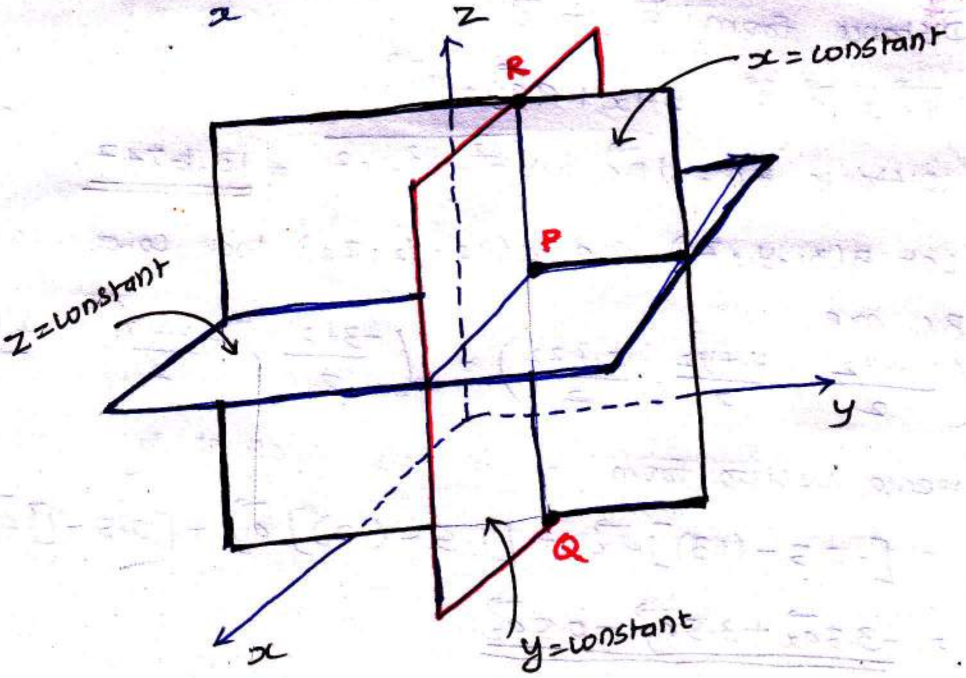
ALTERNATE METHOD TO DEFINE A POINT IN CARTESIAN SYSTEM:

Alternate method to consider three surfaces namely $x =$ constant, $y =$ constant and $z =$ constant. The common intersection of all these three surfaces is the point to be defined and the constants indicate the co-ordinates of that point. as shown in figures given below.









Problem

Given three points in cartesian co-ordinate system as

$A(3,-2,1)$, $B(-3,-3,5)$ and $C(2,6,-4)$

Find (i) The vector from A to C

(ii) The unit vector from B to A

(iii) The distance from B to C.

(iv) The vector from A to the midpoint of the straight line joining B to C.

Solution:

The position vectors for the given points are

$\vec{A} = 3\vec{a_x} - 2\vec{a_y} + 1\vec{a_z}$ $\quad \vec{B} = -3\vec{a_x} - 3\vec{a_y} + 5\vec{a_z}$ $\quad \vec{C} = 2\vec{a_x} + 6\vec{a_y} - 4\vec{a_z}$

(i) vector from ~~vector~~ $\vec{A}$ to $\vec{C}$

$$\vec{AC} = \vec{C} - \vec{A} = [2-3]\vec{a_x} + [+6-(-2)]\vec{a_y} + [-4-(+1)]\vec{a_z}$$

$$= -\vec{a_x} + 8\vec{a_y} - 5\vec{a_z}$$

(ii) unit Vector from ~~from~~ $\vec{B}$ to $\vec{A}$

$$\vec{BA} = \vec{A} - \vec{B} = 6\vec{a_x} + \vec{a_y} - 4\vec{a_z}$$

$$|\vec{BA}| = \sqrt{6^2+1^2+4^2} = 7.2801.$$

$$\vec{a}_{BA} = \frac{\vec{BA}}{|\vec{BA}|} = \frac{6\vec{a_x} + \vec{a_y} - 4\vec{a_z}}{7.2801} = 0.8241\vec{a_x} + 0.1373\vec{a_y} - 0.5494\vec{a_z}$$

(iii) Distance from $\vec{B}$ to $\vec{C}$

$$\vec{BC} = \vec{C} - \vec{B} = 5\vec{a_x} + 9\vec{a_y} - 9\vec{a_z}$$

$$\text{Distance } BC = |\vec{BC}| = \sqrt{5^2+9^2+9^2} = 13.6747.$$

(iv) Let $B(x_1, y_1, z_1)$ and $C(x_2, y_2, z_2)$ then co-ordinates of midpoint of BC are

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = \left(\frac{-3+2}{2}, \frac{-3+6}{2}, \frac{+5-4}{2}\right) = (-0.5, 1.5, 0.5)$$

Hence vector from A to this midpoint is

$$= [-0.5-(+3)]\vec{a_x} + [1.5-(-2)]\vec{a_y} + [0.5-1]\vec{a_z}$$

$$= -3.5\vec{a_x} + 3.5\vec{a_y} - 0.5\vec{a_z}$$

The magnitude of vectors interms of three mutually $\perp r$ components are given by

$$|\vec{r}_{OP}| = \sqrt{(x_1)^2 + (y_1)^2 + (z_1)^2}$$

if Point P has co-ordinates (1, 2, 3) then its Position vector is

$$\boxed{\vec{r}_{OP} = 1\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z} \qquad |\vec{r}_{OP}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} = 3.7416.$$

Now consider two points in a cartesian coordinate system, P and Q with the co-ordinate (x_1, y_1, z_1) & (x_2, y_2, z_2) respectively. The points are shown in fig 1. The individual position vectors of the points are

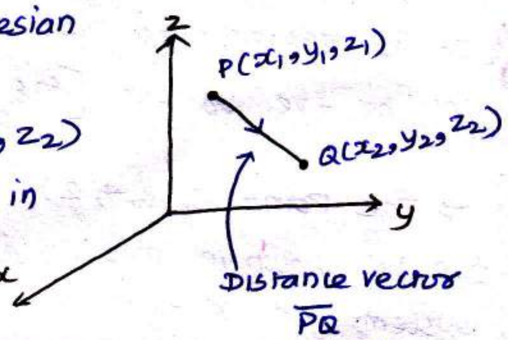


Fig 1

$$\boxed{\begin{aligned} \vec{P} &= x_1\vec{a}_x + y_1\vec{a}_y + z_1\vec{a}_z \\ \vec{Q} &= x_2\vec{a}_x + y_2\vec{a}_y + z_2\vec{a}_z \end{aligned}}$$

Then the distance or displacement from P to Q is represented by a distance vector $\vec{PQ}$ and is given by

$$\boxed{\vec{PQ} = \vec{Q} - \vec{P} = [x_2 - x_1]\vec{a}_x + [y_2 - y_1]\vec{a}_y + [z_2 - z_1]\vec{a}_z}$$

This is also called seperation vectors.

The magnitude of this vector is given by

$$\underbrace{|\vec{PQ}|}_{\text{length of PQ}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

distance formula

~~Direction of the~~ unit vector along direction of PQ is

$$\vec{a}_{PQ} = \frac{\vec{PQ}}{|\vec{PQ}|}.$$

PROBLEM:

1. Obtain the unit vector in the direction from the origin towards the point $P(3,-3,2)$

Solution:

The origin $O(0,0,0)$ while $P(3,-3,2)$ hence the distance vector $\overrightarrow{OP}$ is

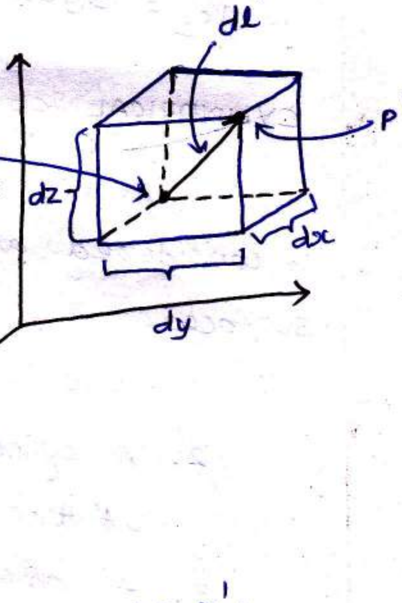
$$\overrightarrow{OP} = (3-0)\vec{a}_x + (-3-0)\vec{a}_y + (-2-0)\vec{a}_z$$
$$= 3\vec{a}_x - 3\vec{a}_y - 2\vec{a}_z$$
$$|\overrightarrow{OP}| = \sqrt{3^2+(-3)^2+(-2)^2} = 4.6904.$$

Hence the unit vector along the direction OP is

$$\vec{a}_{OP} = \frac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \frac{3\vec{a}_x - 3\vec{a}_y - 2\vec{a}_z}{4.6904}$$
$$= 0.6396\,\vec{a}_x - 0.6396\,\vec{a}_y - 0.4264\,\vec{a}_z$$

DIFFERENTIAL ELEMENTS IN CARTESIAN CO-ORDINATOR SYSTEM:

Consider a point $P(x,y,z)$ in the rectangular coordinate system. Let us increase each co-ordinate by differential amount. A new point P' will be obtained. having co-ordinates $(x+dx, y+dy, z+dz)$



dx = Differential length in x dir.

dy = Differential length in y dir

dz = Differential length in z dir.

Hence differential vector length also called elementary vector length can be represented as

$$\boxed{\vec{dl} = dx\,\vec{a}_x + dy\,\vec{a}_y + dz\,\vec{a}_z}$$

→ This is the vector joining P to new point P'.

The distance of P' from P is given by magnitude of the differential vector length.

$$\boxed{|\vec{dl}| = \sqrt{(dx)^2+(dy)^2+(dz)^2}}$$

Hence the differential volume of the rectangular Parallelepiped is given by,

$$\boxed{dv = dx\,dy\,dz}$$

Note: $\vec{dl}$ is a vector but dv is a scalar.

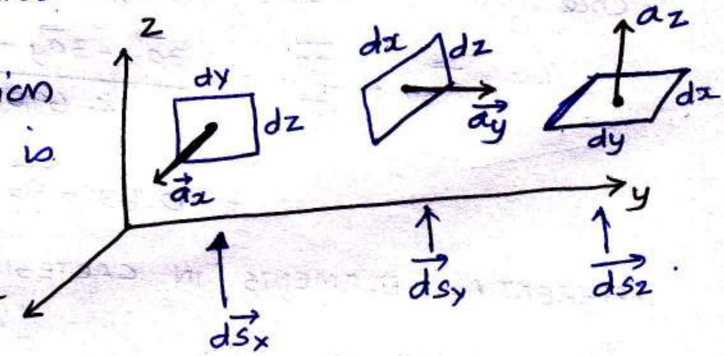
Let us define differential surface areas, the differential surface element $\vec{ds}$ is represented as

$$\vec{ds} = ds\,\vec{a}_n$$

where ds = Differential surface area of the element.

$\vec{a}_n$ = unit vector normal to surface ds.

The vector representation of these three elements is given as,



$$\vec{ds}_x = dy\,dz\,\vec{a}_x$$
$$\vec{ds}_y = dx\,dz\,\vec{a}_y$$
$$\vec{ds}_z = dy\,dx\,\vec{a}_z$$

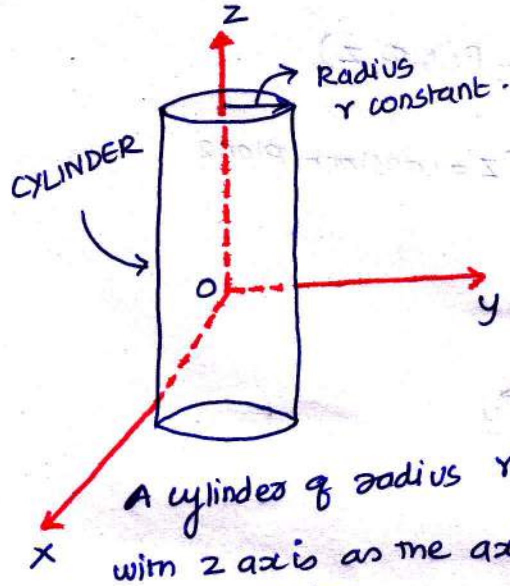
CYLINDRICAL CO-ORDINATE SYSTEM:-

In this system of co-ordinates, any point in a space is considered as the point of intersection of the following surfaces.

1. Plane of constant z which is $\parallel^{el}$ to xy plane
2. A cylinder of radius r with z axis as the axis of the cylinder
3. A half plane $\perp^{r}$ to xy plane and at an angle ϕ w.r.to xz plane. The angle ϕ is called azimuthal angle

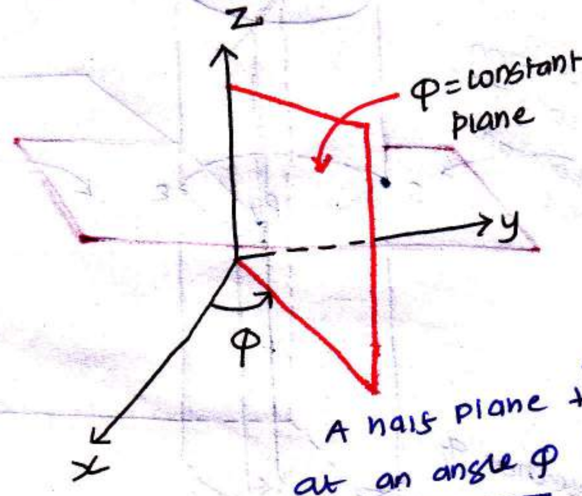
The range of variables are

$$\left.\begin{array}{l} 0 \leq r \leq \infty \\ 0 \leq \varphi \leq 2\pi \\ -\infty \leq z \leq \infty \end{array}\right\} - (1)$$



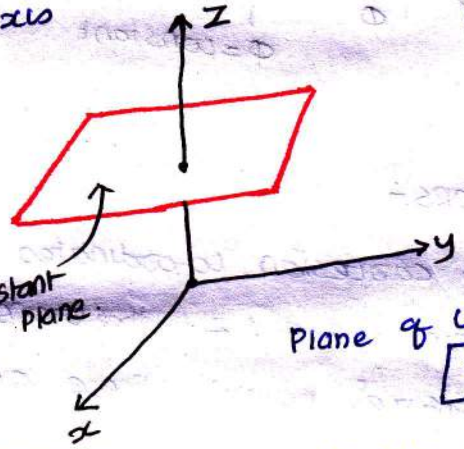
A cylinder of radius r with z axis as the axis of the cylinder

fig 1



A half plane ⊥r to xy plane at an angle φ w.r.to xz plane

fig 2



Plane of constant z

fig 3

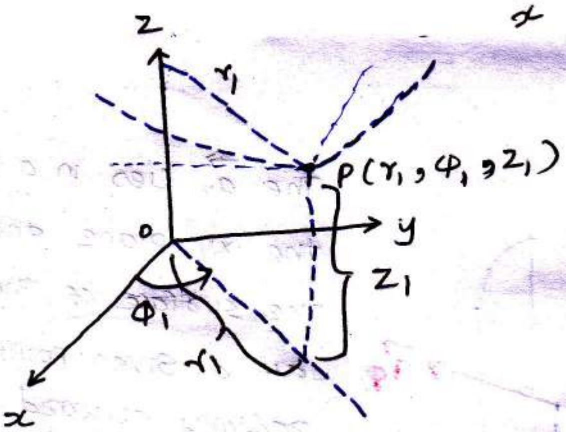
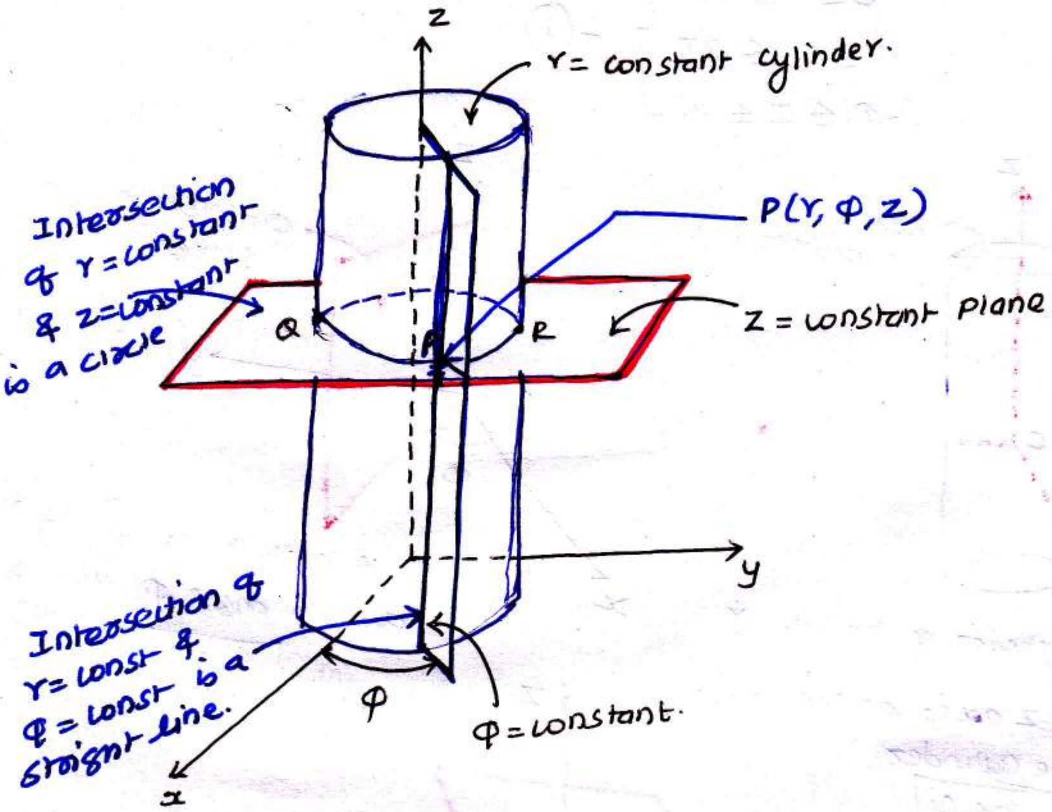


fig 4

The point P in cylindrical co-ordinate system has three co-ordinates r, φ and z whose values lie in the respective ranges as given in (1).

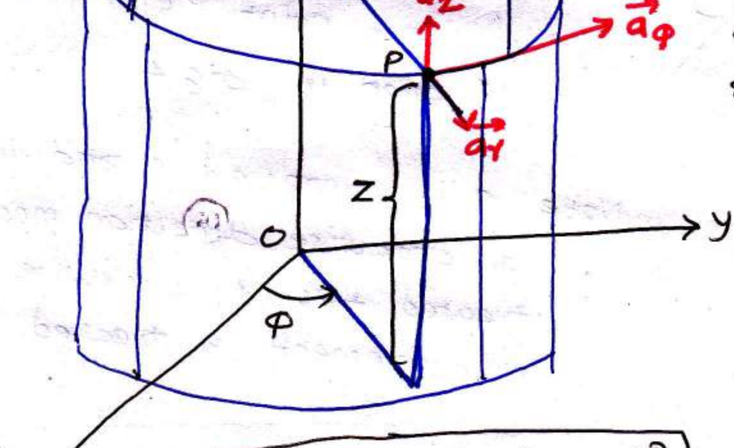
The point $P(r_1, \varphi_1, z_1)$ can be seen in fig 4

Note: φ is expressed in radians and for φ, anticlockwise direction measurement is treated +ve & clockwise direction measurement is treated -ve.



BASE VECTORS:-

$\parallel^r$ to cartesian co-ordinates system, there are three unit vectors in the r, φ and z directions denoted as $\vec{a}_r, \vec{a}_\varphi$ and $\vec{a}_z$ as shown in figure below. These are mutually $\perp^r$ to each other.



$$\boxed{\vec{P} = P_r\vec{a}_r + P_\varphi\vec{a}_\varphi + P_z\vec{a}_z}$$

The $\vec{a}_r$ lies in a plane $\parallel^{el}$ to the xy plane and is $\perp^r$ to the surface of the cylinder at a given point comming radially outward.

The unit vector $\vec{a}_\varphi$ also lies in a plane $\parallel^{el}$ to xy plane but it is tangent to the cylinder, pointing in the direction of increasing φ, at the given point.

The unit vector $\vec{a}_z$ is $\parallel^{el}$ to z axis and directed towards increasing 'z'.

$\vec{P} = P_r \vec{a}_r + P_\phi \vec{a}_\phi + P_z \vec{a}_z$

where P_r is radius r

P_ϕ is angle ϕ

P_z is co-ordinate of point P.

Key note:

In the cartesian co-ordinate system, the unit vectors are not dependent on the co-ordinates. But in cylindrical co-ordinate system $\vec{a}_r$ and $\vec{a}_\phi$ are functions of ϕ co-ordinate as their direction changes as ϕ changes. Hence the differentiation or integration w.r.to ϕ components in $\vec{a}_r$ and $\vec{a}_\phi$ should not be treated as constants.

DIFFERENTIAL ELEMENTS IN CYLINDRICAL CO-ORDINATE SYSTEM:

Consider a point $P(r, \phi, z)$ in a cylindrical co-ordinate system. Let each co-ordinate is increased by the differential amount. The differential increments in r, ϕ, z are dr, $d\phi$ and dz respectively.

→ Now there are two cylinders of radius r and $r+dr$

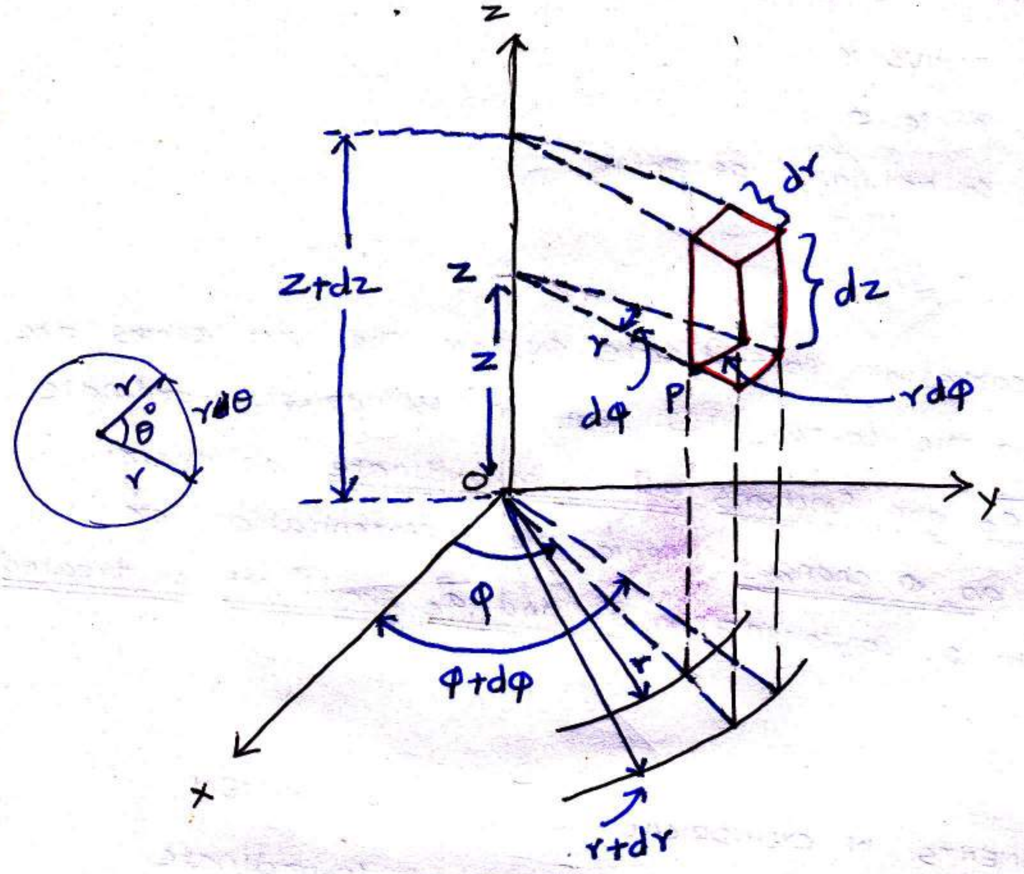
→ There are two radial planes at the angles ϕ and $\phi + d\phi$

→ Two Horizontal Planes at the heights z and $z+dz$.

→ Differential lengths in r and z directions are dr and dz respectively.

→ In ϕ direction, $d\phi$ is the change in angle ϕ and is not the differential length.

→ Due to this change $d\phi$, there exists a differential arc length in ϕ direction. This differential length, due to $d\phi$, in ϕ direction is $r d\phi$ as shown in fig.



Hence the differential vector length in cylindrical co-ordinate system is given by,

$$\boxed{\vec{dl} = dr\,\vec{a_r} + r\,d\phi\,\vec{a_\phi} + dz\,\vec{a_z}}$$

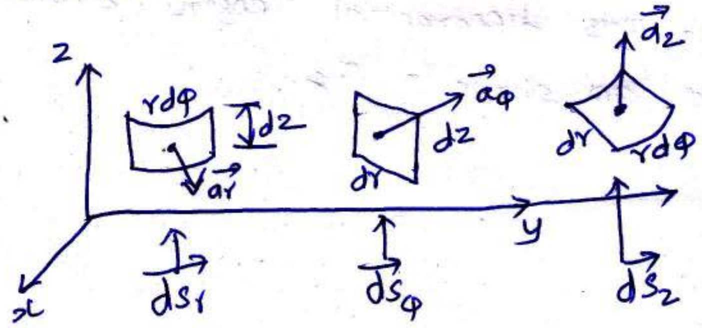
The magnitude of the differential length vector is given by,

$$\boxed{|\vec{dl}| = \sqrt{(\vec{dr})^2 + (r\,d\phi)^2 + (dz)^2}}$$

The differential volume of the differential element formed is given by,

$$\boxed{dv = dr \times r\,d\phi \times dz}$$

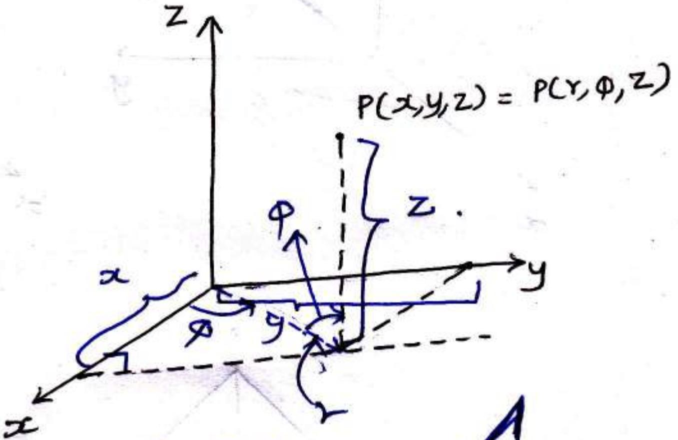
The differential surface areas in the three directions are shown in fig below.



$$\vec{ds_r} = r\,d\phi\,dz\,\vec{a_r}$$
$$\vec{ds_\phi} = dr\,dz\,\vec{a_\phi}$$
$$\vec{ds_z} = r\,d\phi\,dr\,\vec{a_z}$$

RELATIONSHIP B/W CARTESIAN & CYLINDRICAL SYSTEMS:

Consider a point 'p' whose cartesian co-ordinates are x, y and z while the cylindrical co-ordinates are r, φ and z as shown in fig below.



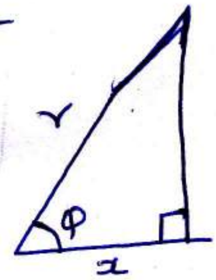
$$\boxed{\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ z &= z. \end{aligned}}$$

It can be seen that, r can be expressed interms of x and y as.

$$r = \sqrt{x^2 + y^2}$$

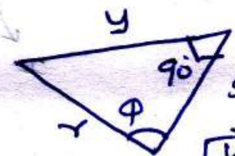
while $\boxed{\tan\varphi = \frac{y}{x}}$

From the planes



$\cos\varphi = \frac{x}{r}$

$\boxed{x = r\cos\varphi}$



$\sin\varphi = \frac{y}{r}$

$\boxed{y = r\sin\varphi}$

Thus the transformation from cartesian to cylindrical can be obtained from the equations.

$$\boxed{r = \sqrt{x^2+y^2}, \quad \varphi = \tan^{-1}\frac{y}{x} \quad \text{and} \quad z = Z}$$

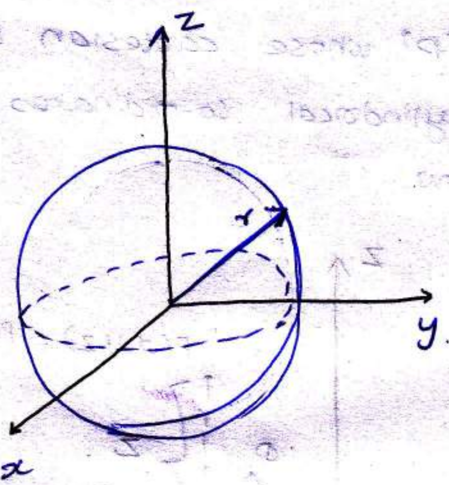
SPHERICAL CO-ORDINATE SYSTEM:

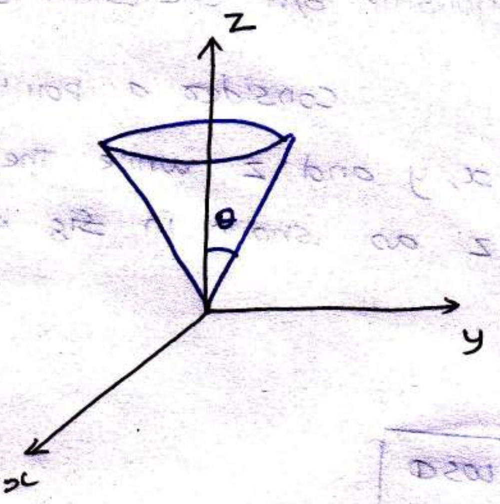
The surfaces which are used to define the spherical co-ordinate system on the three cartesian axes are

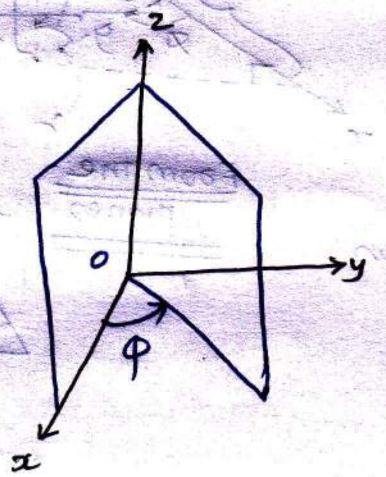
(i) sphere of radius r, origin as the centre of the sphere

(ii) A right circular cone with its apex at the origin and its axis as z axis. Its half angle is θ. It rotates about z axis and θ varies from $0°$ to $180°$

(iii) A half plane $\perp r$ to xy plane containing z axis, making an angle φ with the xz plane.





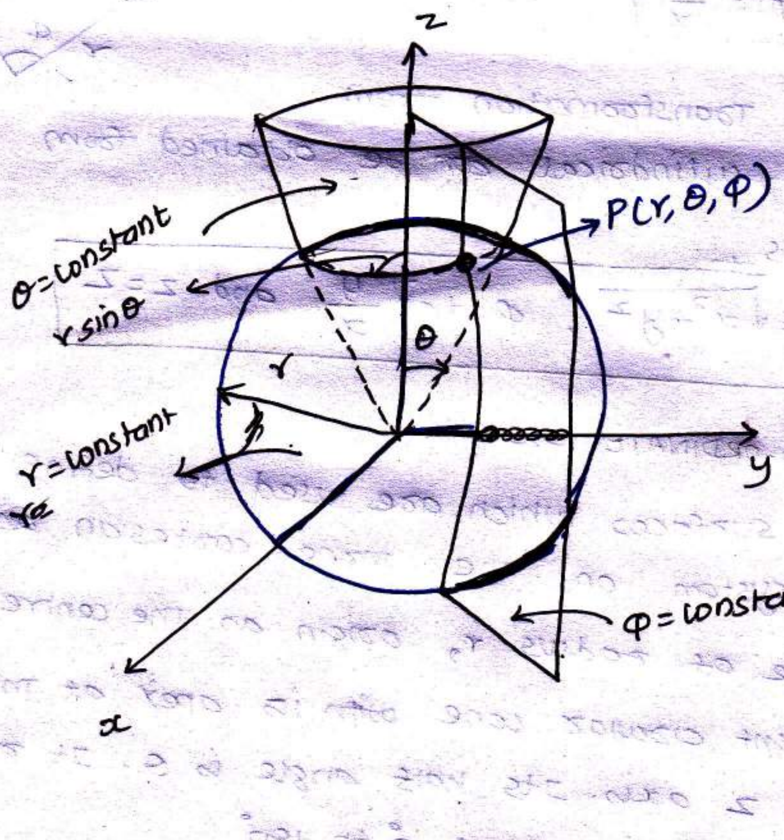


The range of the variables are

$0 \leq r \leq \infty$

$0 \leq \phi \leq 2\pi$

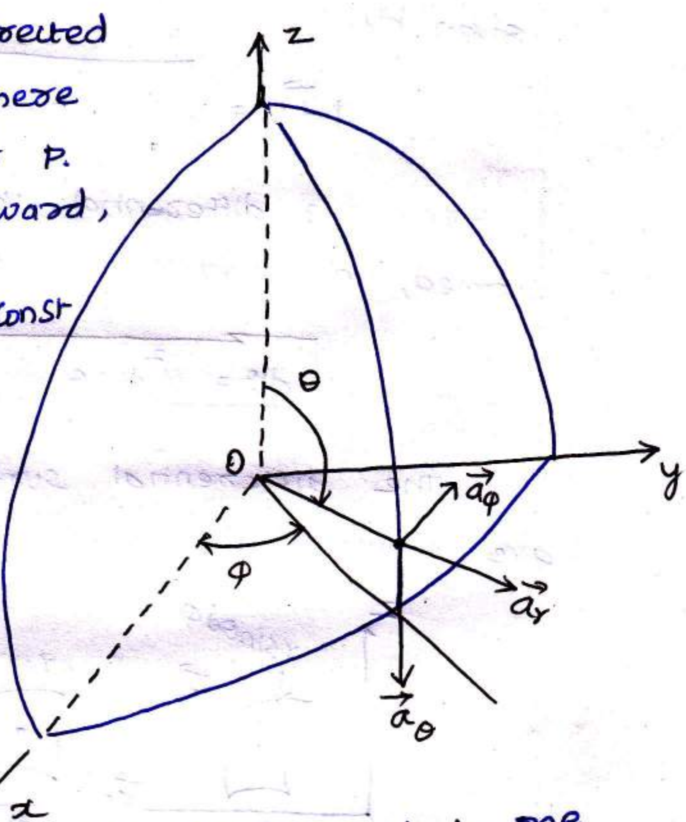
$0 \leq \theta \leq \pi$ as 1/2 angle.



BASE VECTORS:

→ The unit vector $\vec{a}_r$ is directed from the centre of the sphere i.e. origin to the given point P. It is directed radially outward, normal to the sphere.

It lies in the cone θ = const and plane ϕ = constant.



→ The unit vector $\vec{a}_\theta$ is tangent to the sphere and oriented in the direction of increasing θ. It is normal to the conical surface

→ The 3rd unit vector $\vec{a}_\phi$ is tangent to the sphere and also tangent to the conical surface. It is oriented in the direction of increasing ϕ. It is same as defined in the cylindrical co-ordinate system.

Here the vector of point 'p' can be represented as

$$\boxed{\vec{P} = P_r \vec{a}_r + P_\theta \vec{a}_\theta + P_\phi \vec{a}_\phi}$$

DIFFERENTIAL ELEMENTS IN SPHERICAL CO-ORDINATE SYSTEMS:

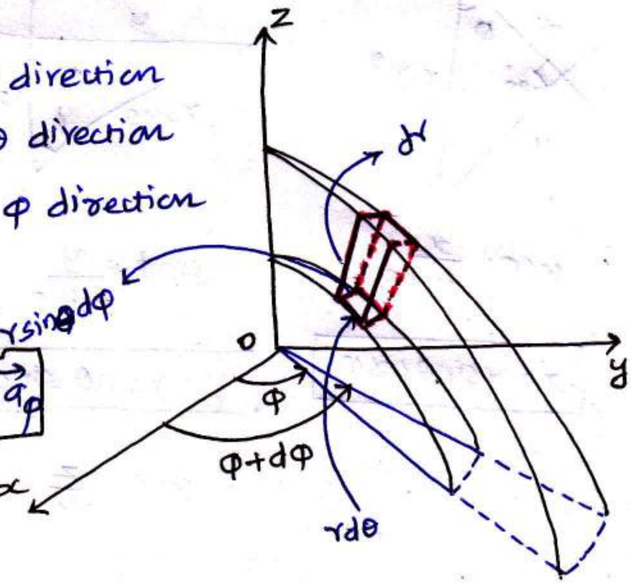
dr = Differential length in r direction

$r d\theta$ = Differential length in θ direction

$r\sin\theta d\phi$ = Differential length in ϕ direction

∴ differential vector length

$$\boxed{\vec{dl} = dr\vec{a}_r + r d\theta\, \vec{a}_\theta + r\sin\theta\, d\phi\, \vec{a}_\phi}$$



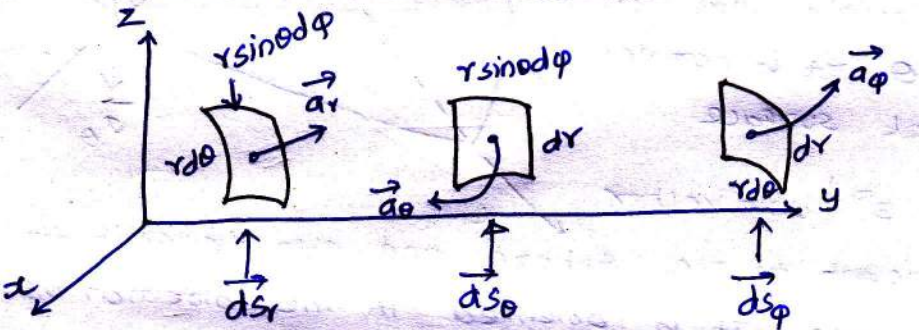
~~Hence~~ The magnitude of the differential length vector is given by,

$$|\vec{dl}| = \sqrt{(dr)^2 + (r d\theta)^2 + (r\sin\theta\, d\phi)^2}$$

Hence the differential volume of the differential element formed, in spherical co-ordinate system is given by,

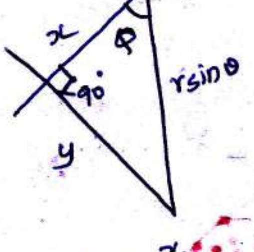
$$\boxed{dv = r^2 \sin\theta \, dr \, d\theta \, d\phi}$$

The differential surface areas in the three directions are shown in fig below.



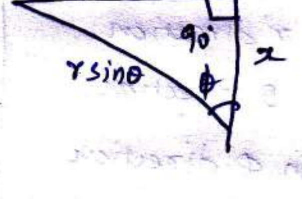
$\vec{ds_r} = r^2 \sin\theta \, d\theta \, d\phi \qquad \vec{ds_\theta} = r\sin\theta \, dr \, d\phi \qquad \vec{ds_\phi} = r \, dr \, d\theta$.

RELATIONSHIP B/W CARTESIAN & SPHERICAL SYSTEMS :-



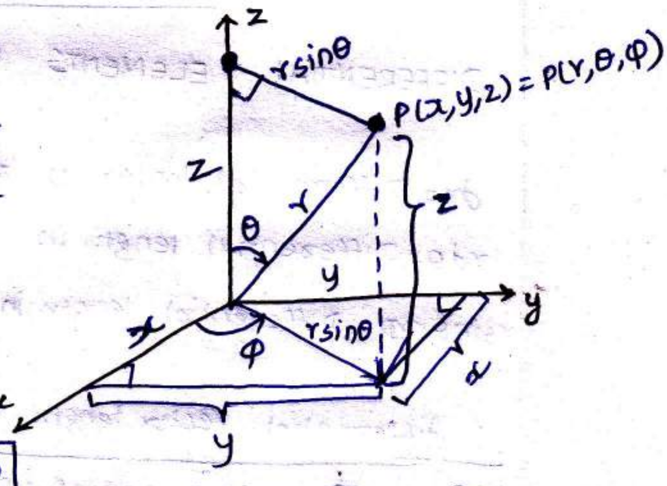
$\cos\phi = \dfrac{x}{r\sin\theta}$

$\boxed{x = r\sin\theta\cos\phi}$



$\sin\phi = \dfrac{y}{r\sin\theta}$

$\boxed{y = r\sin\theta\sin\phi}$



$\cos\theta = \dfrac{z}{r}$

$\boxed{z = r\cos\theta}$

Hence the transformation from spherical to cartesian can be obtained from equations,

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta . \end{aligned}}$$

Now r can be expressed as,

$$x^2+y^2+z^2 = r^2\sin^2\theta\cos^2\varphi + r^2\sin^2\theta\sin^2\varphi + r^2\cos^2\varphi$$

$$= r^2\sin^2\theta\left[\cos^2\varphi+\sin^2\varphi\right] + r^2\cos^2\varphi$$

$$= r^2\left[\sin^2\theta+\cos^2\theta\right]$$

$$= r^2$$

$$\boxed{r = \sqrt{x^2+y^2+z^2}}$$

While $\tan\varphi = \frac{y}{z}$ and $\cos\theta = \frac{z}{r}$.

As r is known, θ can be obtained.

Thus the Transformation from cartesian to spherical co-ordinates system can be obtained from the following equations.

$$r = \sqrt{x^2+y^2+z^2}\ ,\quad \theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right] \text{ and } \varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

TRANSFORMATION OF VECTORS:

TRANSFORMATION OF VECTORS FROM CARTESIAN TO CYLINDRICAL

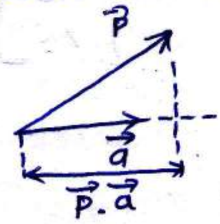
Consider a vector $\vec{A}$ in cartesian co-ordinate system as,

$$\vec{A} = A_x\vec{a_x} + A_y\vec{a_y} + A_z\vec{a_z}$$

While the same vector in cylindrical co-ordinate system can be represented as

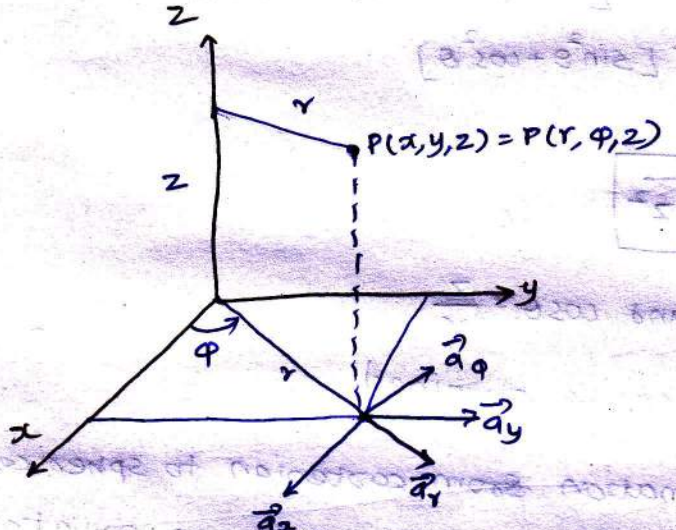
$$\vec{A} = A_r\vec{a_r} + A_\varphi\vec{a_\varphi} + A_z\vec{a_z}.$$

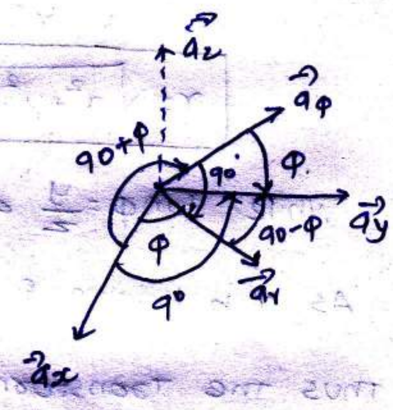
From the dot product it is known that the component of vector in the direction of unit vector is it dot product with that unit vector.



Hence the component of $\vec{A}$ in the direction of $\vec{a_r}$ is the dot product of $\vec{A}$ with $\vec{a_r}$. This component is nothing but A_r.

$$\therefore A_r = \vec{A} \cdot \vec{a_r}$$
$$= [A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}] \cdot \vec{a_r}$$
$$= A_x \vec{a_x} \cdot \vec{a_r} + A_y \vec{a_y} \cdot \vec{a_r} + A_z \vec{a_z} \cdot \vec{a_r}$$





$$\vec{a_x} \cdot \vec{a_r} = (1)(1)\cos\phi_{xr} = (1)(1)\cos\phi = \cos\phi.$$
$$\vec{a_x} \cdot \vec{a_\phi} = (1)(1)\cos(90+\phi) = -\sin\phi$$
$$\vec{a_y} \cdot \vec{a_r} = (1)(1)\cos(90-\phi) = \sin\phi$$
$$\vec{a_y} \cdot \vec{a_\phi} = (1)(1)\cos\phi = \cos\phi.$$

$$\vec{a_z} \cdot \vec{a_r} = \vec{a_z} \cdot \vec{a_\phi} = 0$$

$$\vec{a_z} \cdot \vec{a_z} = 1 \quad \Rightarrow A_r = A_x \cos\phi + A_y \sin\phi.$$

IIIy

$$A_\phi = \vec{A} \cdot \vec{a_\phi} \Rightarrow \vec{A_\phi} = -A_x \sin\phi + A_y \cos\phi$$
$$\vec{A_z} = \vec{A} \cdot \vec{a_z} = A_z \Rightarrow A_z = A_z$$

$$\therefore \begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

TRANSFORMATION OF VECTORS FROM CYLINDRICAL TO CARTESIAN

Now it is necessary to find the transformation from cylindrical to cartesian. Hence we assume $\vec{A}$ is known in cylindrical system.

Thus component of $\vec{A}$ in a_x direction is given by

$$\vec{A}_x = \vec{A}\cdot\vec{a_x} = [A_r\vec{a_r} + A_\varphi\vec{a_\varphi} + A_z\vec{a_z}]\cdot\vec{a_x}$$

$$= A_r\,\vec{a_r}\cdot\vec{a_x} + A_\varphi\,\vec{a_\varphi}\cdot\vec{a_x} + A_z\,\vec{a_z}\cdot\vec{a_x}$$

As dot product is commutative

$$\vec{a_r}\cdot\vec{a_x} = \vec{a_x}\cdot\vec{a_r} = \cos\varphi$$

Hence

$$\vec{A}_x = A_r\cos\varphi + (-\sin\varphi)A_\varphi$$

$$\vec{A}_x = A_r\cos\varphi - A_\varphi\sin\varphi$$

|||ly

$$\vec{A}_y = \vec{A}\cdot\vec{a_y} = \sin\varphi\, A_r + \cos\varphi\, A_\varphi$$

$$A_z = A_z$$

$$\therefore \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\varphi \\ A_z \end{bmatrix}$$

TRANSFORMATION OF VECTORS FROM CARTESIAN TO SPHERICAL

Let

$$\vec{A} = A_x\vec{a_x} + A_y\vec{a_y} + A_z\vec{a_z}$$

$$A_r = \vec{A}\cdot\vec{a_r} = [A_x\vec{a_x} + A_y\vec{a_y} + A_z\vec{a_z}]\cdot\vec{a_r}$$

$$A_r = A_x\,\vec{a_x}\cdot\vec{a_r} + A_y\,\vec{a_y}\cdot\vec{a_r} + A_z\,\vec{a_z}\cdot\vec{a_r}$$

$$A_\theta = \vec{A}\cdot\vec{a_\theta} = [A_x\vec{a_x} + A_y\vec{a_y} + A_z\vec{a_z}]\cdot\vec{a_\theta}$$

$$A_\theta = A_x\,\vec{a_x}\cdot\vec{a_\theta} + A_y\,\vec{a_y}\cdot\vec{a_\theta} + A_z\,\vec{a_z}\cdot\vec{a_\theta}$$

$$A_\phi = \vec{A}\cdot\vec{a}_\phi = \left[A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z\right]\cdot\vec{a}_\phi$$

$$= A_x\,\vec{a}_x\cdot\vec{a}_\phi + A_y\,\vec{a}_y\cdot\vec{a}_\phi + A_z\,\vec{a}_z\cdot\vec{a}_\phi$$

The dot product of spherical unit vectors are given below.

$\cdot$	$\vec{a}_r$	$\vec{a}_\theta$	$\vec{a}_\phi$
$\vec{a}_x$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\vec{a}_y$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\vec{a}_z$	$\cos\theta$	$-\sin\theta$	0

$$\therefore \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

TRANSFORMATION OF VECTORS FROM SPHERICAL TO CARTESIAN

$$\vec{A} = A_r\vec{a}_r + A_\theta\vec{a}_\theta + A_\phi\vec{a}_\phi$$

$$A_x = \vec{A}\cdot\vec{a}_x = A_r\,\vec{a}_r\cdot\vec{a}_x + A_\theta\,\vec{a}_\theta\cdot\vec{a}_x + A_\phi\,\vec{a}_\phi\cdot\vec{a}_x$$

$$A_y = \vec{A}\cdot\vec{a}_y = A_r\,\vec{a}_r\cdot\vec{a}_y + A_\theta\,\vec{a}_\theta\cdot\vec{a}_y + A_\phi\,\vec{a}_\phi\cdot\vec{a}_y$$

$$A_z = \vec{A}\cdot\vec{a}_z = A_r\,\vec{a}_r\cdot\vec{a}_z + A_\theta\,\vec{a}_\theta\cdot\vec{a}_z + A_\phi\,\vec{a}_\phi\cdot\vec{a}_z$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\phi & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

DISTANCE OF ALL CO-ORDINATE SYSTEMS

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} \quad \text{- cartesian}$$

$$d = \sqrt{r_2^2 + r_1^2 - 2r_1r_2\cos(\phi_2-\phi_1) + (z_2-z_1)^2} \quad \text{- cylindrical}$$

$$d = \sqrt{r_2^2 + r_1^2 - 2r_1r_2\cos\theta_2\cos\theta_1 - 2r_1r_2\sin\theta_2\sin\theta_1\cos(\phi_2-\phi_1)}$$

- spherical.

TRANSFORMATION OF VECTORS FROM SPHERICAL TO CYLINDRICAL

Let

$$\vec{A} = A_r \vec{a_r} + A_\theta \vec{a_\theta} + A_\phi \vec{a_\phi}$$

$$A_\rho = A_r \vec{a_r}\cdot\vec{a_\rho} + A_\theta \vec{a_\theta}\cdot\vec{a_\rho} + A_\phi \vec{a_\phi}\cdot\vec{a_\rho}$$

$$A_\phi = A_r \vec{a_r}\cdot\vec{a_\phi} + A_\theta \vec{a_\theta}\cdot\vec{a_\phi} + A_\phi \vec{a_\phi}\cdot\vec{a_\phi}$$

$$A_z = A_r \vec{a_r}\cdot\vec{a_z} + A_\theta \vec{a_\theta}\cdot\vec{a_z} + A_\phi \vec{a_\phi}\cdot\vec{a_z}$$

$\vec{a_r}\cdot\vec{a_\rho} = \sin\theta$ $\quad\vec{a_\theta}\cdot\vec{a_\rho} = \cos\theta$ $\quad\vec{a_\phi}\cdot\vec{a_\rho} = 0$

$\vec{a_r}\cdot\vec{a_\phi} = 0$ $\quad\vec{a_\theta}\cdot\vec{a_\phi} = 0$ $\quad\vec{a_\phi}\cdot\vec{a_\phi} = 1$

$\vec{a_r}\cdot\vec{a_z} = \cos\theta$ $\quad\vec{a_\theta}\cdot\vec{a_z} = -\sin\theta$ $\quad\vec{a_\phi}\cdot\vec{a_z} = 0.$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

TRANSFORMATION OF VECTORS FROM CYLINDRICAL TO SPHERICAL

Let

$$\vec{A} = A_\rho \vec{a_\rho} + A_\phi \vec{a_\phi} + A_z \vec{a_z}$$

$$A_r = A_\rho \vec{a_\rho}\cdot\vec{a_r} + A_\phi \vec{a_\phi}\cdot\vec{a_r} + A_z \vec{a_z}\cdot\vec{a_r}$$

$$A_\theta = A_\rho \vec{a_\rho}\cdot\vec{a_\theta} + A_\phi \vec{a_\phi}\cdot\vec{a_\theta} + A_z \vec{a_z}\cdot\vec{a_\theta}$$

$$A_\phi = A_\rho \vec{a_\rho}\cdot\vec{a_\phi} + A_\phi \vec{a_\phi}\cdot\vec{a_\phi} + A_z \vec{a_z}\cdot\vec{a_\phi}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

TYPES OF INTEGRAL RELATED TO ELECTROMAGNETIC THEORY

In electromagnetic theory a charge can exist in Point ~~form~~, line form, surface form ~~and~~ or volume form.

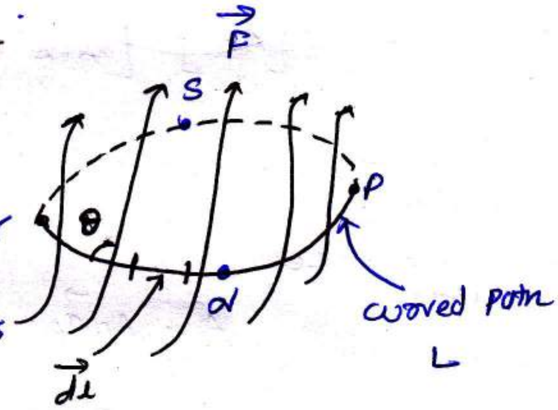
Hence for charge distribution analysis, the following types of integrals are required.

1. Line integral
2. surface integral
3. Volume integral.

LINE INTEGRAL

- A line can exist as a straight line or it can be distance travelled along a curve
- ~~The~~ From mathematical Point of view, a line is a curved path in a space.

Consider a vector field $\vec{F}$ shown in fig. The curved path shown in the field is P-r. This is called Path of integration and corresponding integral can be defined as.



$$\int_L \vec{F}.\vec{dl} = \int_P^r |F|\, dl \cos\theta \quad \text{[using dot product definition]}.$$

where

$dl \rightarrow$ Elementary length.

This is called line integral of $\vec{F}$ around the curved path ~~P-r~~ L.

The curved path can be of two types.

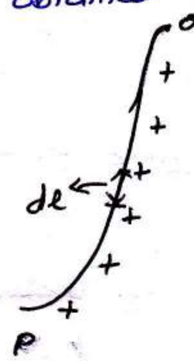
(i) open Path as P-r shown in fig

(ii) closed path as P-a-r-s-P.

The closed path is also called contour. The corresponding integral is called contour integral, closed integral (or) circular integral, and mathematicalled defined as.

$$\oint_L \vec{F}.\vec{dl} = \text{circular integral.}$$

If there exists a charge along a ~~straight~~ line as shown in figure, then the total charge is obtained by calculating a line integral.



$$Q = \int_L \rho_L \cdot dl$$

ρ_L = Line charge density (or) charge per unit length (C/m).

SURFACE INTEGRAL:

In electromagnetic theory a charge may exist in a distributed form. It may be spreaded over a surface as shown in figure(a) below.

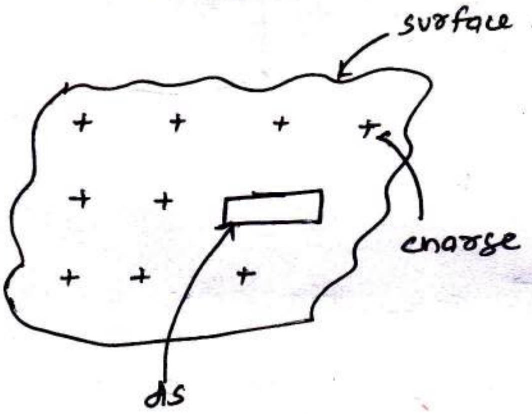


fig (a)

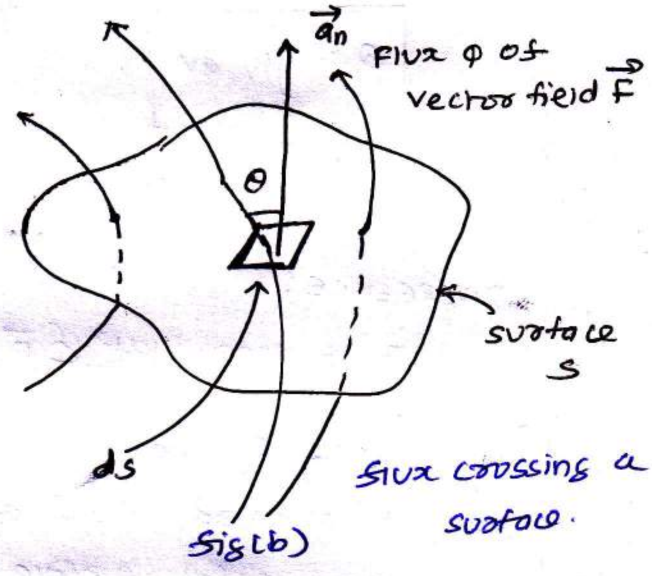


fig (b)

similarly a flux φ may pass through a surface as shown in fig(b). While doing analysis of such cases an integral is required called <u>SURFACE INTEGRAL</u>, to be carried out over a surface related to a vector field.

For a charge distribution shown in fig (a), we can write total charge existing on the surface as

$$Q = \int_S \rho_s \, ds. \quad \rho_s \rightarrow \text{surface charge density in } C/m^2$$

$ds \rightarrow$ Elementary surface area.

$|||^{ly}$ from fig (b), the total flux crossing the surface S can be expressed as

$$\varphi = \int_S \vec{F} \cdot \vec{ds} = \int_S |\vec{F}| \, ds \cos\theta$$

If the surface is closed, then it defines a volume and corresponding surface integral is given by,

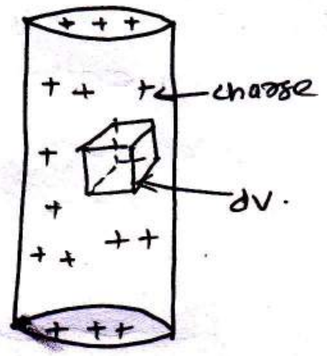
$$\varphi = \oint_S \vec{F}.\vec{ds}.$$

VOLUME INTEGRAL

If the charge distribution exists in a three dimensional volume form as shown in figure below, then a volume integral is required to calculate the total charge.

Thus if ρ_v is the volume charge density over volume V. then the volume integral is defined as

$$\boxed{Q = \int_V \rho_v \cdot dv}$$



dv = Elementary volume.

DIVERGENCE:

It is seen that $\oint_S \vec{F}.\vec{ds}$ gives the flux flowing across the surface S. Then mathematically divergence is defined as the net outward flow of the flux per unit volume over a closed incremental surface. It is denoted as div $\vec{F}$ and given by

$$\text{div } \vec{F} = \lim_{\Delta v \to 0} \frac{\oint \vec{F}.\vec{ds}}{\Delta v} = \text{Divergence of } \vec{F}.$$

symbolically it is denoted as

$$\nabla.\vec{F} = \text{Divergence of } \vec{F}$$

where ∇ = Vector operator $= \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}$

But $\vec{F} = F_x\vec{a_x} + F_y\vec{a_y} + F_z\vec{a_z}$

$$\therefore \boxed{\nabla.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}} \rightarrow \text{In cartesian form}$$

iii) Divergence in other co-ordinates.

$$\nabla\cdot\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\varphi}{\partial \varphi} + \frac{\partial F_z}{\partial z} \quad \text{cylindrical}$$

$$\nabla\cdot\vec{F} = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi} \quad \text{spherical.}$$

(X) The vector field having its divergence zero is called solenoidal field

$$\boxed{\nabla\cdot\vec{A} = 0 \text{ for } \vec{A} \text{ to be solenoidal}}$$

GRADIENT OF A SCALAR:

consider that in space let w be the unique function of x, y and z co-ordinates in the cartesian system. This is the scalar function and denoted as $w(x,y,z)$. Consider a vector operator in cartesian system denoted as ∇ (called del). It is defined as.

$$\nabla(\text{del}) = \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}$$

The operation of the vector operator del(∇) on a scalar function is called gradient of a scalar.

$$\text{Grad } w = \nabla w = \left(\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right)w$$

$$\boxed{\nabla w = \frac{\partial w}{\partial x}\vec{a_x} + \frac{\partial w}{\partial y}\vec{a_y} + \frac{\partial w}{\partial z}\vec{a_z}} \quad \text{cartesian.}$$

In cylindrical co-ordinates.

$$\nabla w = \frac{\partial w}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial w}{\partial\varphi}\vec{a_\varphi} + \frac{\partial w}{\partial z}\vec{a_z}$$

spherical co-ordinates

$$\nabla w = \frac{\partial w}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial w}{\partial\theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial w}{\partial\varphi}\vec{a_\varphi}$$

CURL OF A VECTOR:

$$\nabla \times \vec{F} = \text{curl of } \vec{F}$$

curl indicates the rotational property of vector field. If curl of vector is zero, the vector field is irrotational.

$$\nabla \times \vec{F} = \left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right]\vec{a_x} + \left[\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right]\vec{a_y} + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]\vec{a_z}$$

$$\nabla \times \vec{F} = \begin{bmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{bmatrix} \quad \text{CARTESIAN}$$

$$\nabla \times \vec{F} = \begin{bmatrix} \vec{a_r} & r\vec{a_\varphi} & \vec{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ F_r & F_\varphi & F_z \end{bmatrix} \quad \text{CYLINDRICAL}$$

$$\nabla \times \vec{F} = \begin{bmatrix} \vec{a_r} & r\vec{a_\theta} & r\sin\theta\,\vec{a_\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ F_r & rF_\theta & r\sin\theta\, F_\varphi \end{bmatrix} \quad \text{SPHERICAL}$$

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QUESTION BANK (DESCRIPTIVE)

Subject with Code : EMF(**19EE0207**) **Course & Branch**: B.Tech - EEE
Year & Sem: II-B.Tech & II-Sem **Regulation:** R19

UNIT-I
INTRODUCTION TO VECTOR CALCULUS

1. a) Convert point P (1,3,5) from cartesian to cylindrical and spherical co-ordinates system. 5 M
   b) Given the two points A (X=2, Y=3, Z=-1) and B= (r=4, θ=25 and ϕ=120°). Find the spherical co-ordinates of A and cartesian co-ordinates of B 5 M
2. Point P and Q are located at (0,2,4) and (-3,1,5) calculated: 1. The Position vector P , 2. The distance vector from P and Q, 3. The distance between P and Q and 4. A vector parallel to PQ with magnitude of 10. 10 M
3. Express vector B in cartesian and cylindrical systems. Given $B= 10/r\ a_r + r \cos\theta\ a_\theta + a_\phi$. Find the B at (-3,4,0) and (5,π/2, -2) 10 M
4. a) Transform the vector field $W=10\ a_x -8\ a_y +6\ a_z$ to cylindrical co-ordinate system at point P (10 ,-8, 6) 5 M
   b) Express $B= r^2\ a_r + \sin\theta\ a_\phi$ in the cartesian co-ordinates. Hence obtain B at P (1,2,3) 5 M
5. If $B= y\ a_x + (x+z)\ a_y$ and a point Q is located at (-2,6,3) express.1 The Point Q in cylindrical and spherical co-ordinates and 2) B in spherical coordinates. 10 M
6. a) Given point P (-2,6,3) and $A=y\ a_x +(x+z)\ a_y$. Express A in Cylindrical coordinates. 5 M
   b) Transform the vector A= 3i-2j-4K at P (x=2, y=3, Z=3) to cylindrical coordinates 5 M
7. a) Given the two coplanar vectors $A= 3\ a_x + 4\ a_y- 5\ a_z$ and $B= -6a_x +2\ a_y +4\ a_z$. Obtain the unit vector normal to the plane containing the vector A and B 5 M
   b) The Three fields are given by $A=2a_x -a_z$, $B= 2\ a_x-a_y+2a_z$, $C= 2a_x-3a_y +a_z$. Find the scalar and vector triple product. 5 M
8. Determine the divergence of these vector fields:
   i).$P=x^{-2}yz\ a_x +xz\ a_z$, ii) $Q= r \sin\phi\ a_r +r^2 z\ a_\phi + z \cos\phi\ a_z$ and iii) $T= (1/r^2) \cos\theta\ a_r + r \sin\theta \cos\phi\ a_\theta+\cos\theta\ a_\phi$ 10 M
9. Find the gradient of the following scalar fields
   i) $V= e^{-z} \sin 2x \cosh y$, ii) $U= r^2 z \cos\phi$ and iii) $W= 10r \sin^2\theta \cos\phi$ 10 M
10. Determine the curl of the vector fields:
   i).$P=x^2yz\ a_x +xz\ a_z$, ii) $Q= r \sin\phi\ a_r +r^2 z\ a_\phi + z \cos\phi\ a_z$ and iii) $T= (1/r^2) \cos\theta\ a_r + r \sin\theta \cos\phi\ a_\theta+\cos\theta\ a_\phi$ 10 M

UNIT –II
STATIC ELECTRIC FIELD

1. (a) State and explain Coulomb's law indicating clearly the units of quantities in the equation of force? 5M
   (b) State and prove Gauss's law and write limitations of Gauss's law? 5M
2. Three concentrated charges of 0.25 μC are located at the vertices of an equilateral triangle of 10 cm side . Find the magnitude and direction of the force on one charge due to other two charges. 10 M
3. a) Determine the Electric filed intensity at P(-0.2, 0, -2.3) m due to a point charge of 5 nc at Q (0.2,0.1, -2.5) m in air. 5 M
   b) An infinitely long uniform line charge is located at y=3, Z=5. If ρ_L = 30 n C/m, find the filed intensity E at i) origin , ii) P(0,6,1) and iii) P (5,6,1) 5 M
4. Line charge density ρ_L= 24 n C/m is located in free space on the line y=1 and Z=2 m
   a) Find E at the point P(6,-1,3) , b) What point charge Q_a should be located at A (-3,4,1) to make y component of total E zero at point P? 10 M

5.a) Find E at (0,0,2) m due to charged circular disc in x-y plane with ρ_S=20 n C/m^2 and radius 1m. 5M
   b) A circular disc of 10 cm radius is charged uniformly with total charge of 100μc . Find E at a point 20cm on its axis. 5 M

6.The Electric flux density is given as D= (r/4) a_r n C/m^2 in free space. Calculate:
   The Electric field intensity at r=0.25 m , The total charge within a sphere of r=0.25 m 10 M

7.Given that A= 30 e^{-r} a_r-2 z a_z in the cylindrical co-ordinates. Evaluate both sides of the divergence theorem for the volume enclosed by r=2, z=0 and Z=5 10 M

8. a)An electric potential is given by V=(60 sinθ /r^2) v . Find V and E at P(3,60°,25°) 5 M
   b) In free space V= $x^2y(z+3)$. Find E at (3, 4, -6) and The charge within the cube $0<x,y,z<1$. 5 M

9.a) The potential field in free space is given by V=(50/r) , a<r< b (spherical) show that ρ_v=0 for a<r<b and find the energy stored in the region a<r<b 5 M
   b) Two pint charges 1.5nC at (0,0,0.1) and -1.5nC at (0,0,-0.1) are in free space . Treat the two charges as a dipole at the origin and find the potential at p(0.3,0,0.4) 5 M

10. a) What is the relation between electric flux density and electric field intensity? 2M
   b) Define dipole moment? 2M
   c) Define an electric dipole? 2M
   d) State vector form of coulombs law? 2M
   e) Derive Maxwell second equation? 2M

UNIT –III

CONDUCTORS, DIELECTRICS AND CAPACITANCE

1. (a) Derive the continuity equation. What is its physical significance? 5M
   (b) Derive the point form of ohms law? 5M
2. Explain the boundary conditions of two perfect dielectrics materials? 10M
3. Explain the boundary conditions between conductor and free space? 10M
4. a) In cylindrical coordinates $J=10\ e^{-100r}\ a_\phi\ A/m^2$. Find the current crossing through the region $0.01<r<0.02$ m and $0<z<1$ m and intersection of this region with the ϕ = constant plane 5 M
   b) An aluminum conductor is 2000 ft long and has a circular cross section with a diameter of 20 mm. If there is a DC voltage of 1.2 V between the ends . Find a) The current density b) The current , C power dissipated form the l=knowledge of circuit theory. Assume $\sigma=3.82 *10^7$ mho/m for aluminum . 5 M
5. a) Find the magnitude of D and P for a dielectric material in which E=0.15 mV/m and $\chi=4.25$ 5 M
   b) Find the polarization in dielectric material with $\varepsilon_r=2.8$ if $D=3*10^{-7}\ C/m^2$ 5 M
6. Explain the phenomenon of polarization when a dielectric slab is subjected to an electric field? 10M
7. a) Derive the expression for parallel plate capacitor and capacitance of a co-axial cable? 6 M
   c) A parallel plate capacitor has an area of 0.8 m^2 separation of 0.1 mm with a dielectric for which $\varepsilon_r = 1000$ and a field of 10^6 V/m. Calculate C and V 4 M
8. Find V at P (2,1,3) for the field of two coaxial conducting cones, with V=50 V at $\theta=30$ and V=20 V at $\theta=50$. 10 M
9. Two parallel conducting disc are separated by distance 5 mm at z=0 and z=5 mm . If V=0 and V=100 v at z=5 mm, find the charge densities on the disc. 10 M
10. a)Determine whether or not the following potential fields satisfy the Laplace's equation
    i) $V=x^2-y^2+z^2$ ii) $V= r\cos\phi +z$ and 5 M
    b) Derive Laplace's and Poisson's Equation 5 M

UNIT –IV

STATIC MAGNETIC FIELDS

1. Using Biot-savart's law. Find $\vec{H}$ and $\vec{B}$ due conductor of finite length? 10M
2. a) Explain maxwell's second equation? 5M
   b) State and explain ampere's circuital law? 5M
3. Evaluate both sides of the stokes theorem for the filed H=6xy a_x -3y^2 a_y A/m and the rectangular path around the region 2<x<5, -1<y<1 , Z=0 . Let the positive direction of ds be a_z. 10 M
4. a) Find the flux passing the portion of the plane $\phi=\pi/4$ defined by 0.01<r<0.05 m and 0<z<2 m. A current filament of 2.5 A is along the z axis in the a_z direction in free space. 5 M
   b) In cylindrical coordinates B= (2.0/r) a_ϕ tesla. Determine the magnetic flux ϕ crossing the plane surface defined by 0.5<r<2.5 m and 0<z<2m. 5 M
5. In cylindrical co-ordinates A=50 r^2 a_z wb/m is a vector magnetic potential in a certain region of free space. Find H, B, J and using J find the total current I crossing the surface 0<r<1 , 0<ϕ<2π and Z=0. 10 M
6. a) A Point charge of Q=-1.2 C has a velocity V=(5 a_x +2 a_y -3a_z)m/s. Find the magnitude of the force exerted on the charge if i) E= -18 a_x +5 a_y -10 a_z V/m and ii) B=-4 a_x +4 a_y +3 a_z T 3 M
   b) A magnetic field B= 3.5*10^{-2} a_z exerts a force on a 0.3 m long conductor along x axis. IF a current of 5 A flows in -a_x direction, determine what force must be applied to hold conductor in position. 3 M
   c) Determine the force per meter length between two long parallel wires A and B separated by distance 5 cm in air and carrying currents of 40 A in the same direction. 4 M
7. A rectangular loop in Z=0 plane has corners at (0,0,0), (1,0,0),(1,2,0) and (0,2,0). The loop carries a current of 5 A in a_x direction. Find the total force and torque on the loop produced by the magnetic field B=2 a_x+2a_y-4a_z wb/m^2. 10 M
8. Derive the expression for self-inductance of solenoid, toroid and coaxial cable 10M
9. a) Calculate the inductance of a solenoid of 200 turns wound tightly on a cylindrical tube of 6 cm diameter. The length of the tube is 60 cm and the solenoid is in air. 5 M
   b) Find inductance per unit length of a co-axial cable if radius of inner and outer conductors are 1 mm and 3 mm respectively. Assume relative permeability unity. 5 M
10. a) Calculate the inductance of a 10 m length of coaxial cable filled with a material for which μ_r = 80 and radii inner and outer conductors are 1 mm and 4 mm respectively. 5 M
    b) A straight long wire is situated parallel to one side of a square coil. Each side of the coil has a length of 10 cm. The distance between straight wire and the center of the coil is 20 cm. Find the mutual inductance of the system . 5 M

UNIT –V

TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

1. Write Maxwell's equation in good conductors for time varying fields and static fields both in differential and integral form? 10M
2. Explain faradays law of electromagnetic induction and there from derive maxwell's equation in differential and integral form? 10M
3. Derive the equation of Continuity for time varying fields? 10M
4. Derive an expression for motional and transformer induced emf? 10M
5. What is displacement current? Explain physical significance of displacement current? 10M
6. Derive expressions for integral and point forms of poynting Theorem? 10M
7. Explain faradays law of electromagnetic induction and derive the expression for induced e.m.f? 10M
8. a) Define skin depth? 2M
   b) Define displacement current? 2M
   c) State Faraday's law of electromagnetic induction? 2M
   d) Write Maxwell equations in time varying fields? 2M
   e) Define pointing vector? 2M
9. A Parallel plate capacitor with plate area of 5 cm^2 and plate separation of 3mm has a Voltage of 50 sin 10^3 t volts applied to its plates. Calculate the displacement current Assuming $\varepsilon=2\varepsilon_0$ 10 M
10 An area of 0.65 m^2 in the plane Z=0 encloses a filamentary conductor. Find the induced voltage if B= 0.05 cos 10^3 t (a_y+a_z)/√2 tesla. 10 M

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QUESTION BANK (OBJECTIVE)

Subject with Code : EMF(18EE0203) **Course & Branch**: B.Tech - EEE
Year & Sem: II-B.Tech & II-Sem **Regulation:** R18

UNIT-I
INTRODUCTION TO VECTOR CALCULUS

1. In three dimensional coordinate systems, coordinates are
   A)perpendicular to each other B) parallel to each other
   C) same direction for each other D)opposite direction for each other

2. Three dimensional coordinate system is one in which coordinates intersect each other at
   A) negative points B) zero points C)positive points D) absolute points

3. Rectangular coordinate system is also known as
   A) Space coordinate system B) Polar coordinate system
   C) Cartesian coordinate system D)Planar coordinate system

4. The range of azimuthal angle φ in the spherical polar coordinates is
   A) [0,2π] B) [0,π] C) [0,π/2] D) [-π, +π]

5. The equation to a surface in spherical coordinates is given by θ=π/3. The surface is a.
   A) sector of a circle
   B) A cone making an angle of π/3 with the z-axis
   C) A vertical plane making an angle of π/3 with the z-axis
   D) A vertical plane making an angle of π/3 with the x-axis

6. The equation to a surface in spherical coordinates is given by φ=π/3. The surface is a.
   A) sector of a circle
   B) A cone making an angle of π/3 with the z-axis
   C) A vertical plane making an angle of π/3 with the z-axis
   D) A vertical plane making an angle of π/3 with the x-axis

7. Expressed in spherical coordinates, the equation $x^2+y^2+z^2= 4z$ becomes
   A) 4 cos𝜽sinΦ B) 4 sin𝜽cosΦ C) 4 cos𝜽 D) 4sin𝜽

8. The cylindrical coordinate system is also referred to as
   A) Cartesian system B) Circular system C) Spherical system D) Space system

9. Transform the point (-2,6,3) into cylindrical coordinates.
   A) (6.325,-71.57,3) B) (6.325,71.57,3) C) (6.325,73.57,3) D) (6.325,-73.57,3)

10. A charge located at point p $(5,30^0,2)$ is said to be in which coordinate system?
    A) Cartesian system B) Cylindrical system C) Spherical system D) Space system

11. Transform the spherical system B = (10/r)i + (10cos θ)j + k into cylindrical form at (5, π/2, -2)
A) 2.467i + j + 1.167k B) 2.467i – j + 1.167k
C) 2.467i – j – 1.167k D) 2.467i + j – 1.167k

12. Convert the given rectangular coordinates A(2,3,1) into corresponding cylindrical coordinates
A) (3.21,56.31,1) B) (3.21,57.31,0) C) (3.61,57.31,0) D) (3.61,56.31,1)

13. Convert the point (3,4,5) from Cartesian to spherical coordinates
A) ($7.07,45^0,53^0$) B) ($0.707,45^0,53^0$) C) ($7.07,54^0,63^0$) D) ($0.707,54^0,63^0$)

14. Find the spherical coordinates of A(2,3,-1)
A) ($3.74, 105.5^0, 56.13^0$) B) ($3.74, 105.5^0, 56.31^0$)
C) ($3.74, 106.5^0, 56.13^0$) D) ($3.74, 106.5^0, 56.31^0$)

15. Find the Cartesian coordinates of B($4,25^0,120^0$)
A) (0.845, 1.462, 3.625) B) (-0.845, 1.462, 3.625)
C) (-8.45, 2.462, 6.325) D) (8.45, 2.462, 6.325)

16. Given B= (10/r)i+(rcos θ) j+k in spherical coordinates. Find Cartesian points at (-3,4,0)
A) -2i + j B) 2i + k C) i + 2j D) –i – 2k

17. The scalar factor of spherical coordinates is
A) 1, r, r sin θ B) 1, r, r C) r, r, 1 D) r, 1, r

18. Transform the vector (4,-2,-4) at (1,2,3) into spherical coordinates.
A) 3.197i – 2.393j + 4.472k B) -3.197i + 2.393j – 4.472k
C) 3.197i + 2.393j + 4.472k D) -3.197i – 2.393j – 4.472k

19. Cylindrical systems have the following scalar values respectively
A) 1, ρ ,1 B) 1, 1, 1 C) 0,1,0 D) 1,0,0T

20. he volume of a parallelepiped in Cartesian is
A) dV = dx dy dz B) dV = dx dy C) dV = dy dz D) dV = dx dz

21. Transform the vector A = 3i – 2j – 4k at P(2,3,3) to cylindrical coordinates
A) -3.6j – 4k B) -3.6j + 4k C) 3.6j – 4k D) 3.6j + 4k

22. Which of the following criteria is used to choose a coordinate system?
A) Distance B) Intensity C) Magnitude d) Geometry

23. Vector transformation followed by coordinate point substitution and vice-versa, both given the same result. Choose the best answer.
A) Possible, when the vector is constant B) Possible, when the vector is variable
C) Possible in all cases D) Not possible

24. The polar form of Cartesian coordinates is
A) Circular coordinates B) Spherical coordinates
C) Cartesian coordinates D) Space coordinates

25. The cross product of the vectors 3i + 4j – 5k and –i + j – 2k is,
A) 3i – 11j + 7k B) -3i + 11j + 7k C) -3i – 11j – 7k D) -3i + 11j – 7k

26. Which of the following are not vector functions in Electromagnetics?
A) Gradient B) Divergence
C) Curl D) There is no non- vector functions in Electromagnetics

27. The work done of vectors force F and distance d, separated by angle θ can be calculated using,
A) Cross product B) Dot product C) Addition of two vectors D) Cannot be calculated

28. Find whether the vectors are parallel, (-2,1,-1) and (0,3,1)
A) Parallel B) Collinearly parallel C) Not parallel D) Data insufficient

29. When two vectors are perpendicular, their
A) Dot product is zero B) Cross product is zero
C) Both are zero D) Both are not necessarily zero

30. Find the gradient of $t = x^2y + e^z$ at the point p(1,5,-2)
A) i + 10j + 0.135k B) 10i + j + 0.135k C) i + 0.135j + 10k D) 10i + 0.135j + k

31. Curl of gradient of a vector is
A) Unity B) Zero C) Null vector D) Depends on the constants of the vector

32. Find the gradient of the function given by, $x^2 + y^2 + z^2$ at (1,1,1)
A) i + j + k B) 2i + 2j + 2k c) 2xi + 2yj + 2zk D) 4xi + 2yj + 4zk

33. Find the gradient of the function sin x + cos y.
A) cos x i – sin y j B) cos x i + sin y j C) sin x i – cos y j D) sin x i + cos y j

34. Compute the divergence of the vector xi + yj + zk.
A) 0 B) 1 C) 2 D) 3

35. Find the divergence of the vector yi + zj + xk.
A) -1 B) 0 C) 1 D) 3

36. Given $D = e^{-x}\sin y\ i – e^{-x}\cos y\ j$ Find divergence of D.
A) 3 B) 2 C) 1 D) 0

37. Find the divergence of the vector $F= xe^{-x}\ i + y\ j – xz\ k$
A) $(1 – x)(1 + e^{-x})$ B) $(x – 1)(1 + e^{-x})$ C) $(1 – x)(1 – e)$ D) $(x – 1)(1 – e)$

38. Determine the divergence of $F = 30\ i + 2xy\ j + 5xz^2\ k$ at (1,1,-0.2) and state the nature of\ the field.
A) 1, solenoidal B) 0, solenoidal C) 1, divergent D) 0, divergent

39. Find whether the vector is solenoidal, E = yz i + xz j + xy k
A) Yes, solenoidal B) No, non-solenoidal
C)) Solenoidal with negative divergence D) Variable divergence

40. Identify the nature of the field, if the divergence is zero and curl is also zero.
A) Solenoidal, irrotational B) Divergent, rotational
C) Solenoidal, irrotational D) Divergent, rotational

41. The curl of a curl of a vector gives a
A) Scalar B) Vector C) Zero value D) Non zero value

42. Find the curl of A = (y cos ax)i + (y + e^x)k
A) 2i – ex j – cos ax k B) i – ex j – cos ax k
C) 2i – ex j + cos ax k D) i – ex j + cos ax k

43. Find the curl of the vector A = yz i + 4xy j + y k
A) xi + j + (4y – z)k B) xi + yj + (z – 4y)k
C) i + j + (4y – z)k D) i + yj + (4y – z)k

UNIT –II
STATIC ELECTRIC FIELD

1. A Quantity which gives only direction is called [ ]
A) Vector B) Scalar C) Unit Vector D) None

2. The charge of an electron is [ ]
A) 1.60219×10^{-19} C B) -1.60219×10^{19} C C) -1.60219×10^{-19} C D) 1.60219×10^{19} C

3. The two equal and opposite point charges are separated by a very small distance is known as [ ]
A) Dipole moment B) Potential gradient C) Dipole D) None

4. Find the Laplacian of the Potential function $V=x^2+y^2+z^2$ [ ]
A) $2V/m^2$ B) $6\ V/m^2$ C) $4\ V/m^2$ D) $8\ V/m^2$

5. The _ is defined as the tangential force times the radial distance at which it acts [ ]
A) Power B) Energy C) Torque D) Magnetic flux density

6. Steady magnetic fields are governed by ___________ law. [ ]
A) Biot–Savart's B)Ampere's Circuital C)Both (A) and (B) D)None of these

7. Four fundamental equations of electromagnetics are grouped under [ ]
A) Fleming's laws B) faraday's laws C) lorentz equations D) maxwell's equation

8. According to poisson's equation, if V is the potential function, then [ ]
A) $\nabla^2 V=-\rho/\varepsilon$ B) $\nabla^2 V=-\rho/E$ C) $\nabla^2 V=0$ D)none of these

9. According to Gauss law ψ= [ ]
A) Q_{end} B) $\int_S D.\,dS$ C) $\int_V \rho_V\, dV$ D) ALL

10. Which of the following is a vector quantity? [ ]
A) Electrical potential B) Electrical field intensity
C) Electrical charge D) none of the above

11. An infinite number of charge each equal to q are placed along the x-axis at x=1,x=2,x=3 and so on . The potential at x=0 due to this set of charges will be [ ]
A) q B) 3 q/2 C) 2 q D) 4 $q/4$

12. An infinite number of charges, each equal to 1 q are placed at n=1, 3, 9, 27, 81. . . The electronic potential at n= 0 will be [ ]

A) q B) 3/2 q C) 2 q D) 5 $q/2$

13. A tiny particle carrying a charge of 0.2 coulomb is accelerated through a P.D of 1000 V. The K.E. acquired by the particle will be [ ]

A) 100 J B) 200 J C) 300 J D) 400 J

14. Given V=2x^2 y-12z, V at (0, 0 ,6) is…………….. [ ]

A) -72V B) 62V C) 70 V D) 0 V

15. The unit of electric field intensity is [ ]

A) A/m B) V/m C) V/m D) A/sec

16. The total flux out of a closed surface is equal to the net charge with in the surface. This statement an expression of a [ ]

A) gauss law B)divergence theorem C)faraday's law D)Maxwell's equations

17. In homonogenous linear, isotropic and stationary media, for a plane electromagnetic wave [ ]

A) $\nabla \cdot D=\rho$ B) $\nabla .D=\rho$ C) $\nabla *D=\rho$ D) none

18. It is given that electric flux density (D) in acertain region is expressed by D= (1/r)a_r in spherical co-ordinates. The charge density (u) in this region is given by [ ]

A) l/r B) l/r^2 C) $-l/r^2$ D) r^2

19. The electric field intensity (E) and electric potential (V) are interrelated by [ ]

A) E= -Divergence of V B) E= Divergence of V C) E=-gradient of V D) none of these

20. For an infinite line charge [ ]

A) E= $\rho_S /2\varepsilon$ B) E= $\rho_S /2 \pi \varepsilon$ C) E= $\rho_S /4 \pi \varepsilon$ D) None

21. Potential at R due to a point charge Q is V = [ ]

A) V=Q/4π εR B) V=Q/4π ε R^2 C) V=QR/4π ε D) None

22. Point charges 30nc,-20nc and 10nc are located at (-1, 0, 2), (0, 0, 0) and (1, 5,-1) respectively. The total flux leaving a cube of side 6 m centered at the origins is [ ]

A) 20nc B)-2nc C) 10nc D) -10nc

23. Inside a hollow spherical conductor [ ]

A) Electrical field is zero B) Electrical field is constant

C) Electrical field changes with the magnitude of charge given to the conductor

D) None of the above

24. A sphere of one meter radius can attain a maximum potential of [ ]

A) 1000 V B) 2 KV C) 30 KV D) 3 million volts

25. Surface integral of electric field intensity is [ ]

A) Electrical charge B) differential of volume flux

C) Net flux emanating from surface D) none of these

26. A plane z=10 m carries charge 20 nc/m^2.Electric field intensity at the origin is [ ]

A) -15 a_z V/m B) -36 π a_z V/m C) -72 pia_z V/m D) -360 π a_z V/m

27. Point charges Q_1=1 nC and Q_2 =2 Nc are at a distance apart. Which of the following statements are correct? [ ]

A) The force on Q_1 is repulsive B) the force on Q_2 is the same in magnitude as that-on Q_1

C) As the distance between them decreases, the force on Q_1 increases linearly

D) All the above

28. Find the Laplacian of the Potential function $V=2x^2+y^2+z^2$ [ ]

A) 2V/m2 B) 6 V/m2 C) 4 V/m2 D) 8 V/m2

29. The unit of electric flux is [ ]

A) Coulomb B) Coulomb/m^2 C) Weber D) Newton/ Coulomb

30. Coulomb's law States that [ ]

A) $F=Q1Q2/4\pi \varepsilon R^2$ B) F=Q1/4π ε R C) F=Q2/4π R D) None

31. The electric flux density D is related to E as [ ]

A) D=E B) εD=E C) D=εE D) None

32. The electric displacement current density is measured in [ ]

A) coulombs/meter B) coulombs /meter ^{2} C) volts/m D) amp/m^2

33. Conductivity is measured in [ ]

A) ohm-m B) ohms/m C)mho-m D) mhos/m

34. The relation between electric polarization and susceptibility indicates that electric Polarizations is [ ]

A) Independent of susceptibility B) inversely proportional to susceptibility

C) Proportional to square root of susceptibility D) proportional to susceptibility

35. The divergence theorem applies to a [ ]

A) Static field only B) time varying field only C) both A & B D) magnetic fields only

36. Find the Laplacian of the Potential function $V=x^2+y^2-z$ [ ]

A) 2V/m2 B) 6 V/m2 C) 4 V/m2 D) 8 V/m2

37. The electric flux density (D) and the electric field intensity (E) interrelated by [ ]

A) D=εE B) D=E/ε C) $D=\varepsilon E^2$ D) D=μE

38. First Maxwell's equation is [ ]

A) ρ_V=V.D B) ρ_V=V.E C) both A & B D) None

39. Laplaces equation $\nabla^2 V=$ [ ]

A) $-\rho_v/\varepsilon$ B) ρ_v C) 1 D) 0

40. The unit of field intensity is [ ]

A) Coulomb B) Coulomb/m^2 C) Weber D) Newton/ Coulomb

UNIT –III

CONDUCTORS, DIELECTRICS AND CAPACITANCE

1. The conductivity of a material usually depends on [ ]

   A) Temperature B) Frequency C) Temperature and Frequency D) Length

2. The electric field inside the conductor is [ ]

   A) Maximum B) Zero C) both a and b D) infinity

3. Convection current occurs when current flows through an insulating medium such as [ ]

   A) Liquid B) Copper C) Resistor D) Air

4. Charges in dielectric material are called [ ]

   A) Bound charges B) free charges C) polar charges D) none

5. The expression for Electric displacement in Dielectrics, D= [ ]

   A) $\epsilon_0 E - P$ B) $\epsilon_0 E + P$ C) $P - \epsilon_0 E$ D) both b & c

6. The phenomena of polarization happens in [ ]

   A) Dielectrics B) conductors C) insulators D) none

7. Point form of ohm's law is [ ]

   A) $E=\sigma J$ B) $J=\sigma E$ C) $E=\sigma/J$ D) $E=J$

8. For steady current, the continuity equation [ ]

   A) $\nabla \cdot \bar{J} = 0$ B) $\nabla \cdot \bar{J} = 1$ C) $\nabla \times \bar{J} = 0$ D) none

9. On the two sides of the boundary, the tangential components of **E** are [ ]

   A) Same B) Discontinuous C) Zero D) Infinity

10. A dielectric material is Isotropic if € does not change with [ ]

    A) Point to point B) E C) V D) Direction

11. The law of refraction is [ ]

    A) $\frac{Tan\,\theta 1}{Tan\,\theta 2} = \frac{\epsilon r1}{\epsilon r2}$ B) $\frac{Tan\,\theta 2}{Tan\,\theta 1} = \frac{\epsilon r1}{\epsilon r2}$ C) $\frac{Tan\,\theta 1}{Tan\,\theta 2} = \frac{\epsilon r2}{\epsilon r1}$ D) $\frac{Tan\,\theta 2}{Tan\,\theta 1} = \frac{\epsilon ro}{\epsilon r1}$

12. The energy density W_m can write [ ]

    A) W=1/2 D.E B) $W=1/2\epsilon E^2$ C) $W=D^2/2\epsilon$ D) All

13. Which is not an example of convection current [ ]

    A) A moving charged belt B) Electronic movement in vacuum tube

C) an electron beam in a television tube C) Electric current flowing in a copper wire

14. Unit of permittivity [ ]

A) F/m B) m/F C) F.m D) F/m^2

15. Dielectric strength is the _______value of electric field at which dielectric breakdown occurs[ ]

A) Maximum B) Minimum C) Zero D) Infinity

16. If no free charges exist at interface then [ ]

A) D_{1n}-D_{2n}= ρ_s B) D_{1n}-D_{2n}= 0 C) D_{1n}-D_{2n}= ∞ D) None

17. A material is said to be a conductor if [ ]

A) □/ω□<<1 B) □/ω□>>1 C) □/ω□=1 D)□/ω□=0

18. If a dielectric material of ε_r =4 is kept in an electric field **E**=3**a**$_x$+2**a**$_y$+**a**$_z$, V/m, find electric susceptibility. [ ]

A) 1 B) 2 C) 3 D)4

19. When an electric field **E** is applied, the force on an electron with charge –e is [ ]

A) **F**=-e**E** B)**F**=e**E** C)**F**=-e/**E** D)**F**=e/**E**

20. ______________is current at a given point trough a unit normal area at that point. [ ]

A) Current density B) Flux density C) Both D) Electric field

21. At boundary condition of two dielectrics D_n1= [ ]

A) D_{n2}/ε B) D_{n2} C) $\varepsilon\ D_{n2}$ D) none

22. At boundary condition of two dielectrics E_t1= [ ]

A) E_{t2}/ε B) E_{t2} C) $\varepsilon\ E_{t2}$ D) None

23. The flux passing through a $2m^2$ area that is normal to the xx-axis at x=4.5m for D=10x $\bar{a}_x$ is [ ]

A) 60 C B) 30 C C) 90 C D) 120 C

24. Dipole moment of two equal & opposite charges separated with equal distance d is [ ]

A) p=Q/d B) p=d/Q C) p=Qd D) None

25. In a capacitor, the conduction current and displacement currents are -------- [ ]

A) Equal B) Zero C) not Equal D) depends on area of capacitor plate

26. The displacement current density is given by [ ]

A) $J_D=\frac{\partial D}{\partial t}$ B) $J_D=-\frac{\partial D}{\partial t}$ C) $J_D=-\frac{\partial B}{\partial t}$ D) $J_D=\frac{\partial B}{\partial t}$

27. Polarization of dielectric materials results in [ ]

A) Production of eddy currents B) Creation of dielectric dipoles

C) Release of protons D) absorption of electrons

28. The unit of Polarization is the same as that of [ ]

A) Electric field density (D) B) electric intensity (E) C) charge D) dielectric flux

29. The Polarization of dielectric material is given by [ ]

A) $P=\varepsilon_r E$ B) $P=(\varepsilon_r-1)E$ C) $P=(\varepsilon_r-1)E\varepsilon_o$ D) $P=(\varepsilon_r-1)\varepsilon_o$

30. The capacitance of an insulated conducting sphere of radius R in vacuum is [ ]

A) $2\pi\varepsilon_0 R$ B) $4\pi\varepsilon_0 R$ C) $4\pi\varepsilon_0 R^2$ D) $4\pi\varepsilon_0/R$

31. The conductivity of an ideal conductor is [ ]

A) Zero B) infinite C) 100C D) 50nF

32. The continuity equation of the current is based on [ ]

A) Conservation of charge B) Conservation of momentum

C) Conservation of motion D) Conservation of velocity

33. Capacitance is measured in _____ [ ]

A) Coulomb/ amp B) amp/Coulomb C) Coulomb/ volt D) volt/ Coulomb

34. The maximum value of applied electric field at which the dielectric break down occurs is called [ ]

A) dielectric field B) dielectric intensity C) dielectric strength D) none

35. Dielectrics can store the energy due to [ ]

A) magnetization B) Polarization C) density D) electrons

36. The conductivity of ideal conductor is [ ]

A) Zero B) infinite C) +250C D) +100C

37. Current density is ___________ [ ]

A) Scalar quantity B) vector quantity C) both D) none

38. In Dielectrics displacement current is under the influence of [ ]

A) Magnetic field B) magnetic field intensity C) electric field D) electric field intensity

39. The phenomena of polarization happen in [ ]

A) Dielectrics B) conductors C) insulators D) none

40. Energy stored in capacitor is____ [ ]

A) $\frac{1}{2} cv^2$ B) $\frac{1}{2} Lv^2$ C) $\frac{1}{2} cI^2$ D) $\frac{1}{2} LI^2$

UNIT –IV

STATIC MAGNETIC FIELDS

1. In steady magnetic field $\nabla \times \vec{H}$ = --- [ ]

A) Zero B) $\vec{J}$ C) $-\frac{\partial B}{\partial t}$ D) $\frac{\partial D}{\partial t}$

2. The line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path is given by ------------- law [ ]

A) Gauss B) Faraday's C) Biot-savart D) Amperes

3. The magnetic force F_m on a moving charge is given by--------------- [ ]

A) F =QE B) F= V×B `C) F= Q V×B D) F=0

4. The Lorentz force equation is given by------------------------- [ ]

A) F =QE B) F= Q (E+V×B) C) F= Q V×B D) none

5. The Maxwell equation in time variant field is given by------------------ [ ]

A) $\nabla \times \vec{H} = \bar{J}$ B) $\nabla \times \vec{H} = \bar{J} + \frac{\partial D}{\partial t}$ C) $\nabla \times \vec{H} = \bar{J} + \frac{\partial E}{\partial t}$ D) $\nabla \times \vec{H} = 0$

6. The faraday's law in differential form is given by [ ]

A) $\nabla \times \vec{E} = \bar{J}$ B) $\nabla \times \vec{E} = \frac{\partial D}{\partial t}$ C) $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$ D) $\nabla \times \vec{E} = \frac{\partial B}{\partial t}$

7. In general magnetic field intensity is directly proportional to [ ]

A) Voltage B) current C) distance D) None

8. In general magnetic field intensity is inversely proportional to [ ]

A) Voltage B) current C) distance D) None

9. A conductor 6m long lies along Z direction with a current of 2A in a direction. Find the force experienced by the conductor if $\vec{B}$ = 0.08 a_x Tesla. [ ]

A) 0.9 a_y B) 0.96 a_y C) 0.96 a_z D) 0.96 a_x

10. The magnetic field intensity at the centre of a long solenoid is--------------- [ ]

A) H= $N\frac{I^2}{l}$ B) $\frac{NI}{l}$ C) $\frac{NI}{l^2}$ D) $\frac{N^2I}{l}$

11. The total magnetic flux coming out of closed surface is----------------------- [ ]

A) infinite B) finite C) zero D)None

12. The MFI due to an infinitely long straight conductor carrying a current I is----- [ ]

A) H= $\frac{I}{2\pi d}$ B) H = $\frac{I}{2d}$ C) H = $\frac{I}{d}$ D) 2dl

13. The line integral of H about any closed path is exactly equal to the ------- enclosed by that path [ ]

A) field B) potential C)current D)None

14. The MFI at the centre of the square current carrying wire is [ ]

A) H= $\frac{I}{a}$ B) H = $\frac{\sqrt{2}I}{a}$ C) H = $\frac{2I}{\pi a}$ D) $\frac{\sqrt{2}I}{\pi a}$

15. The expression for biot-savarts law in integral form is [ ]

A) H= $\int \frac{I.\overline{dl} \times \vec{r}}{4\pi r^2}$ B) H= $\int \frac{I.\overline{dl} \times \vec{r}}{4\pi r^3}$ C) H= $\int \frac{I.\overline{dl} \times \vec{r}}{4\pi r}$ D) $\int \frac{I.\overline{dl} \times \vec{r}}{4r^2}$

16. The Amperes circuital law in integral form is [ ]

A) $\oint \vec{H}.\overline{dl}$ = I B) $\oint \vec{H}.\overline{dl}$ = J C) $\oint \vec{H}.\overline{dl}$ = 0 D) none

17. Point form of Ampere's circuital law is [ ]

A) $\nabla \times \vec{H} = \bar{J}$ B) $\nabla \times \vec{H} = 0$ C) $\nabla \times \vec{B} = \bar{J}$ D) $\nabla \times \vec{H} = 0$

18. The charges is motion produce a------------------------------- [ ]

A) Electric field B) magnetic field C) electro static fields D) None

19. If the particle is at rest in magnetic fields, then it will experience---------------- [ ]

A) Forces B) no forces C) can't say D) none

20. The force on a straight conductor in a magnetic field is given by /F/= [ ]

A) BILsinΘ B) $\vec{F} = I\vec{l} \times \vec{B}$ C) A or B D) none

21. The surface integral of B over a closed surface S in a magnetic field must be [ ]

A) BSCOSΘ B) BS SINΘ C) Zero D) none

22. A differential current loop is carrying current I have a magnetic dipole moment m= [ ]

A) $\frac{I}{A}$ B) IA C) $I^2 A$ D) None

23. Magnetic field intensity interms of magnetic flux density is given as--------------- [ ]

A) $\vec{H} = \mu \vec{B}$ B) $\vec{H} = \frac{\vec{B}}{\mu}$ C) $\vec{H} = \frac{\vec{B}}{\varepsilon\mu}$ D) $\vec{H} = \frac{\vec{B}}{\varepsilon}$

24. The concept of displacement current was a major contribution attributed to [ ]

A) Faraday B) Lenz C) Lorentz D) Maxwell

25. Magnetic fields can exert force on [ ]

A) Moving charges only B) Staionary charges only C) A and B D) None

26. Ampere's law state that the force $\vec{F}$ between two parallel wire carrying current I_1 and I_2 is equal to [ ]

A) $\frac{\mu_0 I_1 I_2}{2\pi d}$ B) $\frac{\mu_0 I_1 I_2}{2\pi d} l$ C) $\frac{\mu_0 I_1 I_2}{2d}$ D) $\frac{\mu_0 I_1 I_2}{2\pi d l}$

27. When a charged particle having charge Q travels with velocity V in magnetic field $\vec{B}$, it will experience a force F_m is given by [ ]

A) $\overrightarrow{F_m} = Q(\vec{V} \times \vec{B})$ B)QVB sinΘ C) A or B D) none

28. The expression for Torque on a current loop placed in a magnetic field is T = [ ]

A) mB sinΘ B) $\vec{m} \times \vec{B}$ C)A or B D) none

29. The unit of magnetic field intensity $\vec{H}$ is ------------------------ [ ]

A) weber B) $\frac{AT}{m}$ C)Tesla D)no units

30. The Curl operator used in ------------fields [ ]

A) Electrostatic B) Magneto static C) both A and B D) none

31. The torque on a magnetic dipole is ($\vec{F} = force\ and\ \vec{R} = moment\ of\ arm$) [ ]

A) $\vec{T} = \vec{R} \times \vec{F}$ B) $\vec{T} = \vec{F} \times \vec{R}$ C) $\vec{T} = \vec{R}.\vec{F}$ D) $\vec{T} = \vec{F}$

32. The MFI at the centre of the circular loop is [ ]

A) H = $\frac{I}{2a}$ B) H = $\frac{I}{a}$ C) L = $\frac{\sqrt{3}}{2a}$ I D) L = $\frac{5I}{2a}$

33. Ampere's law state that the force $\vec{F}$ between two parallel wire carrying current I_1 and I_2 is equal to [ ]

A) $\frac{\mu_0 I_1 I_2}{2\pi d}$ B) $\frac{\mu_0 I_1 I_2}{2\pi d} l$ C) $\frac{\mu_0 I_1 I_2}{2d}$ D) $\frac{\mu_0 I_1 I_2}{2\pi d l}$

34. When a charged particle having charge Q travels with velocity V in magnetic field $\vec{B}$, it will experience a force F_m is given by [ ]

A) $\overrightarrow{F_m} = Q(\vec{V} \times \vec{B})$ B) QVB sinΘ C) A or B D) none

35. The line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current

enclosed by that path is given by ------------- law [ ]

A) Gauss B) Faraday's C) Biot savart D) Amperes

36. In the expression $\vec{B} = \times \bar{A}$, is $\vec{A}$ is called -------------------------- [ ]

A) Area of the field B) vector magnetic potential C) scalar magnetic potentials D) None

37. The expression for biot-savarts law in integral form is [ ]

A) H= $\int \frac{I.\overline{dl} \times \vec{r}}{4\pi r^2}$ B) H= $\int \frac{I.\overline{dl} \times \vec{r}}{4\pi r^3}$ C) H=$\int \frac{I.\overline{dl} \times \vec{r}}{4\pi r}$ D) $\int \frac{I.\overline{dl} \times \vec{r}}{4 r^2}$

38. The faraday's law in integral form is given by [ ]

A) $emf = -\int_s \frac{\partial B}{\partial t}.ds$ B) $emf = \int_s \frac{\partial B}{\partial t}.ds$ C) $emf = -\int_s \frac{\partial D}{\partial t}.ds$ D) none

39. The force of ------------ is experienced between two parallel conductors carrying current in opposite direction. [ ]

A) Attraction B) Repulsion C) Zero D) None

40. The force of ------------ is experienced between two parallel conductors carrying current in same direction. [ ]

A) Attraction B) Repulsion C) Zero D) None

UNIT –V

TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

1. The inductance of a solenoid is given by [ ]

A) L = $\frac{N\mu A}{l}$ B) L = $\frac{N\mu}{l}$ C) L = $\frac{N^2\mu A}{l}$ D) L = $\frac{N^2\mu A}{2\pi R}$

2. The inductance of a Torroid is given by [ ]

A) L = $\frac{N\mu A}{l}$ B) L = $\frac{N\mu}{l}$ C) L = $\frac{N^2\mu A}{l}$ D) L = $\frac{N^2\mu A}{2\pi R}$

3. The divergence of magnetic flux density $\nabla.\vec{B}$ is --------------- [ ]

A) $\nabla.\vec{B} = \rho_v$ B) $\nabla.\vec{B} = -\rho_v$ C) $\nabla.\vec{B} = 0$ D) none

4. What is the energy density in free space on account of field intensity H= 1000A/m? [ ]

A) 0.2 J/m^3 B) 0.628 J/m^3 C) 0.735 J/m^3 D) 0

5. The scalar magnetic potentials satisfy the --------------------- equation [ ]

A) Poisson B) Laplace C) Both A &B D) None

6. The vector magnetic potentials satisfy the --------------------- equation [ ]

A) Poisson B) Laplace C) Both A &B D) None

7. What is the value of permeability constant μ_0 in free space [ ]

A) 8.54×10^{-12} H/m B) $4\pi\times10^{-12}$ H/m C) $4\pi\times10^{-7}$ H/m D) 0

8. The numan's formulae for finding the mutual inductance is given by [ ]

A) M=$\frac{\mu}{4\pi}\iint_{c1c2}\frac{dl_1.dl_2}{r}$ B) M=$\frac{\mu}{4\pi}\iint_{c1c2}\frac{dl_1}{r}$ C) M=$\frac{\mu I}{4\pi}\iint_{c1c2}\frac{dl_1.dl_2}{r}$ D) none

9. If the two coils L_1 and L_2 are connected in series aiding the total inductance is [ ]

A) L_1+L_2 B) L_1+L_2-2M C) L_1+L_2+2M D) $M = \frac{L1L2}{L1+L2}$

10. If the two coils L_1 and L_2 are connected in series opposing the total inductance is [ ]

A) L_1+L_2 B) L_1+L_2-2M C) L_1+L_2+2M D) $M = \frac{L1L2}{L1+L2}$

11. If the two coils L_1 and L_2 are connected in parallel aiding the total inductance is [ ]

A) L_1+L_2 B) L_1+L_2-2M C) $M = \frac{L1L2-M^2}{L1+L2+2M}$ D) $M = \frac{L1L2-M^2}{L1+L2-2M}$

12. If the two coils L_1 and L_2 are connected in parallel opposing the total inductance is [ ]

A) L_1+L_2 B) L_1+L_2-2M C) $M = \frac{L1L2-M^2}{L1+L2+2M}$ D) $M = \frac{L1L2-M^2}{L1+L2-2M}$

13. The energy density in magnetic field is given by [ ]

A) $\frac{1}{2}\mu H^2$ B) $\frac{1}{2}\mu B^2$ C) $\frac{1}{2}\mu H$ D) none

14. The energy stored in magnetic field is given by [ ]

A) $\frac{1}{2}LI$ B) $\frac{1}{2}LI^2$ C) $\frac{1}{2}I^2$ D) none

15. The coefficient of coupling K between two coil is [ ]

A) $K = M\sqrt{L_1L_2}$ B) $K=\frac{M}{\sqrt{L_1L_2}}$ C) $K=\sqrt{\frac{M}{L_1L_2}}$ D) None

16. In free space relative permeability μ_r=------------------- [ ]

A) 0 B) 1 C) infinite D) None

17. What is the unit of Energy density? [ ]

A) Joules B) Weber C) Joules /m^3 D) Weber/m^3

18. in magnetic fields $\nabla.\vec{B}$ is --------------- [ ]

A) $\nabla.\vec{B} = \frac{\rho_v}{\varepsilon}$ B) $\nabla.\vec{B} = -\rho_v$ C) $\nabla.\vec{B} = 0$ D) none

19. The transformer induction equation is given by [ ]

A) emf $=-\oint_s \frac{\partial \vec{B}}{\partial t}$ B) emf $= \oint_s \frac{\partial \vec{B}}{\partial t}$ C) emf $=-\oint_s \frac{\partial \vec{D}}{\partial t}$ D) emf $= \oint_s \frac{\partial \vec{D}}{\partial t}$

20. The emf induced in a coil is directly proportional to [ ]

A) flux B) rate of change of flux C) current D) none

21. Find the coefficient of coupling K between two coil, where $L_1=L_2=M=1H$ [ ]

A) K =1 B) K= 0.5 C) K=2 D) None

22. The inductance of a Torroidal ring is given by [ ]

A) $L = \frac{N\mu A}{l}$ B) $L = \frac{N\mu}{l}$ C) $L = \frac{N^2\mu A}{l}$ D) $L = \frac{N^2\mu A}{2\pi R}$

23. The curl of magnetic field intensity is [ ]

A) $\nabla \times \vec{H} = \vec{J}$ B) $\nabla \times \vec{H} = 0$ C) $\nabla \times \vec{B} = \vec{J}$ D) $\nabla \times \vec{H} = 0$

24. The unit of scalar magnetic potential is [ ]

A) Ampere B) Volt C) Amp/m D) Volt/m

25. Vector magnetic potential exists in regions where **J** is [ ]

A) Absent B) Present C) not related to **J** D) None

26. Vector magnetic potential has applications in [ ]

A) Antennas B) transmission lines C) Microwave ovens D) All

27. Magnetic scalar potential is defined in the region [ ]

A) **J**=0 B)**J**>0 C) **J**<0 D)**E**=0

28. The relation between magnetic flux density **B**and vector magnetic potential **A** is [ ]

A) $\bar{B}=\nabla.\bar{A}$ B) $\bar{A}=\nabla.\bar{B}$ C) $\bar{B}=\bar{A}\,X\,\nabla$ D) $\bar{B}=\nabla\,X\,\bar{A}$

29. If R is the mean radius of toroid with N number of turns and A is the area of cross-section of a toroid then Inductance of toroid is [ ]

A) $L=\frac{\mu NA}{2\pi r}$ B) $L=\frac{\mu NR}{2\pi A}$ C) $L=\frac{\mu N^2 A}{2\pi r}$ D) None

30. If M is the mutual inductance between two magnetically coupled circuits having self-inductances L_1 and L_2 and K is the coefficient of coupling between them then [ ]

A) $M = K\sqrt{L_1L_2}$ B) $K = M\sqrt{L_1L_2}$ C) $M = K\,L_1L_2$ D) None

31. The magnetic field in a solenoid is [ ]

A) H=N/I B) H=n/I C) H=NA/I D) H=I/N

32. A toroid has air core and has a cross-sectional area of 10mm^2. It has 1000 turns and its mean radius is 10 mm. Find its inductance. [ ]

A) 0.02mH B) 0.002mH C) 0.02H D) 0.02mH

33. Energy density in a magnetic field [ ]

A) $W_H=0.5\mu H^2$ B) $W_H=1/2\ \mu H^2$ C) $W_H=1/2\ B.H$ D) All

34. Inductance has equivalent use in magnetics as ____________has in electrostatics, including storage of energy. [ ]

A) Electric filed B) Electric Flux density C) Potential D) Capacitance

35. Self-inductance is defined as the rate of total magnetic flux linkage to the_____________ through the coil. [ ]

A) Current B) energy C) Power D) flux

36. The mutual inductance between two coupled circuit has the property that [ ]

A) $L_{12}>L_{21}$ B) $L_{12}<L_{21}$ C) $L_{12}=L_{21}$ D) $L_{12}\leq L_{21}$

37. If a current of 1.0 amp flowing in an inductor , L=2 henry, the energy stored in an inductance [ ]

A) 2 J B) 1J C) 2J/m D) 0.5J

38. If μ=1.0 μH/m for a medium, H=2.0 A/m, the energy stored in the field is [ ]

A) 0.5 J/m^3 B) 1$\mu J/m^3$ C) 2$\mu J/m^3$ D) 1J/m^3

39. The force produced by B=2wb/m^2on a current element of 2 A-m is [ ]

A) 4 N B) 1 N C) 2 N D) 0.5 N

40. $M_{12}=\frac{N1\ \emptyset 12}{I2}$ is ____________ inductance between two coils [ ]

A) Self B) Mutual C) Series D) Parallel

41. Current passing through the capacitor is called [ ]

A) Conduction current B) Convection current C) Displacement current D) All

42. Electromagnetic fields produced by [ ]

A) Stationary charges B) Steady current C) time-varying currents D) All

43. Except in electrostatics, voltage and potential difference are usually [ ]

A) not equivalent. B) equivalent C) zero D) infinity

44. When a conducting loop is moving in a static B field, an emf is induced in the loop. Such an emf is called as [ ]

A) Motional emf B) flux cutting emf C)Static emf D) a & b

1. In case of time varying fields Gauss law is [ ]

A) Curl H=J+ ∂D/∂t B) Div D= ρ_v C) Div .B = 0 D) Curl E =-∂ B/∂

2. Formula for displacement current [ ]

A) $\partial\mathbf{D}/\partial t$ B) $\mathbf{J}=J+\partial\mathbf{D}/\partial t$ C)$\mathbf{J}=\sigma\mathbf{E}$ D) $\mathbf{J}=\partial\mathbf{B}/\partial t$

3. Who is the founder of electromagnetic theory [ ]

A) Faraday B)Lenz C)Lorentz D)Maxwell

4. A time-harmonic field is one that varies____________with time. [ ]

A) Periodically B) sinusoidally C) non-periodically D)a & b

5. A loop is rotating about the y-axis in a magnetic field B = Ba sin wt$\mathbf{a}_x$Wb/m^2. The voltage induced in the loop is due to [ ]

A) Rotional emf B) Transformer emf

C) A combination of motional and transformer emf D) none of the above

6. The Maxwell's equation $\nabla.\mathbf{B}$=0 is due to [ ]

A) $\mathbf{B}=\mu\mathbf{H}$ B) $B=\mu/\boldsymbol{H}$ C) non-existence of mono pole D) none of these

7. Applications of electromagnetic waves [ ]

A) satellite B) television C)Radars D) All

8. For a uniform plane wave in the x-direction has [ ]

A) $E_x=0$ B) $H_x=0$ C) $E_x=0$ and $H_x=0$ D) $E_z=0$

9. **E.H** of a uniform plane wave is [ ]

A) EH B) 0 C) ηE^2 D) ηH^2

10. The direction of propagation of EM wave is obtained from [ ]

A) **E×H** B) **E.H** C) **E** D) **H**

11. Velocity of the wave in an idle conductor is [ ]

A) Zero B) very large C) moderate D) small

12. Velocity of EM wave in free space is [ ]

A) Independent of frequency (f) B) increase with increase in f

C) Decrease with increase in f D) Zero

13. Pointing vector **P**= [ ]

a) **E×H** B) **E.H** C) ½ **E×H** D) $(\mathbf{E}\times\mathbf{H})^2$

14. Depth of penetration δ= [ ]

A) $1/\beta$ B) $1/\alpha$ C) $1/\gamma$ D) $1/\sigma$

15. In pointing vector **E×H** represents [ ]

A) Electric field per unit area B) magnetic field per unit area

C) power flow per unit area D) All

16. Velocity of EM wave in good dielectric is [ ]

A) $v=\omega/\beta$ B) $v=\omega/\alpha$ C) $v=\omega/\delta$ D) $v=\alpha/\beta$

21. Reciprocal of attenuation constant is called [ ]

A) Skin depth B) pointing vector C) drift current D) displacement current

22. A wave propagating in the +z direction and the wave is called_____ [ ]

A) Forward travelling wave B) backward travelling wave C) wavelength D) none

23. The emf induced in coil is given by [ ]

A) $e=-N\frac{d\emptyset}{dt}$ B) $e=-N\frac{dI}{dt}$ C) $e=-L\frac{dI}{dt}$ D) A and C

24. A wave propagating in the -z direction and the wave is called_____ [ ]

A) Forward travelling wave B) backward travelling wave C) wavelength D) none

25. Skin resistance (Ω/m^2) is defined ____ part of intrinsic impedance for good conductor [ ]

A) Real part B) imaginary part C) zero D) none

26. The field intensity in a conductor rapidly decreases are known as [ ]

A) Skin depth B) skin effect C) pointing field D) wave field

27. Skin depth is also known as [ ]

A) Wave depth B) pointing depth C) penetration depth D) drift current

28. In dielectric medium the displacement current is_____ compared to conduction current [ ]

A) greater B) equal C) lesser D) none

29. The e.m.f is induced in a stationary closed path due to the time varying field is called [ ]

A) Statically induced e.m.f B) dynamically induced e.m.f

C) Motional e.m.f D) none

30. The e.m.f is induced in a stationary closed path due to the static varying field is called [ ]

A) Statically induced e.m.f B) dynamically induced e.m.f

C) Transformer e.m.f D) none

31. Skin Depth δ= [ ]

A) α B) 1/α C)1/β D) β

32. For a time varying fields ∇ X H= _______ [ ]

A) $J + \frac{\partial \vec{B}}{\partial t}$ B) $J + \frac{\partial \vec{D}}{\partial t}$ C) $J + \frac{\partial \vec{E}}{\partial t}$ D) $I + \frac{\partial \vec{D}}{\partial t}$

33. Poynting vector ________ [ ]

A) AXB B) AXE C) EXH D) BXH

34. The induced voltage opposes the flux producing in it is called ________Law [ ]

A) Lenz's B) Faraday's C) Ampere's D) Gauss

35. Time varying fields are due to ____________Charges [ ]

A) Static B) Accelerated C) Decelerated D) Uniform

36. Time varying fields are due to ____________Charges Lenz's [ ]

A) Static B) Accelerated C) Decelerated D) Uniform

37. The induced voltage opposes the flux producing in it is called ________Law [ ]

A) Lenz's B) Faraday's C) Ampere's D) Gauss

38. The induced emf, V_{emf} in any closed circuit is equal to time rate of change of the magnetic flux linkages by the circuit is called ________Law [ ]

A) Gauss's B) Ampere's C) Lenz's D) Faraday's

39. If a moving loop is kept in a static B field, the emf induced is _______. [ ]

A) Rotational B) Motional C) Both D) None of these

40. The ratio of transmitted electric field to incident electric field is called_________ [ ]

A) Transmission B) Reflection C) Both D) None

Prepared by: **J.GOWRISHANKAR AND N.SRAVANTHI**

Q.P. Code: 19EE0207 **R19**

Reg. No: | | | | | | | | | | |

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR

(AUTONOMOUS)

B.Tech II Year II Semester Regular Examinations July-2021

ELECTROMAGNETIC FIELDS

(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 60

(Answer all Five Units **5 x 12 = 60** Marks)

UNIT-I

1	a	Convert point P(1,3,5) from cartesian to cylindrical and spherical co-ordinates system.	L4	6M
	b	Transform the vector field W=10 ax -8 ay +6 az to cylindrical co-ordinate system at point P (10 ,-8, 6)	L1	6M

OR

2	a	Given point P (-2,6,3) and A=y ax +(x+z) ay. Express A in Cylindrical coordinates	L4	6M
	b	Transform the vector A= 3i-2j-4K at P (x=2, y=3, Z=3) to cylindrical coordinates	L4	6M

UNIT-II

3	a	State and explain Coulomb's law indicating clearly the units of quantities in the equation of force?	L1	6M
	b	State and prove Gauss's law and write limitations of Gauss's law?	L2	6M

OR

4	a	Determine the Electric filed intensity at P(-0.2, 0, -2.3) m due to a point charge of 5 nc at Q (0.2,0.1, -2.5) m in air.	L4	6M
	b	An infinitely long uniform line charge is located at y=3, Z=5. If ρL = 30 n C/m, find the filed intensity E at i) origin , ii) P(0,6,1) and iii) P (5,6,1)	L4	6M

UNIT-III

5	a	Derive the continuity equation. What is its physical significance?	L1	6M
	b	Derive the point form of ohms law?	L1	6M

OR

6	a	Derive the expression for parallel plate capacitor and capacitance of a co-axial cable?	L4	6M
	b	A parallel plate capacitor has an area of 0.8 m2 separation of 0.1 mm with a dielectric for which Er = 1000 and a field of 106 V/m. Calculate C and V	L4	6M

UNIT-IV

7	a	Explain maxwell's second equation?	L1	6M
	b	State and explain ampere's circuital law?	L1	6M
		OR		
8	a	A Point charge of Q=-1.2 C has a velocity V=(5 ax +2 ay -3az)m/s. Find the magnitude of the force exerted on the charge if i) E= -18 ax +5 ay -10 az V/m and ii) B=-4 ax +4 ay +3 az T	L4	6M
	b	Determine the force per meter length between two long parallel wires A and B separated by distance 5 cm in air and carrying currents of 40 A in the same direction.	L4	4M

UNIT-V

9	Write Maxwell's equation in good conductors for time varying fields and static fields both in differential and integral form?	L1	12M
	OR		
10	Explain faradays law of electromagnetic induction and there from derive maxwell's equation in differential and integral form?	L1	12M

*** END ***

Q.P. Code: EMF 16EE214 **R16**

Reg. No: | | | | | | | | | | |

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR

(AUTONOMOUS)

B.Tech II Year II Semester Supplementary Examinations July-2021

ELECTROMAGNETIC FIELDS

(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 60

(Answer all Five Units **5 x 12 = 60** Marks)

UNIT-I

1 The three vertices of a triangle are located at A(-1,2,5), B(-4,-2,-3), and C(1,3,-2). (i) Find the length of the perimeter of the triangle. (ii) Find a unit vector that is directed from the mid point of the side AB to the midpoint of the side BC. (iii) Show that this unit vector multiplied by a scalar is equal to the vector from A to C and that the unit vector is therefore parallel to AC . **12M**

OR

2 The surfaces $\rho=3$, $\rho=5$, $\Phi=100o$, $\Phi=130^{O}$, z=3, and z=4.5 define a closed surface. (i) Find enclosed volume; (ii) Find the total area of enclosing surface; (iii) Find the total length of the twelve edges of the surfaces;(iv) Find the length of longest straight line that lies entirely within the volume. **12M**

UNIT-II

3 a State and explain Coulomb's law indicating clearly the units of quantities in the equation of force. **6M**

b Derive the expression for the electric field intensity due to line charge. **6M**

OR

4 a Derive Laplace and Poisson's equation. **6M**

b Derive the expression for torque on electric dipole in the presence of uniform electric field. **6M**

UNIT-III

5 a Derive the expression for capacitance of the spherical condenser. **6M**

b Derive the expression for parallel plate capacitor. **6M**

OR

6 a What is the energy stored in a capacitor made of two parallel metal plates each of 30 cm^2 area separated by 5mm in air? The capacitor is charged to potential difference of 500V **6M**

Given that ε o= $8.854x10^{-12.}$

b i) Define polarization in dielectric materials **6M**

ii) Write the relation between current I and current density.

iii) Write the equation for energy stored in capacitor.

UNIT-IV

7 a Write down maxwell's third equation in point and integral form. **6M**

b Derive the expression for the force between two current carrying wires. **6M**

OR

8 **a** Derive an expression for the force between two current carrying wires. **6M**

b i) Define Magnetic dipole moment. **6M**

ii) Write Lorentz force equation.

iii) State point form of Amperes law.

UNIT-V

9 **a** A toroid has air core and has a cross sectional area of $10mm^2$ it has 1000 turns and its mean radius is 10mm. find its inductance. **6M**

b A coil of 500 turns is wound on a closed iron ring of mean radius 10cm and cross section of 3 cm^2. Find the self inductance of the winding if the relative permeability of iron is 800. **6M**

OR

10 Derive an expression for the force between two straight long and parallel conductors **12M**

*** END ***

Q.P. Code: 18EE0203 **R18**

Reg. No: | | | | | | | | | | |

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR
(AUTONOMOUS)

B.Tech II Year I Semester Supplementary Examinations August-2021

ELECTROMAGNETIC FIELDS

(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 60

PART-A

(Answer all the Questions 5 x 2 = 10 Marks)

1	**a**	What are the types of coordinate system?	**2M**
	b	Define dipole moment.	**2M**
	c	Write the relation between current I and current density J.	**2M**
	d	What is the inductance of Solenoid?	**2M**
	e	Define skin depth.	**2M**

PART-B

(Answer all Five Units 5 x 10 = 50 Marks)

UNIT-I

2 The surfaces $\rho=3$, $\rho=5$, $\Phi=100^o$, $\Phi=130^o$, $z=3$, and $z=4.5$ define a closed surface. (a) Find enclosed volume; (b) Find the total area of enclosing surface; (c) Find thetotal length of the twelve edges of the surfaces; (d) Find the length of longest straight line that lies entirely within the volume. **10M**

OR

3 Three vectors extending from the origin are given as r1 = (7,3,-2), r2=(-2,7,-3) and r3=(0,2,3). Find: (i) a unit vector perpendicular to both r1 and r2 ; (ii) a unit vector perpendicular to the vectors r1-r2 and r2-r3 ; (iii) The area of the triangle defined by r1 and r2; (iv) The area of the triangle defined by the heads of r1,r2, and r3. **10M**

UNIT-II

4 **a** Derive the expression for electric field intensity at a point due to electric dipole. **5M**

b Derive Maxwell first equation. **5M**

OR

5 Four point charges each of 10μC are placed in free space at the points (1, 0, 0),(-1, 0, 0), (0, 1,0) and (0, -1, 0) m respectively. Determine the force on a point charge of 30μC located at a point (0, 0, 1) m? **5M**

UNIT-III

6 **a** Derive the expression for capacitance of a co-axial cable. **5M**

b A parallel plate capacitor has a plate area of 1.5m2 and a plate separation of 5mm. Three are two dielectrics in between the plates. The first dielectric has a thickness of 3mm with a relative permittivity of 6 and the second has a thickness of 2mm with a relative permittivity of 4. Find the capacitor? **5M**

OR

7 A parallel plate capacitor consists of two square metal plates with 500mm side and separated by 10mm. a slab of sulphur (εr= 4) 6mm thick is placed on the lower plate and air gap of 4mm. find capacitance of capacitor? **10M**

UNIT-IV

8 **a** A circular loop is located on X2+Y2=9 and Z=0 carries a direct current of 10A along direction. Determine H at (0, 0, 5) m. **6M**

b State and explain ampere`s circuital law. **4M**

OR

9 Derive an expression for the force between two straight long and parallel conductors. **10M**

UNIT-V

10 Write Maxwell's equation in good conductors for time varying fields and static fields both in differential and integral form. **10M**

OR

11 Explain faradays law of electromagnetic induction and there from derive Maxwell's equation in differential and integral form. **10M**

END

SIDDHARTH INSTITUTE OF ENGINEERING AND TECHNOLOGY: PUTTUR-517 583

Descriptive Test-I

Year &Sem: **II B.Tech II-Semester** Branch : **EEE**

Subject: **ELECTRO MAGNETIC FIELDS (19EE0207)**

Date & Time: **19-07-2021& 09.30 to 11.00 AM** Max. Marks: **30 M**

Answer All the Questions

All Questions Carry Equal Marks

1. Point P and Q are located at (0,2,4) and (-3,1,5) calculate: 1. The Position vector P , 2. The distance vector from P and Q, 3. The distance between P and Q and 4. A vector parallel to PQ with magnitude of 10. **[L4][CO1]][10M]**

OR

2. Express vector B in cartesian and cylindrical systems. Given B= 10/r ar + r cosθ aθ+ aϕ Find the B at (-3,4,0) and (5, π/2,-2) **[L4][CO1][10M]**

3. Three concentrated charges of 0.25μC are located at the vertices of an equilateral triangle of 10 cm side . Find the magnitude and direction of the force on one charge due to other two charges. **[L4][CO2][10M]**

OR

4. a) The potential field in free space is given by V=(50/r) , a<r< b (spherical) show that ρ_v=0 for a<r<b and find the energy stored in the region a<r<b **[L1][CO2][5M]**

b) Two pint charges 1.5nC at (0,0,0.1) and -1.5nC at (0,0,-0.1) are in free space . Treat the two charges as a dipole at the origin and find the potential at p(0.3,0,0.4) **[L4][CO2][5M]**

5. a) In cylindrical coordinates J=10 e^{-100r} a_ϕ A/m^2. Find the current crossing through the region 0.01<r<0.02 m and 0<z<1 m and intersection of this region with the ϕ = constant plane **[L1][CO3][5M]**

b) An aluminium conductor is 2000 ft long and has a circular cross section with a diameter of 20 mm. If there is a DC voltage of 1.2 V between the ends . Find a) The current density b) The current , C power dissipated form the l=knowledge of circuit theory. Assume σ=3.82 *10^7 mho/m for aluminium **[L1][CO3][5M]**

OR

6. Explain the boundary conditions between conductor and free space. **[L1][CO3][10M]**

Paper set by: **J.GOWRISHANKAR**

SIDDHARTH INSTITUTE OF ENGINEERING AND TECHNOLOGY: PUTTUR-517 583
Descriptive Test-I
Year &Sem: **II B.Tech II-Semester** Branch : **EEE**
Subject: **ELECTRO MAGNETIC FIELDS (19EE0207)**
Date & Time: **19-07-2021& 09.30 to 11.00 AM** Max. Marks: **30 M**

Answer All the Questions
All Questions Carry Equal Marks

1.		If B= y ax + (x+z) ay and a point Q is located at (-2, 6, 3) express.1 The Point Q in cylindrical and spherical co-ordinates and 2) B in spherical coordinates.	**[L4][CO1][10M]**
		OR	
2.		Find the gradient of the following scalar fields i) V= e^{-z} sin 2x coshy , ii) U= $r^2 z\cos\phi$ and iii) W= $10r\sin^2\theta\cos\phi$	**[L4][CO1][10M]**
3.	**a)**	Determine the Electric field intensity at P(-0.2, 0, -2.3) m due to a point charge of 5 nc at Q (0.2,0.1, -2.5) m in air	**[L4][CO2][5M]**
	b)	An infinitely long uniform line charge is located at y=3, Z=5. If ρ_L = 30 n C/m, find the field intensity E ati) origin , ii) P(0,6,1) and iii) P (5,6,1)	**[L4][CO2][5M]**
		OR	
4.		The Electric flux density is given as D= (r/4) ar n C/m^2 in free space. Calculate: The Electric field intensity at r=0.25 m , The total charge within a sphere of r=0.25 m	**[L1][CO2][10M]**
5.		Explain the boundary conditions of two perfect dielectrics materials.	**[L4][CO3][10M]**
		OR	
6.		Explain the phenomenon of polarization when a dielectric slab is subjected to an electric field.	**[L1][CO3][10M]**

Paper set by: **J.GOWRISHANKAR**

SIDDHARTH INSTITUTE OF ENGINEERING AND TECHNOLOGY: PUTTUR-517 583

Descriptive Test-I

Year &Sem: **II B.Tech II-Semester** Branch : **EEE**

Subject: **ELECTRO MAGNETIC FIELDS (19EE0207)**

Date & Time: **19-07-2021& 09.30 to 11.00 AM** Max. Marks: **30 M**

Answer All the Questions

All Questions Carry Equal Marks

1.		Determine the curl of the vector fields: i).P=x^2yz ax +xzaz , ii) Q= r sinϕar+r^2z aϕ+ z cosϕaz and iii) T= (1/r^2 cosθa_r + r sinθcosϕ aθ+Cosθaϕ	**[L4][CO1][10M]**
		OR	
2.	**a)**	Convert point P (1, 3, 5) from Cartesian to cylindrical and spherical co-ordinates system.	**[L4][CO1][5M]**
	b)	Given the two points A (X=2, Y=3, Z=-1) and B= (r=4, θ=25°, and ϕ=120) .Find the sphericalco-ordinates of A and Cartesian co-ordinates of B	**[L4][CO1][5M]**
3.		Given that A= 30 $e^{-r}a_r$-2 z a_z in the cylindrical co-ordinates. Evaluate both sides of the divergence theorem for the volume enclosed by r=2, z=0 and Z=5	**[L1][CO2][10M]**
		OR	
4.		Line charge density ρ_L= 24 n C/m is located in free space on the line y=1 and Z=2m (i) Find E at the point P(6,-1,3) , ii) What point charge Qa should be located at A (-3,4,1) to make y component of total E zero at point P?	**[L4][CO2][10M]**
5.	**a)**	Find the magnitude of D and P for a dielectric material in which E=0.15 mV/m and χ=4.25	**[L1][CO3][5M]**
	b)	Find the polarization in dielectric material with ε_r =2.8 if D=3*10-7 C/m2	**[L1][CO3][5M]**
		OR	
6.	**(a)**	What is the relation between electric flux density and electric field intensity?	**[L1][CO2][2M]**
	(b)	Define dipole moment.	**[L1][CO2][2M]**
	(c)	Define an electric dipole.	**[L1][CO2][2M]**
	(d)	State vector form of coulombs law.	**[L1][CO2][2M]**
	(e)	Derive Maxwell second equation.	**[L1][CO3][2M]**

Paper set by: **J.GOWRISHANKAR**

SIDDHARTH INSTITUTE OF ENGINEERING AND TECHNOLOGY: PUTTUR-517 583	
Descriptive Test-I	
Year &Sem: **II B.Tech II-Semester**	Branch : **EEE**
Subject: **ELECTRO MAGNETIC FIELDS (19EE0207)**	
Date & Time: **19-07-2021& 09.30 to 11.00 AM**	Max. Marks: **30 M**

Answer All the Questions

All Questions Carry Equal Marks

1. a) Determine the curl of the vector fields: i).$P=x^2yz\ ax\ +xzaz$, ii) $Q= r\ \sin\phi ar+r^2z\ a\phi+ z\ \cos\phi az$ and iii) $T= (1/r^2 \cos\theta a_r + r \sin\theta\cos\phi\ a\theta+\text{Cos}\theta a\phi$ **[L4][CO1][5M]**

OR

2. a) Given the two coplanar vectors $A= 3\ a_x + 4\ a_y- 5\ a_z$ and $B= -6\ a_x +2\ a_y +4\ a_z$. Obtain the unit vector normal to the plane containing the vector A and B **[L4][CO1][5M]**

b) The Three fields are given by $A=2\ a_x-a_z$, $B= 2\ a_x- a_y+2a_z$, $C= 2\ a_x-3\ a_y +a_z$. Find the scalar and vector triple product. **[L1][CO1][5M]**

3. a) Find E at (0,0,2) m due to charged circular disc in x-y plane with ρ_S = 20 nC/m2 and radius 1m **[L4][CO2][5M]**

b) A circular disc of 10 cm radius is charged uniformly with total charge of 100μc Find E at a point 20cm on its axis. **[L4][CO2[5M]**

OR

4. a) The potential field in free space is given by $V=(50/r)$, a<r< b (spherical) show that ρv=0 for a<r<b and find the energy stored in the region a<r<b **[L1][CO2][10M]**

b) Two pint charges 1.5nC at (0,0,0.1) and -1.5nC at (0,0,-0.1) are in free space . Treat the two charges as a dipole at the origin and find the potential at p(0.3,0,0.4) **[L1][CO2][5M]**

5. a) Find the magnitude of D and P for a dielectric material in which E=0.15 mV/m and χ=4.25 **[L1][CO3][5M]**

b) Find the polarization in dielectric material with ε_r =2.8 if $D=3*10^{-7}$ C/m^2 **[L1][CO3][5M]**

OR

6. Explain the boundary conditions of two perfect dielectrics materials. **[L1][CO3][10M]**

Paper set by: **J.GOWRISHANKAR**

SIDDHARTH INSTITUTE OF ENGINEERING AND TECHNOLOGY: PUTTUR-517 583	
Descriptive Test-II	
Year &Sem: **II B.Tech II-Semester**	Branch : **EEE**
Subject: **ELECTRO MAGNETIC FIELDS (19EE0207)**	
Date & Time: **03-08-2021& 09.30 to 11.00 AM**	Max. Marks: **30 M**

Answer All the Questions
All Questions Carry Equal Marks

1.		Two parallel conducting disc are separated by distance 5 mm at z=0 and z=5 mm .If V=0 and V=100 v at z=5 mm, find the charge densities on the disc.	**[L4][CO3][10M]**
		OR	
2.	**a)**	Derive the expression for parallel plate capacitor and capacitance of a co-axial cable	**[L1][CO3][6M]**
	b)	A parallel plate capacitor has an area of 0.8 m2 separation of 0.1 mm with a dielectric for which ε_r = 1000 and a field of 10^6 V/m. Calculate C and V.	**[L2][CO3][4M]**
3.		In cylindrical co-ordinates A=50 r^2 a_Z wb/m is a vector magnetic potential in a certain region of free space. Find H, B, J and using J find the total current I crossing the surface $0<r<1, 0<\phi<2\pi$ and Z=0.	**[L4][CO4][10M]**
		OR	
4.		Derive the expression for self-inductance of solenoid, toroid and coaxial cable	**[L1][CO4][10M]**
5.		Derive expressions for integral and point forms of poynting Theorem	**[L1][CO5][10M]**
		OR	
6.		Write Maxwell's equation in good conductors for time varying fields and static fields both in differential and integral form.	**[L1][CO5][10M]**

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Answer All the Questions
All Questions Carry Equal Marks

1. Find V at P (2,1,3) for the field of two coaxial conducting cones, with V=50 V at θ=30 and V=20 V at θ=50. **[L4][CO3][10M]**

OR

2. Evaluate both sides of the stokes theorem for the filed H=6xy a_x -3y^2 a_y A/m and the rectangular path around the region 2<x<5, -1<y<1 , Z=0 . Let the positive direction of ds be a_z. **[L4][CO4][10M]**

3. A rectangular loop in Z=0 plane has corners at (0,0,0), (1,0,0),(1,2,0) and (0,2,0). The loop carries a current of 5 A in a_x direction. Find the total force and torque on the loop produced by the magnetic field B=2 a_x+2a_y- 4a_z wb/m2 **[L4][CO4][10M]**

OR

4. Using Biot-savart's law. Find H and B due conductor of finite length. **[L1][CO5][10M]**

5. Explain faradays law of electromagnetic induction and there from derive maxwell's equation in differential and integral form. **[L1][CO6][10M]**

OR

6. Derive an expression for motional and transformer induced emf. **[L1][CO6][10M]**

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Date & Time: **03-08-2021& 09.30 to 11.00 AM** Max. Marks: **30 M**

Answer All the Questions

All Questions Carry Equal Marks

1	**a)**	Determine whether or not the following potential fields satisfy the Laplace's equation i) $V=x^2 - y^2 + z^2$ and ii) $V= r\cos\phi+z$	**[L4][CO3][5M]**
	b)	Derive Laplace's and Poisson's Equation.	**[L4][CO3][5M]**
		OR	
2.		In cylindrical co-ordinates $A=50\, r^2\, a_z$ wb/m is a vector magnetic potential in a certain region of free space. Find H, B, J and using J find the total current I crossing the surface $0<r<1$, $0<\phi<2\pi$ and $Z=0$.	**[L3][CO4][10M]**
3.	**a)**	Explain maxwell's second equation?	**[L1][CO4][5M]**
	b)	State and explain ampere's circuital law?	**[L1][CO4][5M]**
		OR	
4.	**a)**	Calculate the inductance of a 10 m length of coaxial cable filled with a material for which $\mu r = 80$ and radii inner and outer conductors are 1 mm and 4 mm respectively.	**[L3][CO5][5M]**
	b)	A straight long wire is situated parallel to one side of a square coil. Each side of the coil has a length of 10 cm. The distance between straight wire and the center of the coil is 20 cm. Find the mutual inductance of the system .	**[L4][CO5][5M]**
5.		Derive the equation of Continuity for time varying fields	**[L1][CO6][10M]**
		OR	
6.	**(a)**	Define skin depth	**[L1][CO6][2M]**
	(b)	Define displacement current.	**[L1][CO5][2M]**
	(c)	State Faraday's law of electromagnetic induction.	**[L1][CO6][2M]**
	(d)	Write Maxwell equations in time varying fields	**[L1][CO4][2M]**
	(e)	Define pointing vector	**[L1][CO6][2M]**

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Answer All the Questions

All Questions Carry Equal Marks

1.	**a)**	Derive the expression for parallel plate capacitor and capacitance of a co-axial cable	**[L4][CO3][6M]**
	b)	A parallel plate capacitor has an area of 0.8 m2 separation of 0.1 mm with a dielectric for which ε_r = 1000 and a field of 10^6 V/m. Calculate C and V.	**[L4][CO3][4M]**
		OR	
2.	**a)**	Determine whether or not the following potential fields satisfy the Laplace's equation i) $V=x^2 -y^2+z^2$ and ii) $V= r\cos\phi+z$	**[L4][CO3][5M]**
	b)	Derive Laplace's and Poisson's Equation.	**[L2][CO3][5M]**
3.	**a)**	Calculate the inductance of a solenoid of 200 turns wound tightly on a cylindrical tube of 6 cm diameter. The length of the tube is 60 cm and the solenoid is in air.	**[L4[CO4][5M]**
	b)	Find inductance per unit length of a co-axial cable if radius of inner and outer conductors is 1 mm and 3 mm respectively. Assume relative permeability unity.	**[L4][CO4][5M]**
		OR	
4.	**a)**	Find the flux passing through the portion of the plane $\phi=\pi/4$ defined by $0.01<r<0.05$m and $0<z<2$m.A current filament of 2.5A is along the z- axis in the direction of a_z direction in free space.	**[L4][CO5][5M]**
	b)	In cylindrical coordinates B= (2.0/r) $a\phi$ tesla. Determine the magnetic flux ϕ crossing the plane surface defined by $0.5<r<z<2$m. and $0<z<2$m.	**[L4][CO5][5M]**
5.		Derive an expression for motional and transformer induced emf?	**[L1][CO6][10M]**
		OR	
6.		Derive expressions for integral and point forms of poynting Theorem?	**[L1][CO6][10M]**

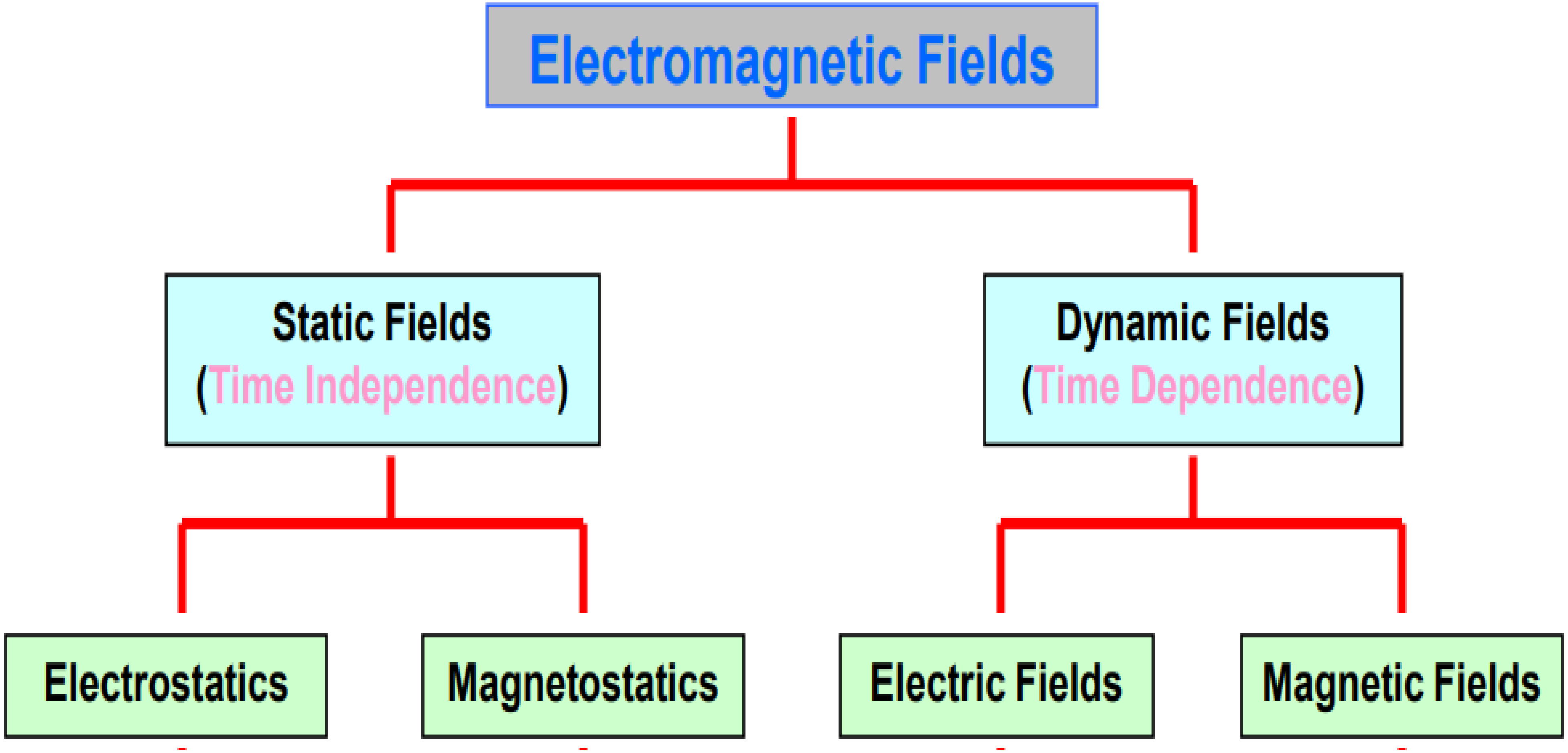
Paper set by: **J.GOWRISHANKAR**

Electromagnetic Field

Dr.J.Gowrishankar
Professor, Department of EEE,
SIETK, PUTTUR

"Our thoughts and feelings have electromagnetic reality. Manifest wisely."

Brief Flow Chart for Electromagnetic Study



Introduction

- Electromagnetics is the study of the effect of charges at rest and charges in motion.
  Some special cases of electromagnetics:
  - ✓ Electrostatics: charges at rest
  - ✓ Magnetostatics: charges in steady motion (DC)
  - ✓ Electromagnetic : charges in time-varying motion (AC)
- Fundamental vector field quantities in electromagnetics:
  - ✓ Electric field intensity - E
    units = volts per meter (V/m)
  - ✓ Electric flux density (electric displacement) -
    D units = coulombs per square meter (C/m^2)
  - ✓ Magnetic field intensity - H
    units = amps per meter (A/m)
  - ✓ Magnetic flux density- B
    units = teslas = webers per square meter ($T = Wb/m^2$)

Basis of Electromagnetic Laws

	Description	Study Chapter	Critical Experiment
Gauss's Law (electric fields)	Charge and Electric fields	Electrostatic / Electric fields	Charge repel and attract different charges. Comply with the inverse square law. Charge on an insulated conductor moves outward surface.
Gauss's Law (magnetic fields)	Magnetic fields	Magnetostatic Magnetic fields	Confirmed that only exist magnetic dipole. (naturedly no magnetic monopole)
Faraday's Law	The electrical effect of a changing magnetic fields	Magnetic induction	The bar magnet is pushed through a closed loop of wire will produce a current in the loop.
Ampere's Law	The magnetic effect of a changing electric fields	Magnetic fields	The current in the wire produces a magnetic field close to the wire.

- Lode stone 400BC, Compass 200BC
- Static electricity, Greek, 400 BC
- Ampere's Law 1823;
- Faraday Law 1838;
- KCL/KVL 1845
- Telegraphy (Morse) 1837;
- Electrical machinery (Sturgeon) 1832;

- Pinhole camera, 400BC, Mozi,
- Ibn Sahl, refraction 984;
- Snell, 1621;
- Huygens/Newton 1660;
- Fresnel 1814;
- Kirchhoff 1883;

  - Maxwell's equations 1864/1865;
  - Heaviside, Hertz, Rayleigh, Sommerfeld, Debye, Mie, Kirchhoff, Love, Lorentz (plus many unsung heroes);
  - Quantum electrodynamics 1927 (Dirac, Feynman, Schwinger, Tomonaga);
  - Electromagnetic technology;

- Nano-fabrication technology;
- Single-photon measurement;
- Quantum optics/Nano-optics 1980s;
- Quantum information/Bell's theorem 1980s;
- Quantum electromagnetics/optics (coming).

Why is Electromagnetics Difficult?

Electric and Magnetic Field:

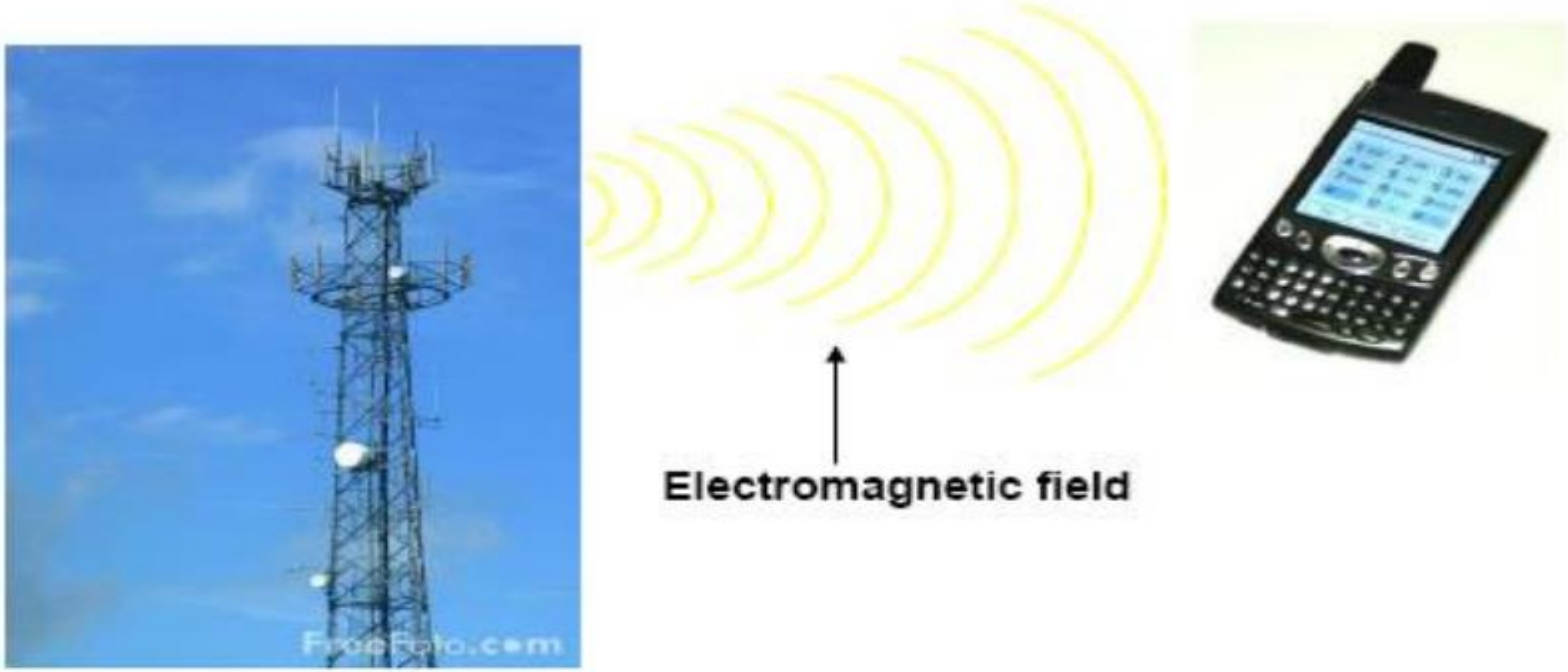
- are 3-dimensional !
- are vectors !
- vary in space and as well as in time !
- are governed by PDEs (partial differential equations)

Therefore →

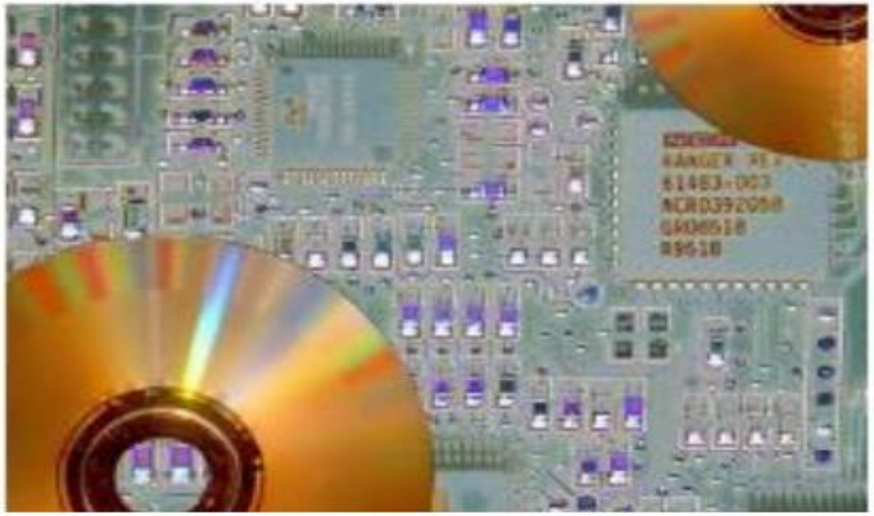
- Solution of electromagnetic problems requires a high level of abstract thinking !
- Students must develop a deep physical understanding !

Math is just a powerful tool !

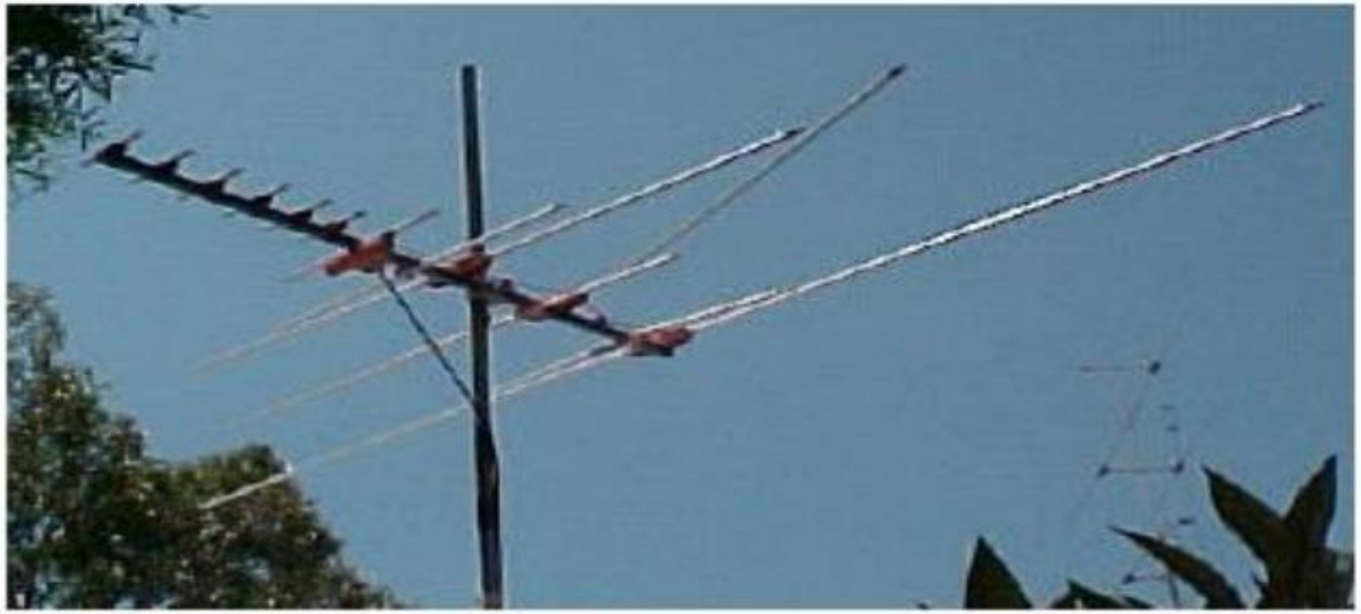
Communication Technology

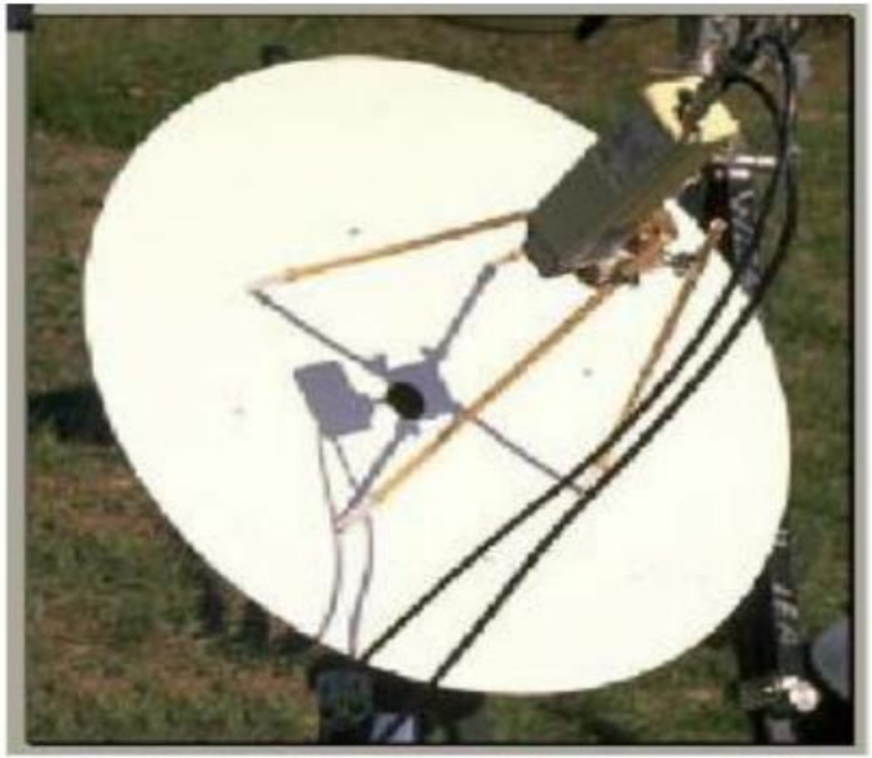


Computer Technology



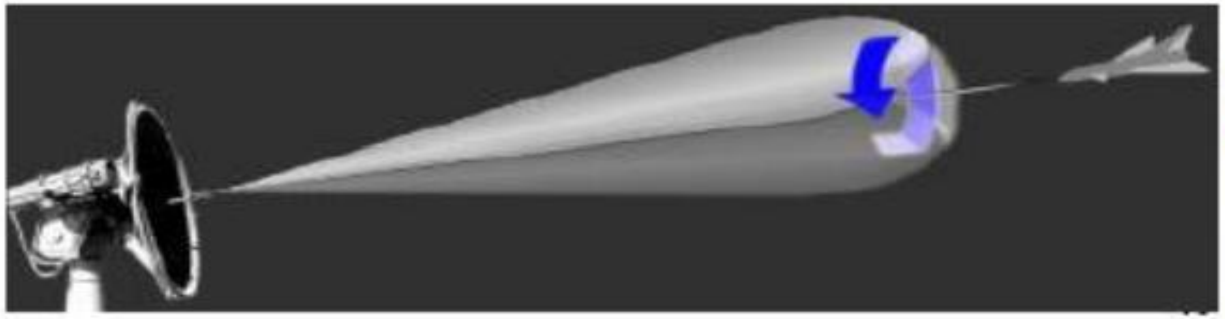
Antenna Technology

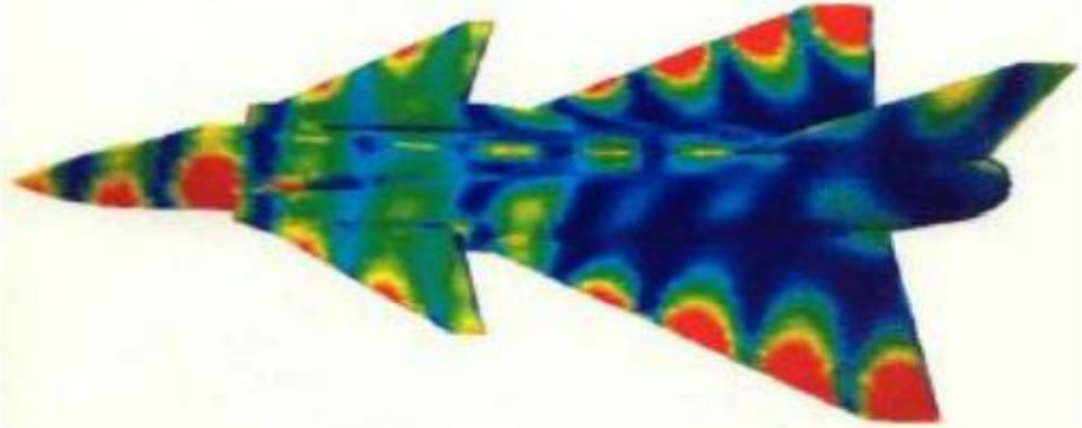




Military and Defense Applications

Radars

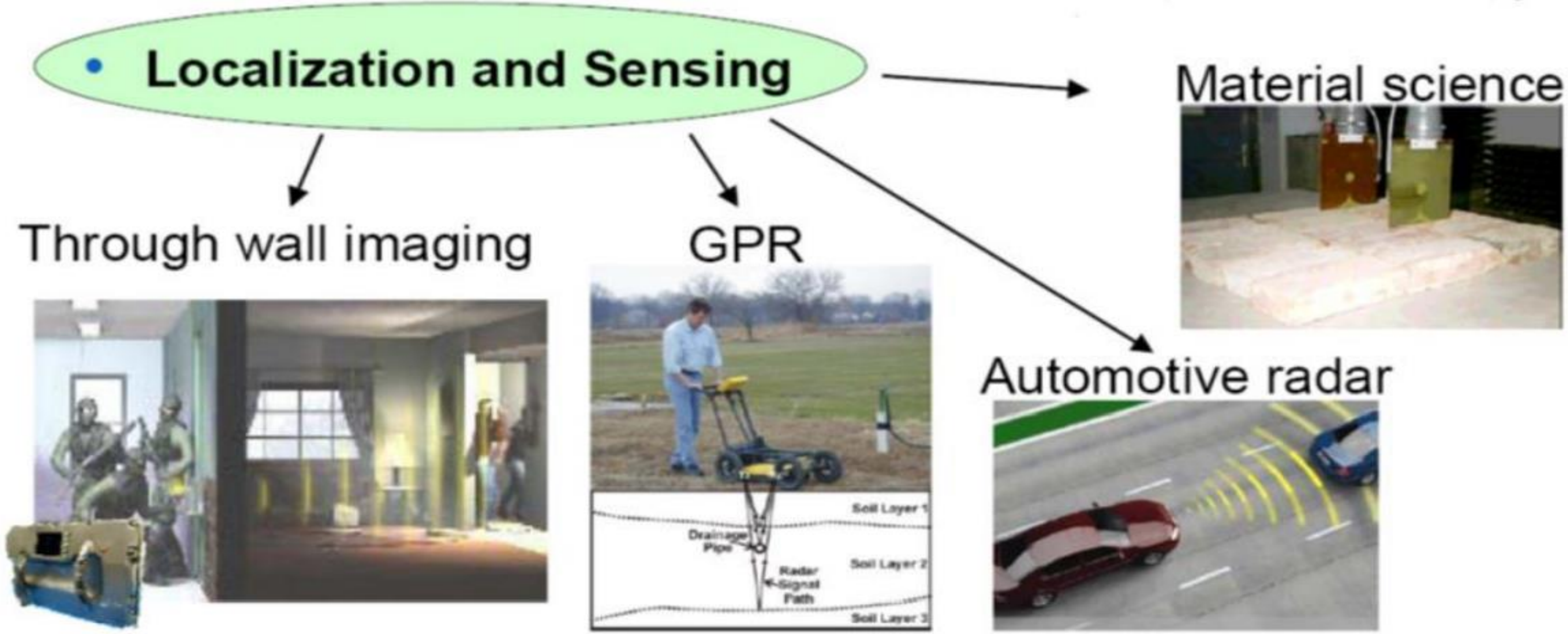


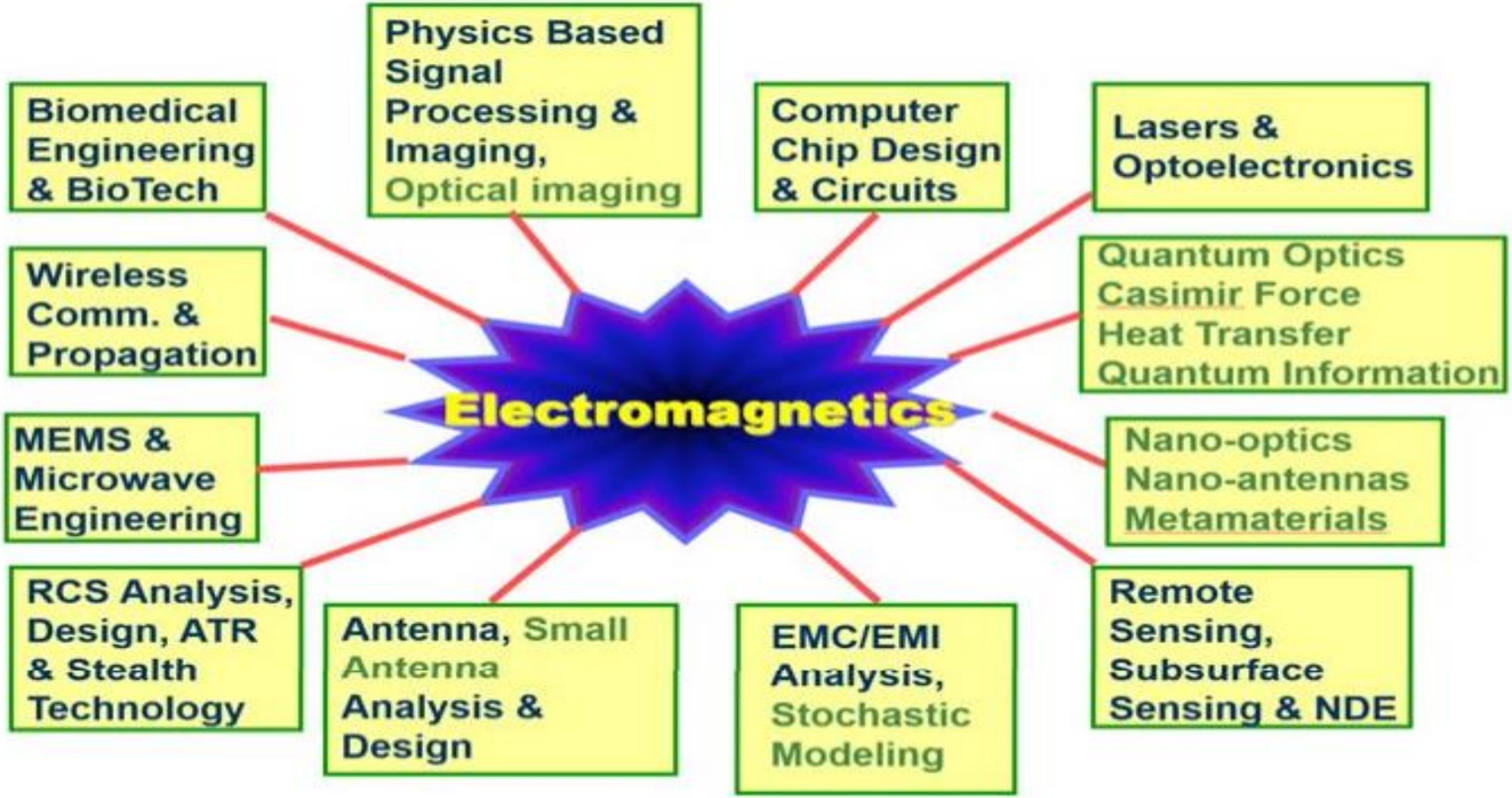


Transportation

- Levitated trains: Maglev prototype

Localization and Sensing

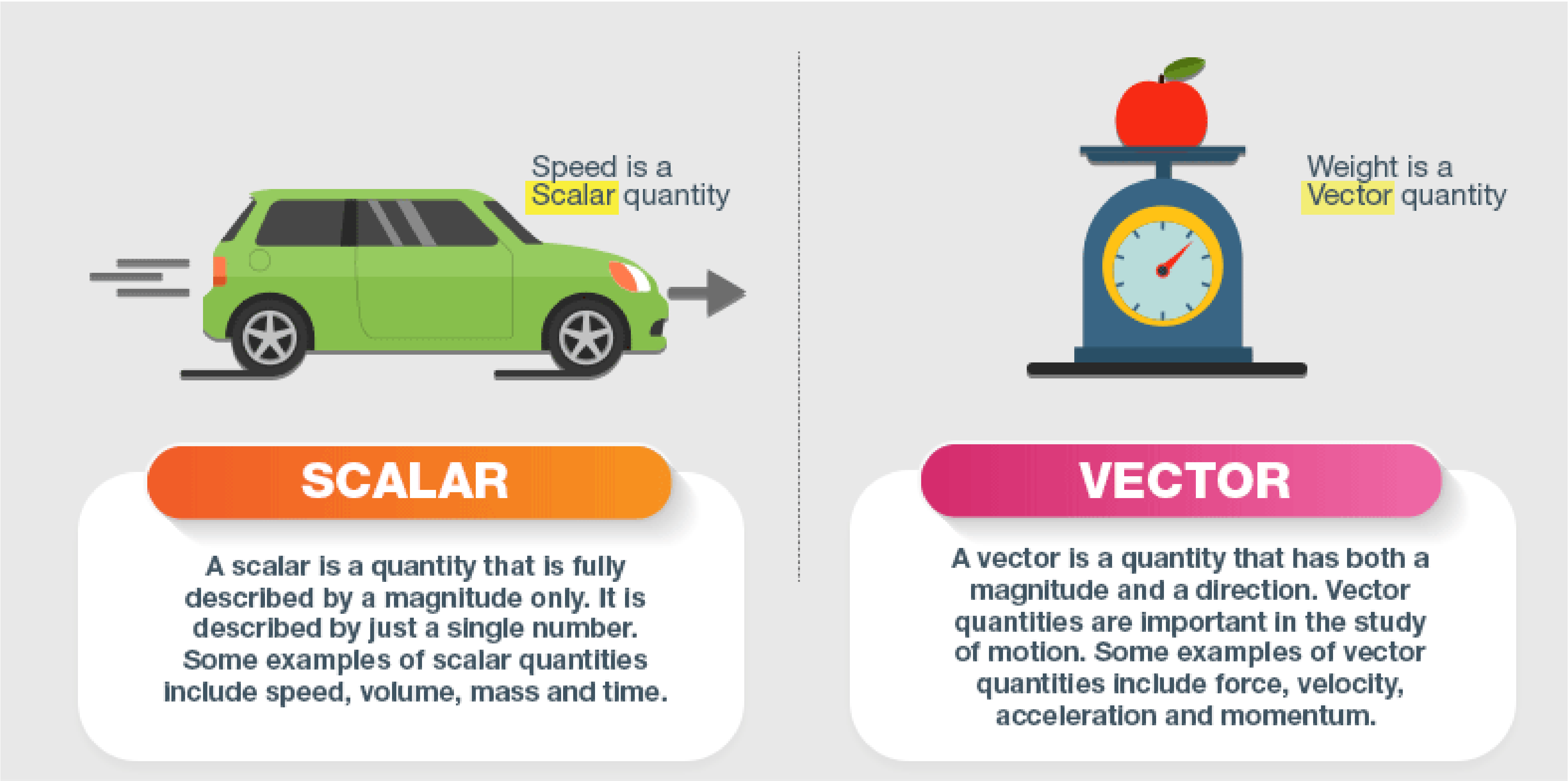



Electromagnetics
Biomedical Engineering & BioTech
Physics Based Signal Processing & Imaging, Optical imaging
Computer Chip Design & Circuits
Lasers & Optoelectronics
Wireless Comm. & Propagation
Quantum Optics Casimir Force Heat Transfer Quantum Information
MEMS & Microwave Engineering
Nano-optics Nano-antennas Metamaterials
RCS Analysis, Design, ATR & Stealth Technology
Antenna, Small Antenna Analysis & Design
EMC/EMI Analysis, Stochastic Modeling
Remote Sensing, Subsurface Sensing & NDE

- When an event in one place has an effect on something at a different location, we talk about the events as being connected by a "field".
- A ***field*** is a spatial distribution of a quantity; in general, it can be either *scalar* or *vector* in nature.

- transmitter and receiver are connected by a "field."

Scalar and Vector



Vector Addition

Q: Say we **add** two vectors $\vec{A}$ and $\vec{B}$ together; what is the **result**?

A: The addition of two vectors results in **another vector**, which we will denote as $\vec{C}$. Therefore, we can say:

$$\boxed{\vec{A}+\vec{B}=\vec{C}}$$

The **magnitude** and **direction** of $\vec{C}$ is determined by the **head-to-tail rule**.

This is not a **provable** result, rather the head-to-tail rule is the **definition** of vector addition. This definition is used because it has many **applications** in physics.

Some important properties of vector addition:

1. Vector addition is **commutative:** $\vec{A}+\vec{B}=\vec{B}+\vec{A}$
2. Vector addition is **associative:** $(\vec{X}+\vec{Y})+\vec{Z}=\vec{X}+(\vec{Y}+\vec{Z})=\vec{K}$

From these two properties, we can conclude that the addition of **several** vectors can be executed in **any order**

Basic Law of Vector (1)

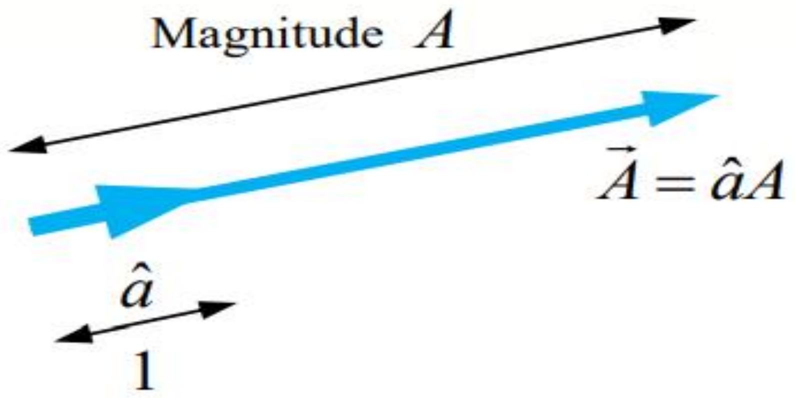
1. A vector $\vec{A}$ has a

   a) magnitude $A = |\vec{A}|$

   b) direction specified by a unit vector $\hat{a}$

2. A vector $\vec{A}$

$$\vec{A} = \hat{a}|\vec{A}| = \hat{a}A$$

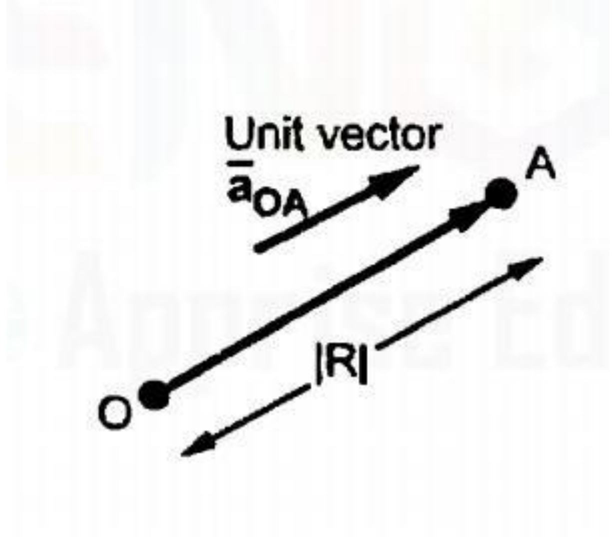


3. The unit vector $\hat{a}$ is given by

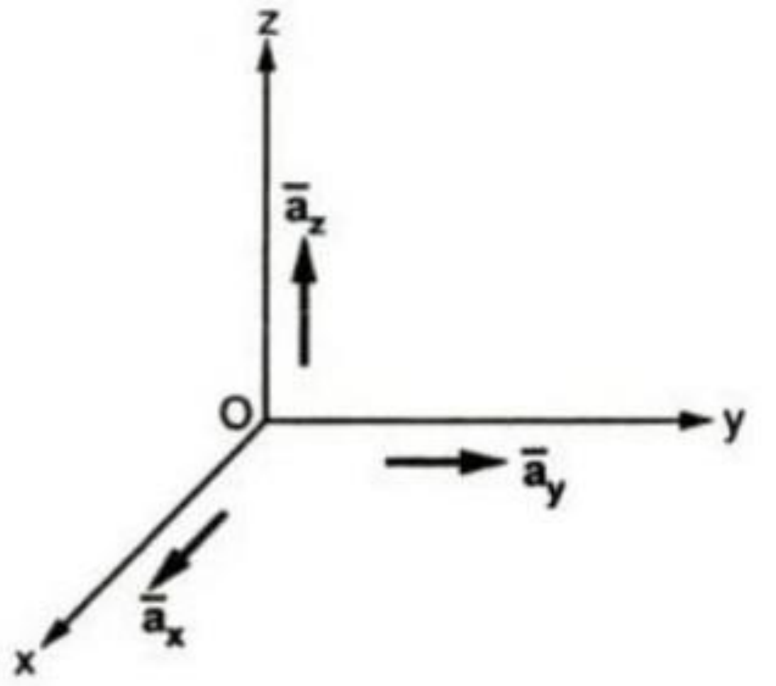
$$\hat{a} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$

Unit Vector / Direction Vector

A **unit vector** is a vector with a magnitude of one unit.
Any vector can be expressed as a scalar multiple of its unit vector.



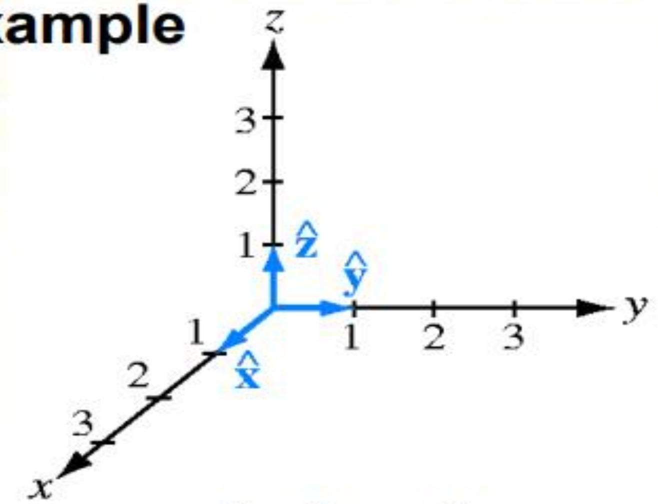
$$\text{Unit vector } \bar{a}_{OA} = \frac{\overline{OA}}{|\overline{OA}|}$$



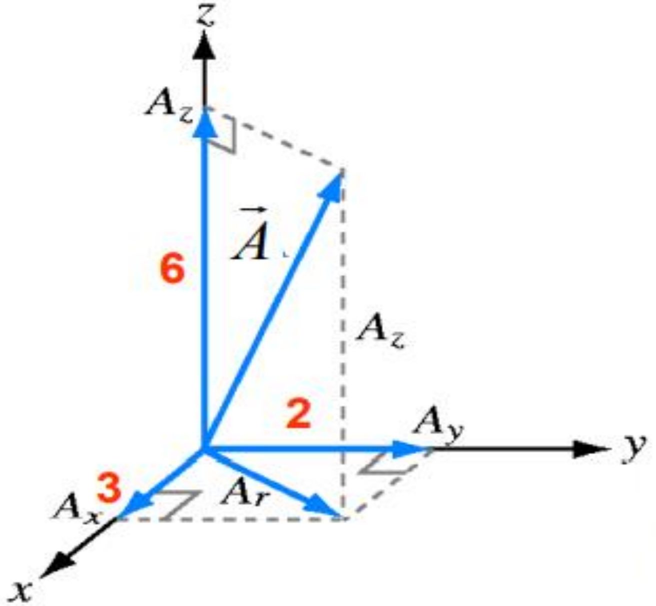
Unit vectors in cartesian system

Basic Law of Vector (2)

Example



Basic vector



Components of vector $\vec{A}$

The vector $\vec{A}$ may be represented as:

$$\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z \qquad \vec{A} = \hat{x}3 + \hat{y}2 + \hat{z}6$$

The magnitude of $\vec{A}$

$$A = \left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$A = \sqrt{3^2 + 2^2 + 6^2} = 7$$

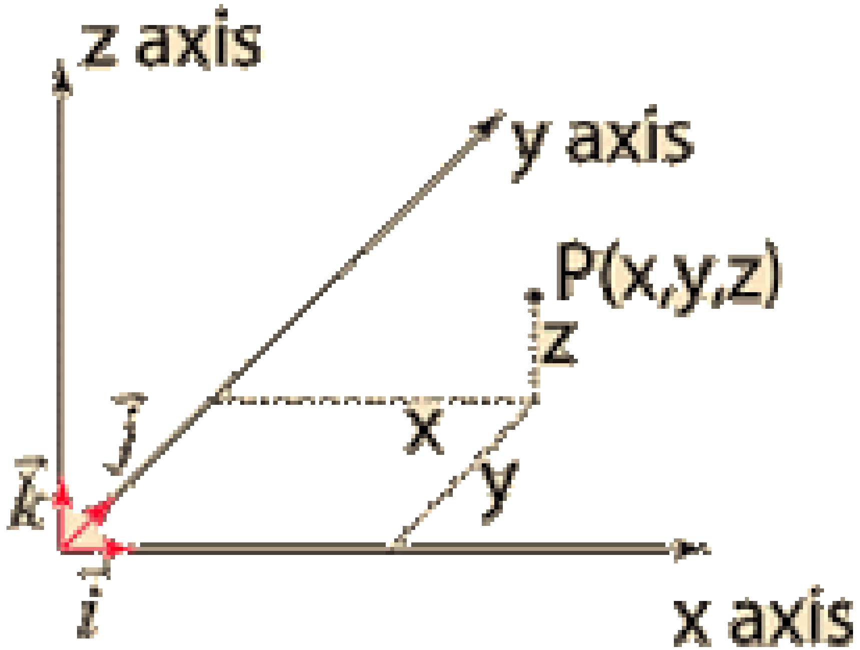
The direction of $\vec{A}$

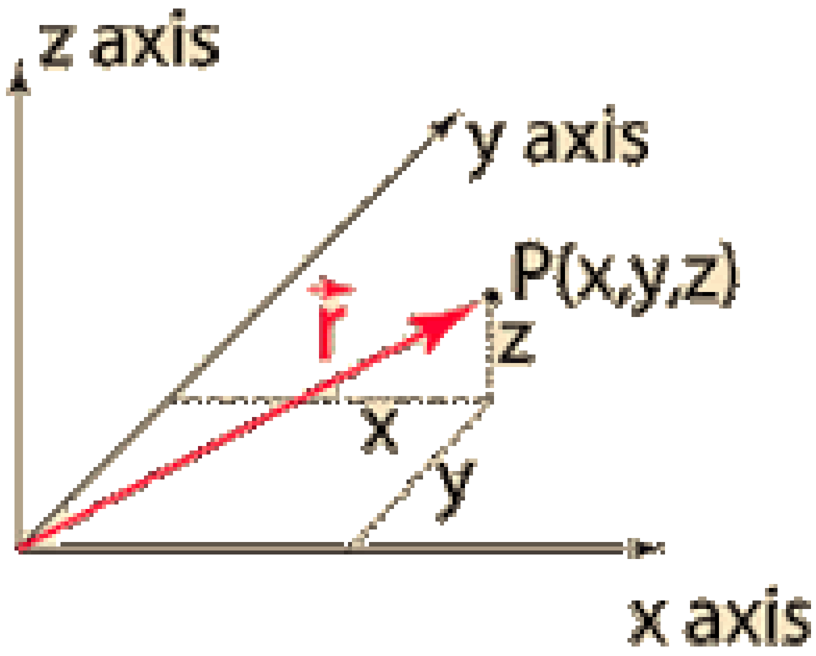
$$\hat{a} = \frac{\vec{A}}{A} = \frac{\hat{x}A_x + \hat{y}A_y + \hat{z}A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\hat{a} = \frac{\hat{x}3 + \hat{y}2 + \hat{z}6}{7} = \hat{x}0.429 + \hat{y}0.286 + \hat{z}0.857$$

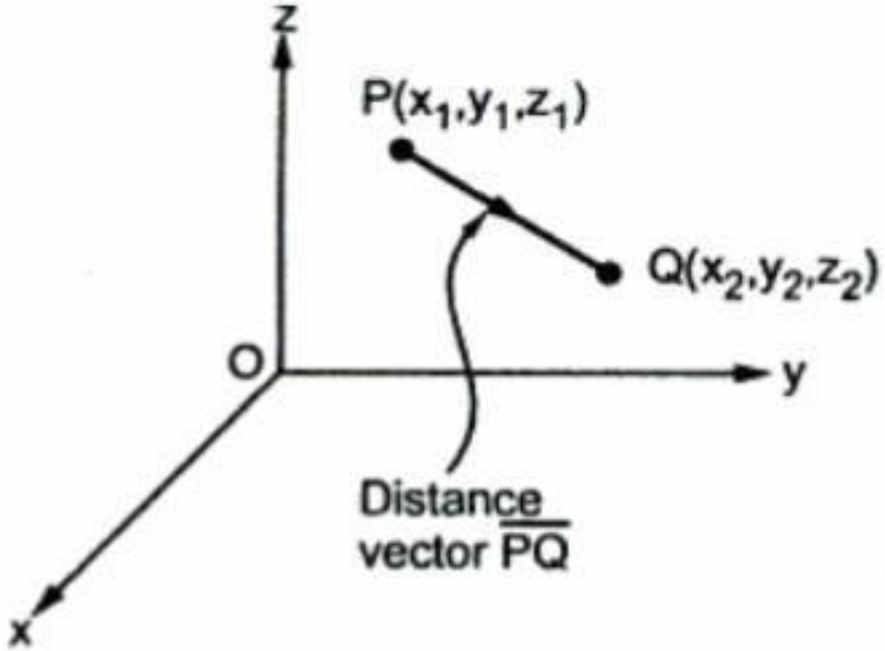
Position Vector

Position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$





Distance Vector



$$\bar{P} = x_1\,\bar{a}_x + y_1\,\bar{a}_y + z_1\,\bar{a}_z$$

$$\bar{Q} = x_2\,\bar{a}_x + y_2\,\bar{a}_y + z_2\,\bar{a}_z$$

$$\boxed{\overline{PQ} = \bar{Q} - \bar{P} = [x_2 - x_1]\,\bar{a}_x + [y_2 - y_1]\,\bar{a}_y + [z_2 - z_1]\,\bar{a}_z}$$

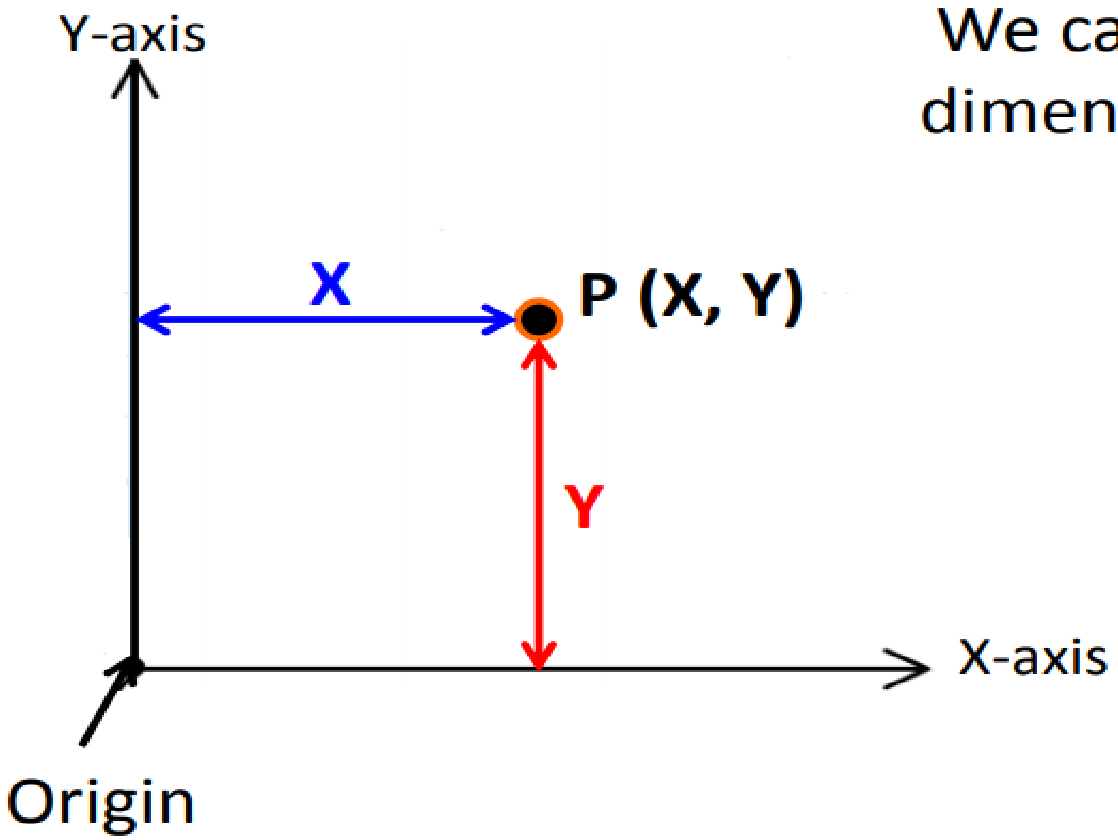
$$\boxed{\left|\overline{PQ}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$\boxed{\bar{a}_{PQ} = \text{Unit vector along PQ} = \frac{\overline{PQ}}{\left|\overline{PQ}\right|}}$$

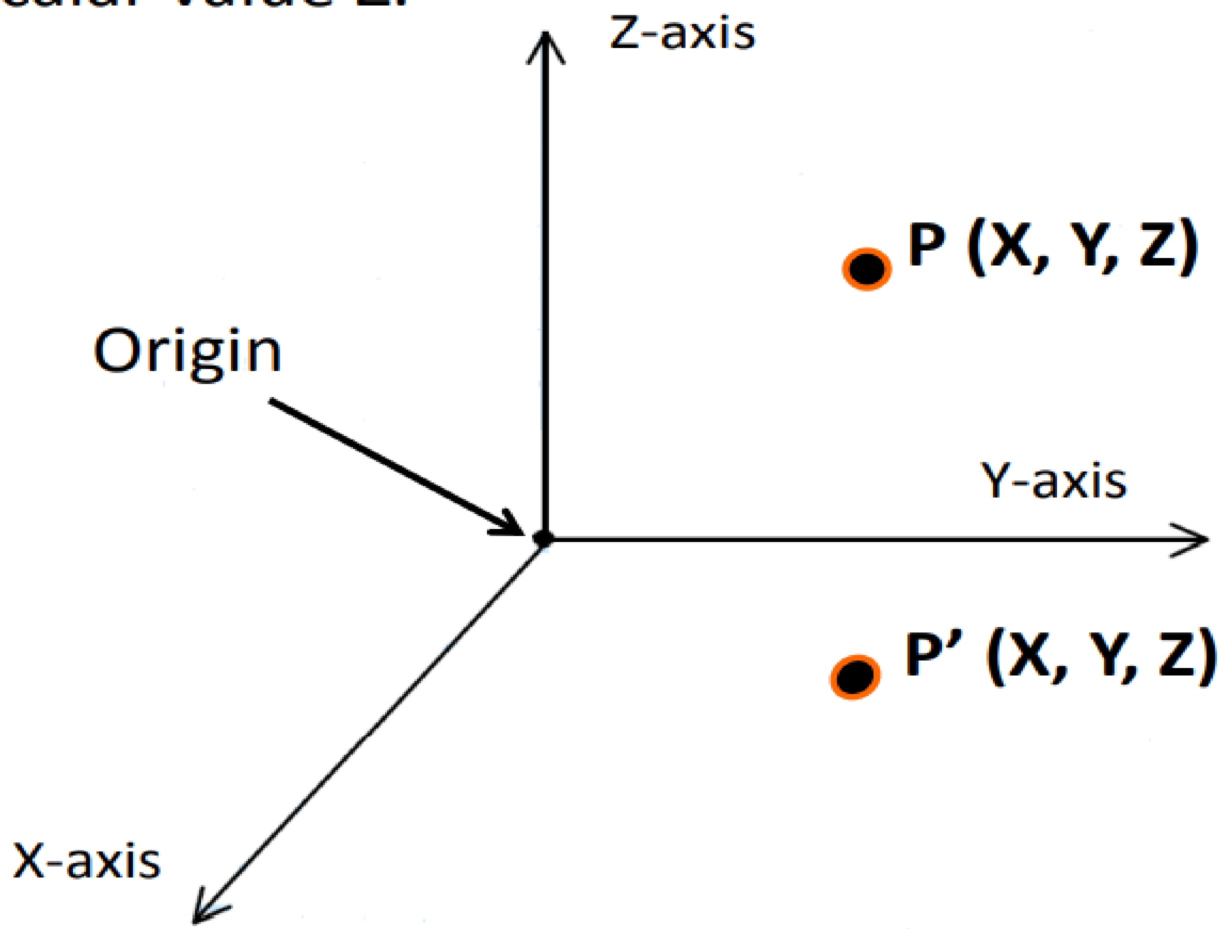
Coordinate System

Cartesian Coordinates

- In **two** dimensions, we can specify a point on a plane using **two** scalar values, generally called X and Y.



We can extend this to **three-** dimensions, by adding a **third** scalar value Z.

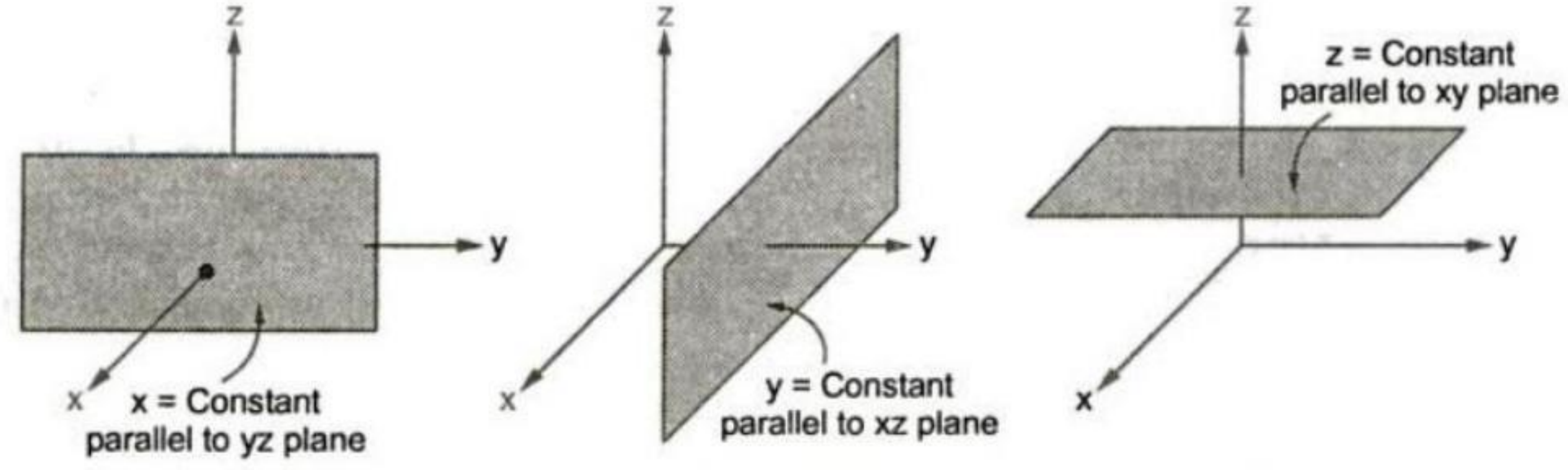


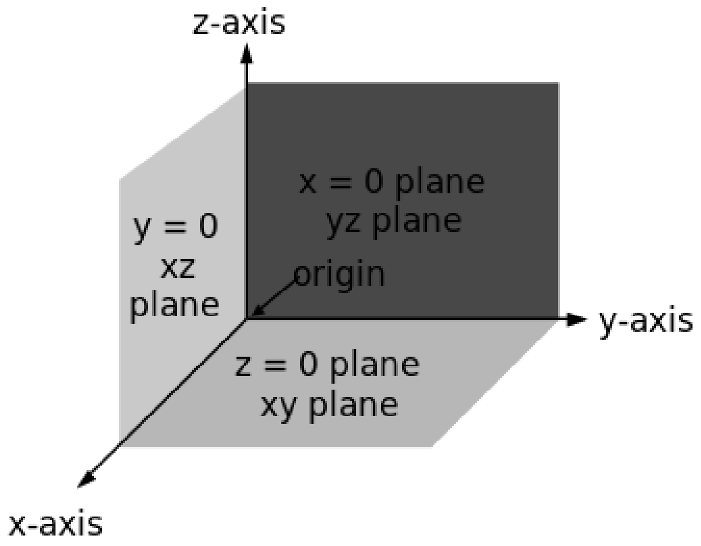
RECTANGULAR CARTESIAN COORDINATE SYSTEM:

The rectangular Cartesian coordinate system is described by three planes which are mutually perpendicular to each other. The three planes intersect at a point 'O' which is called the origin of the coordinate system. There are three coordinate axes which are usually denoted by x,y,z. Values of x,y,z are measured from the origin. The three planes are

- x= constant plane ie yz plane
- y= constant plane ie xz plane
- z= constant plane ie xy plane

Cartesian System

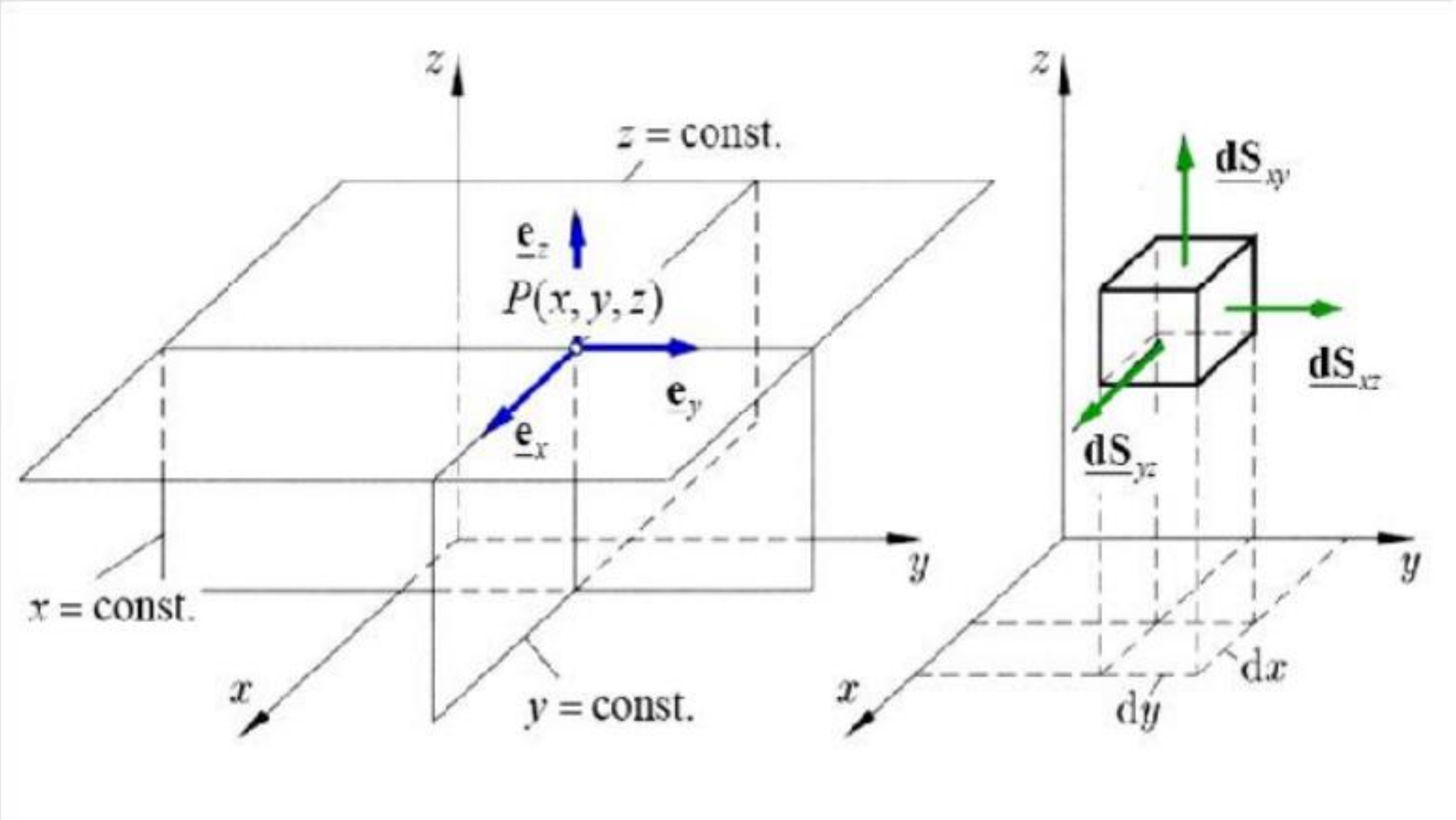




Ranges

x	$-\infty$ to $+\infty$
y	$-\infty$ to $+\infty$
z	$-\infty$ to $+\infty$

Cartesian rectangular coordinate system



A point in rectangular coordinate system is defined by (x, y, z). The limits for the coordinates are

$$-\infty \leq x \leq \infty \tag{2.1}$$

$$-\infty \leq y \leq \infty \tag{2.2}$$

$$-\infty \leq z \leq \infty \tag{2.3}$$

The unit or base vectors are a_x, a_y, a_z. The following relations hold for the dot and cross products of the unit vectors

$$a_x \times a_y = a_z \tag{2.4}$$

$$a_y \times a_z = a_x \tag{2.5}$$

$$a_z \times a_x = a_y \tag{2.6}$$

$$a_x \bullet a_y = 0 \tag{2.7}$$

$$a_y \bullet a_z = 0 \tag{2.8}$$

$$a_z \bullet a_x = 0 \tag{2.9}$$

The differential length element is given by

$$dl = dxa_x + dya_y + dza_z \tag{2.10}$$

Cartesian System

dx = Differential length in x direction

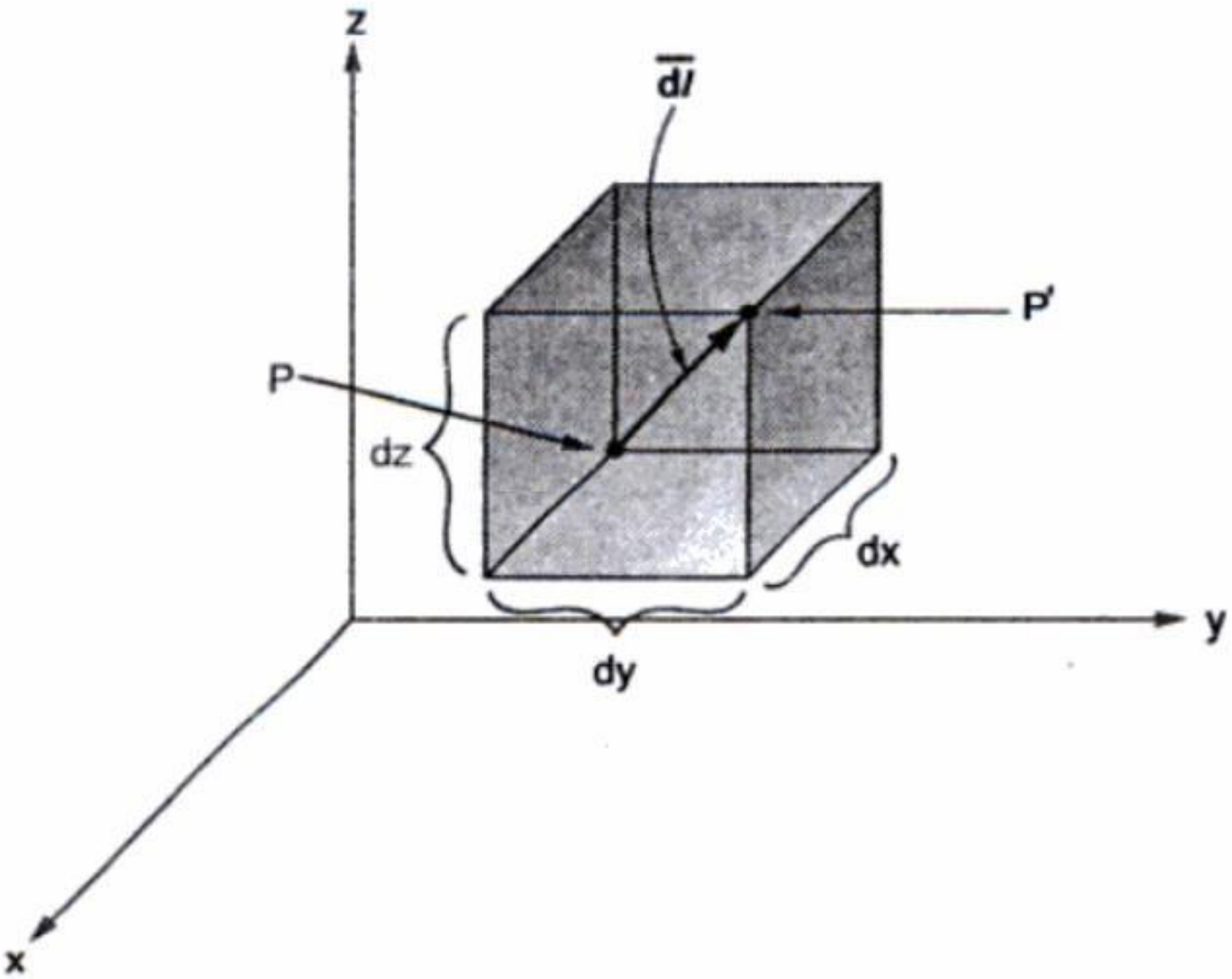
dy = Differential length in y direction

dz = Differential length in z direction

$$\boxed{\overline{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z}$$

$$\boxed{\left|\overline{dl}\right| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}}$$

$$\boxed{dv = dx\ dy\ dz}$$

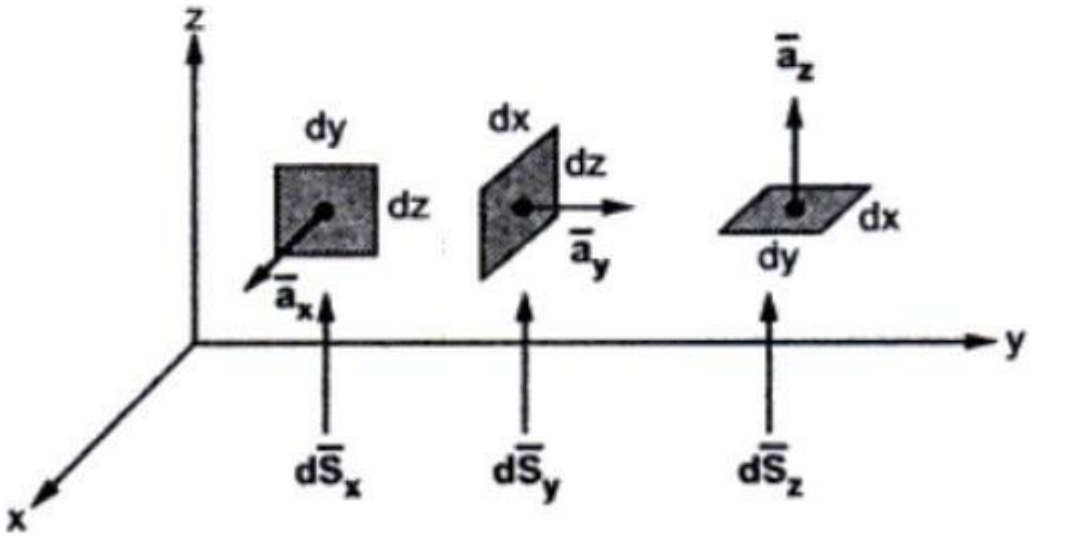


Cartesian System

$$\boxed{d\bar{S} = dS\,\bar{a}_n}$$

where dS = Differential surface area of the element

$\bar{a}_n$ = Unit vector normal to the surface dS



$d\bar{S}_x$ = Differential vector surface area normal to x direction

$= dydz\,\bar{a}_x$

$d\bar{S}_y$ = Differential vector surface area normal to y direction

$= dxdz\,\bar{a}_y$

$d\bar{S}_z$ = Differential vector surface area normal to z direction

$= dxdy\,\bar{a}_z$

CYLINDRICAL COORDINATE SYSTEM:

The cylindrical coordinate system is also defined by three mutually orthogonal surfaces. They are a cylinder and two planes. One of the planes is the same as the $z = constant$ plane in the Cartesian coordinate system. The second plane is orthogonal to the $z = constant$ plane and hence contains the $z-axis$. it makes an angle ϕ with the $xz-plane$. This plane is defined by $\phi = constant$.The third one is a cylinder whose axis is the z axis and has a radius $\rho = constant$ from the $z-axis$. So a point in cylindrical coordinates is defined by (ρ, ϕ, z). The limits for the coordinates are

$$\begin{aligned} 0 &\leq \rho \leq \infty \\ 0 &\leq \phi \leq 2\pi \\ -\infty &\leq z \leq \infty \end{aligned}$$

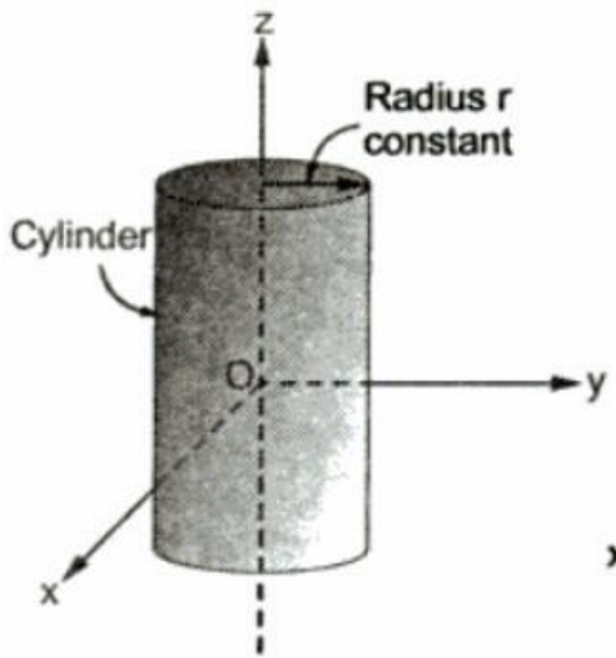
The unit or base vectors are $a_\rho, a_\phi \, and \, a_z$.

Cylindrical System

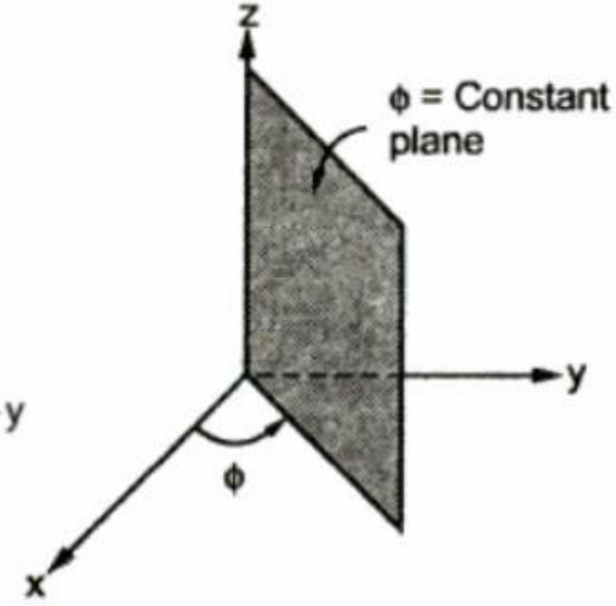
1. Plane of constant z which is parallel to xy plane.

2. A cylinder of radius r with z axis as the axis of the cylinder.

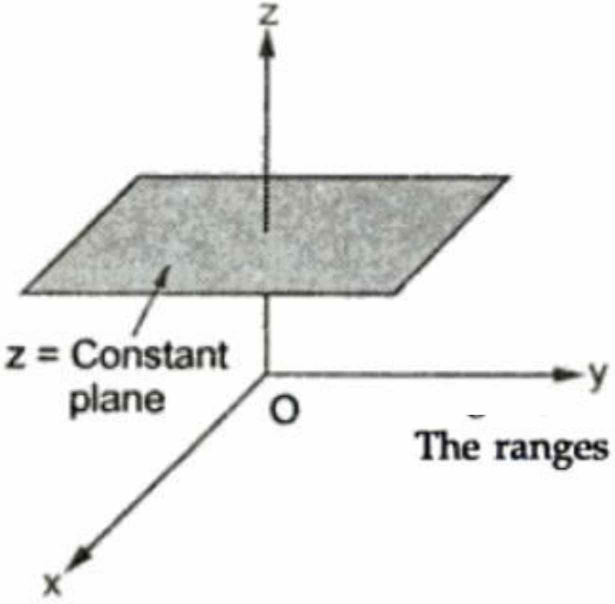
3. A half plane perpendicular to xy plane and at an angle ϕ with respect to xz plane. The angle ϕ is called **azimuthal angle.**



(a) r = Constant



(b) ϕ = Constant

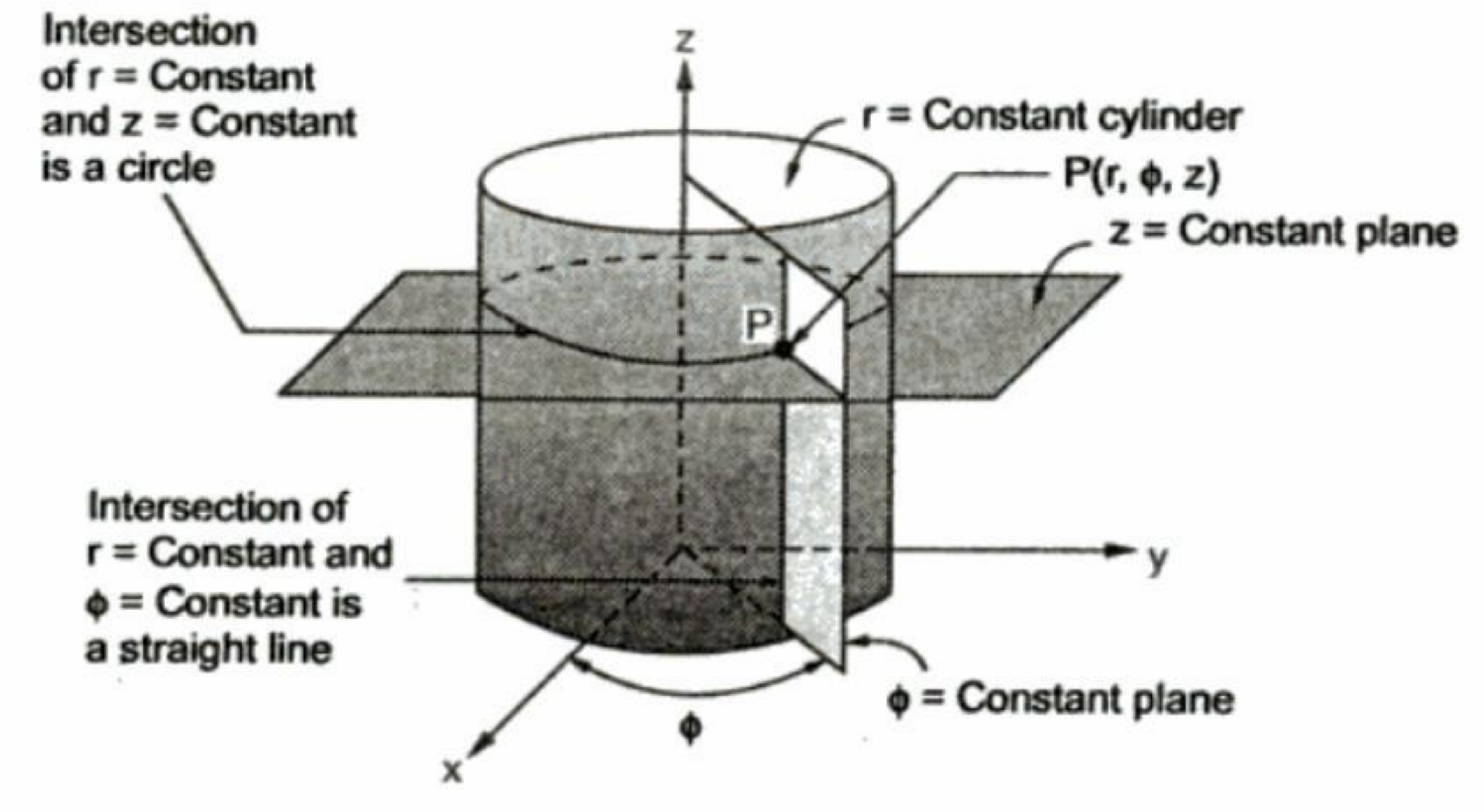


(c) z = Consta...

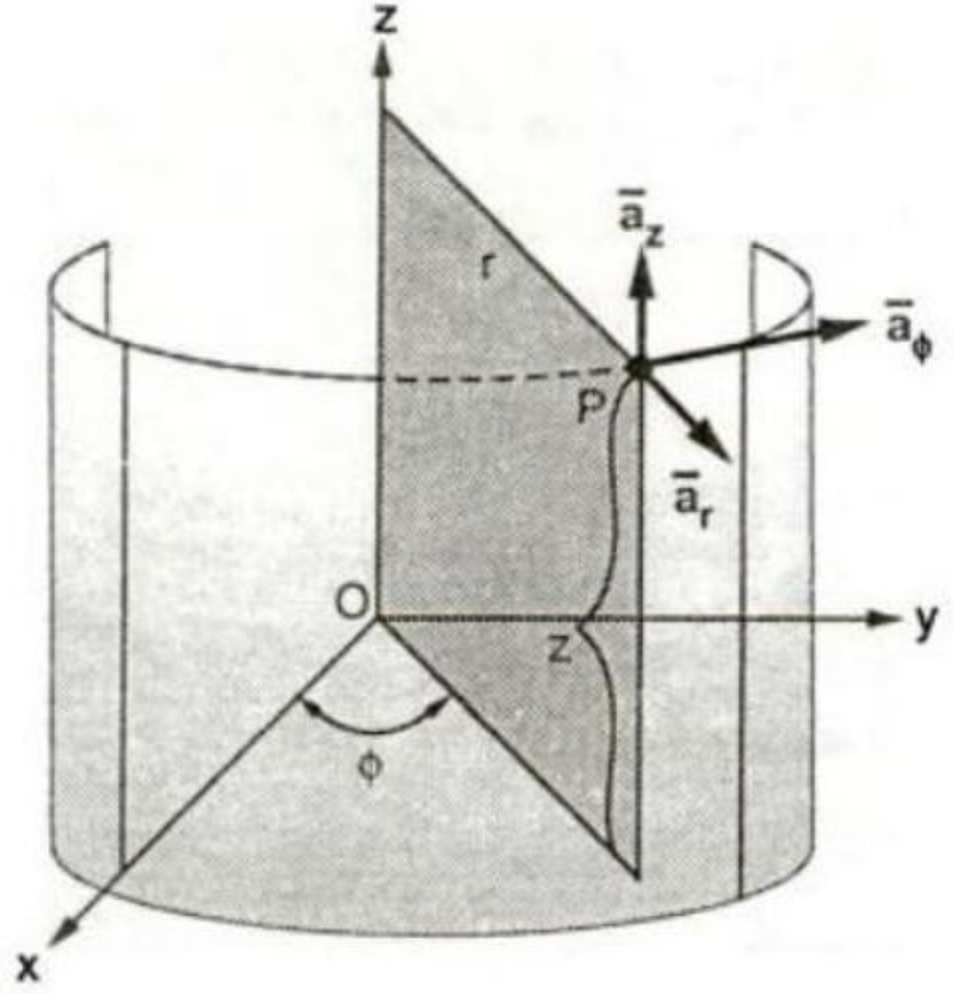
The ranges of the variables are,

$$0 \leq r \leq \infty$$
$$0 \leq \phi \leq 2\pi$$
$$-\infty < z \leq \infty$$

Cylindrical System



Cylindrical System



$$\boxed{\bar{P} = P_r\,\bar{a}_r + P_\phi\,\bar{a}_\phi + P_z\,\bar{a}_z}$$

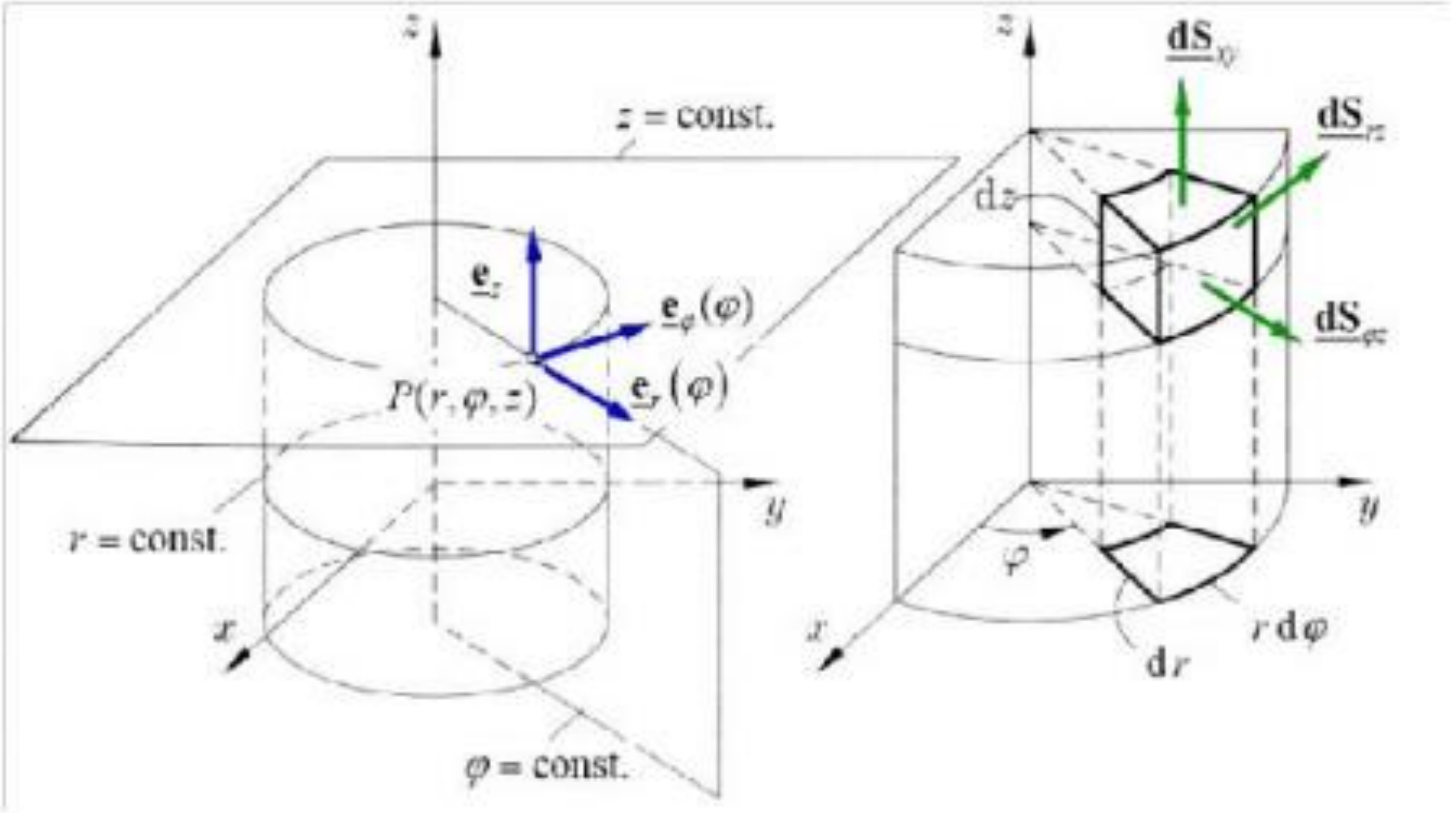


Figure 2.7: Cylindrical coordinate system

Cylindrical System

dr = Differential length in r direction

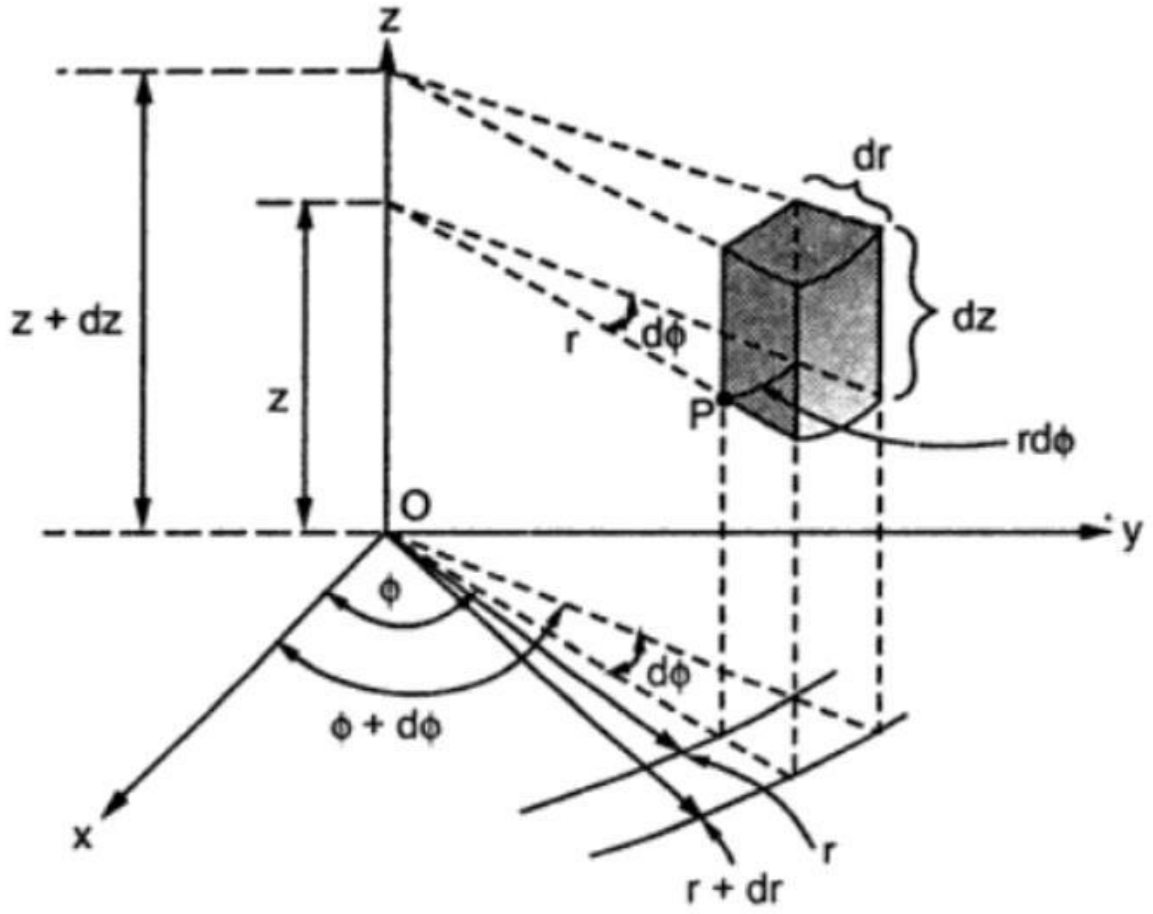
r dϕ = Differential length in ϕ direction

dz = Differential length in z direction

$$\boxed{\overline{\mathbf{d}l} = dr\ \bar{\mathbf{a}}_r + r\ d\phi\ \bar{\mathbf{a}}_\phi + dz\ \bar{\mathbf{a}}_z}$$

$$\boxed{|\overline{\mathbf{d}l}| = \sqrt{(dr)^2 + (r\ d\phi)^2 + (dz)^2}}$$

$$\boxed{dv = r\ dr\ d\phi\ dz}$$



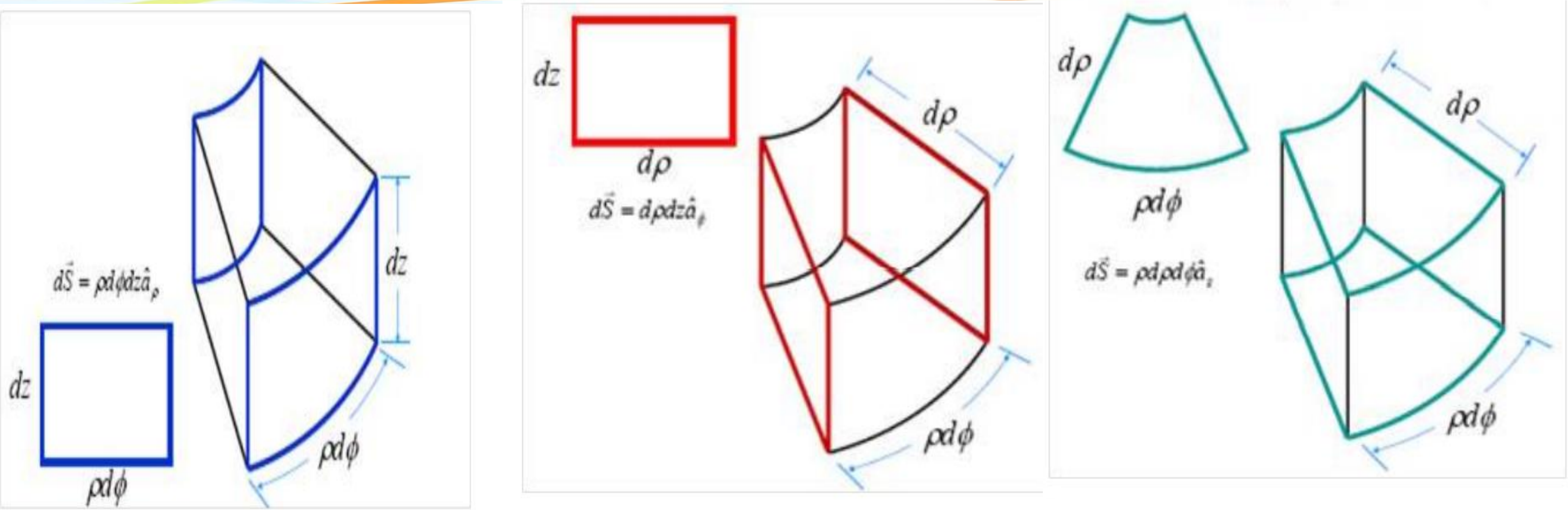
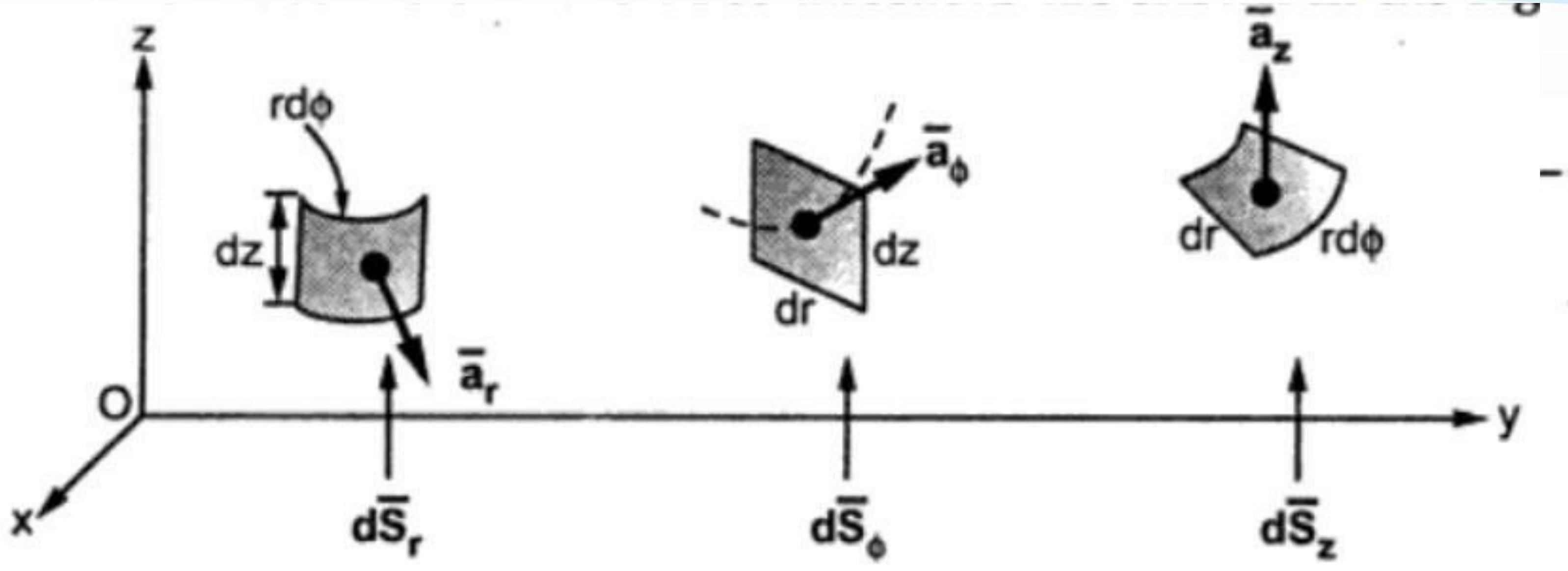


Figure 2.2: Cylindrical area

Cylindrical System



$d\bar{S}_r$ = Differential vector surface area normal to r direction

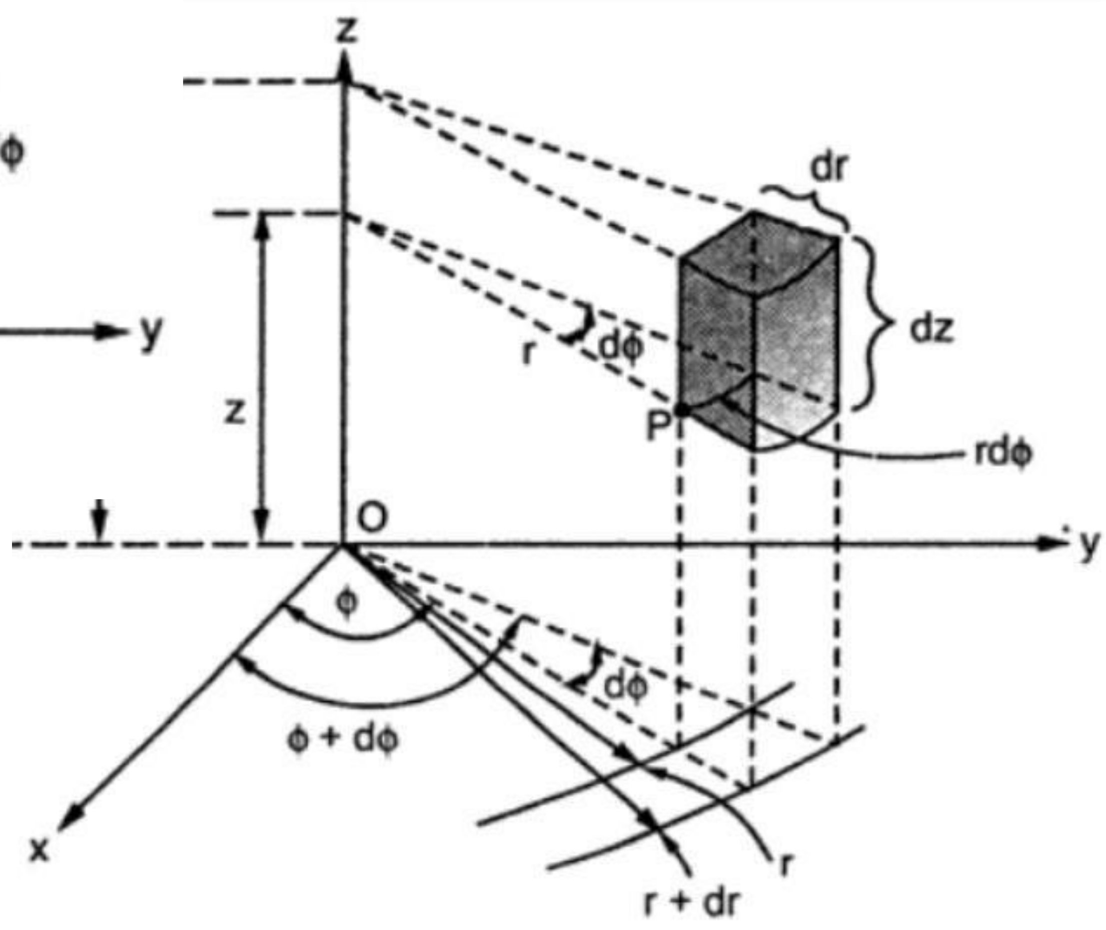
$= r\, d\phi\, dz\, \bar{a}_r$

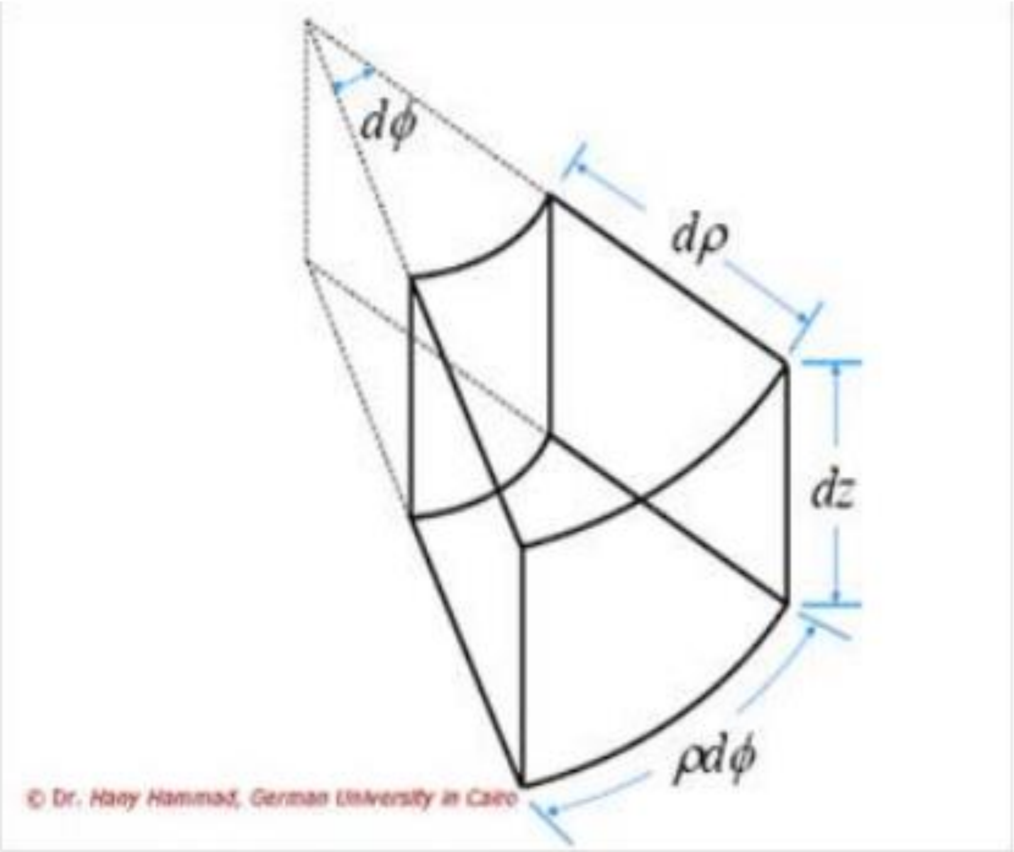
$d\bar{S}_\phi$ = Differential vector surface area normal to ϕ direction

$= dr\, dz\, \bar{a}_\phi$

$d\bar{S}_z$ = Differential vector surface area normal to z direction

$= r\, dr\, d\phi\, \bar{a}_z$





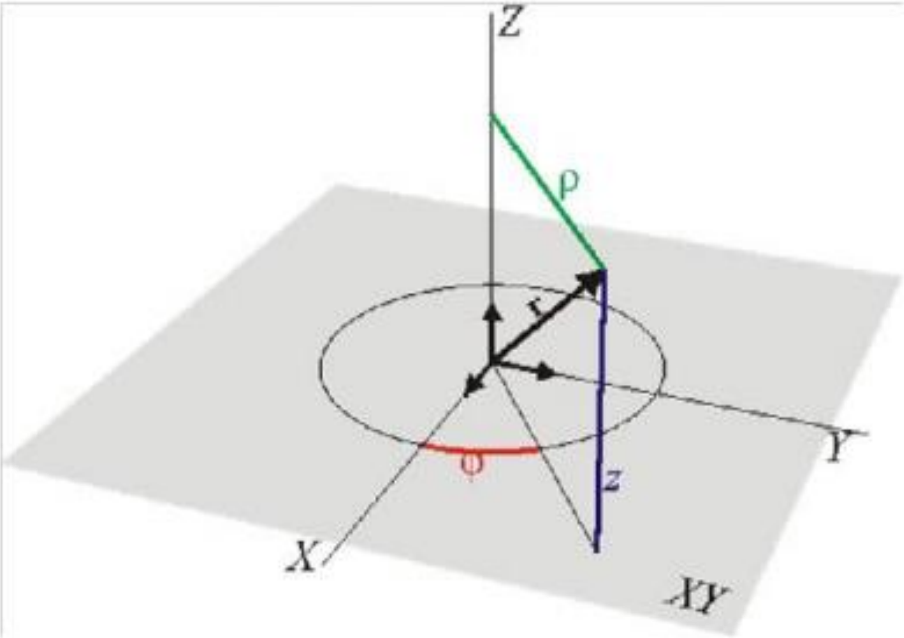
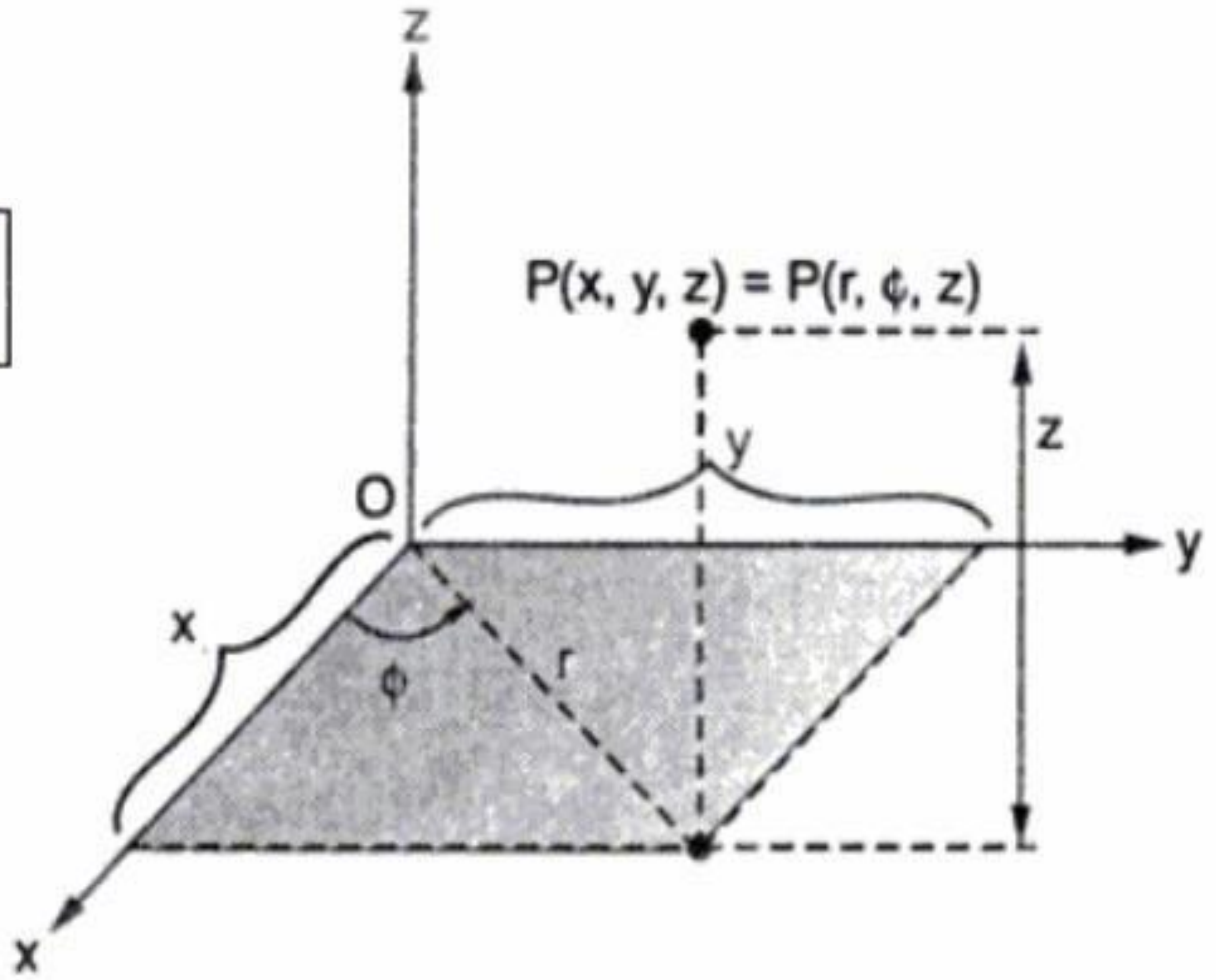


Figure 2.6: Cylindrical coordinate system

Relation between Cartesian and Cylindrical

$$\boxed{x = r\cos\phi, \quad y = r\sin\phi \quad \text{and} \quad z = z}$$

$$\boxed{r = \sqrt{x^2+y^2}, \quad \phi = \tan^{-1}\frac{y}{x} \quad \text{and} \quad z = z}$$

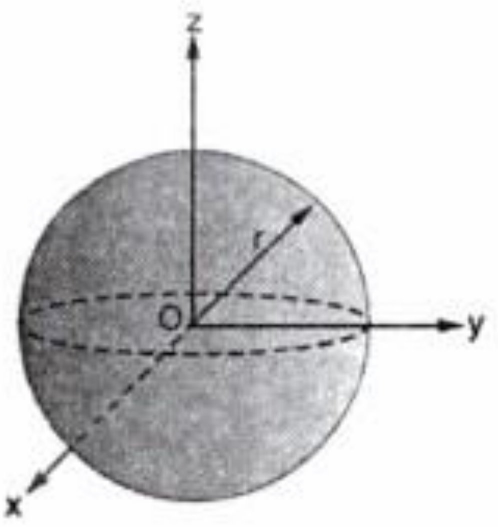


Spherical System

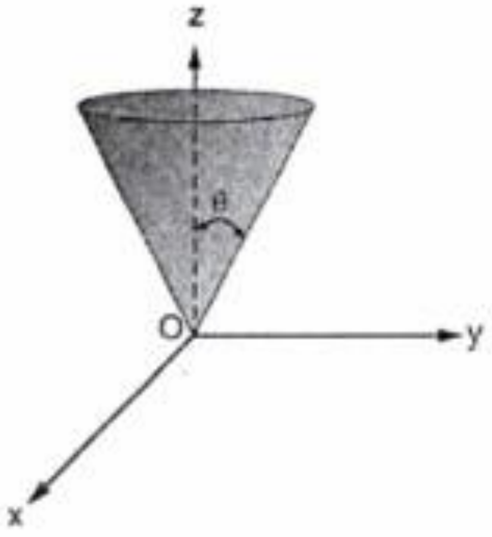
The surfaces which are used to define the spherical co-ordinate system on the three cartesian axes are,

1. Sphere of radius r, origin as the centre of the sphere.
2. A right circular cone with its apex at the origin and its axis as z axis. Its half angle is θ. It rotates about z axis and θ varies from 0 to 180°.
3. A half plane perpendicular to xy plane containing z axis, making an angle ϕ with the xz plane.

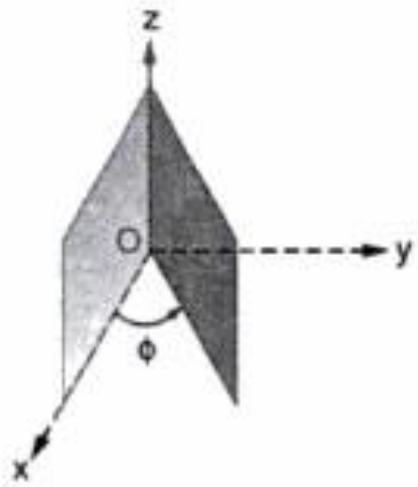
Thus the three co-ordinates of a point P in the spherical co-ordinate system are (r, θ, ϕ).



(a) Sphere of radius r with centre as origin



(b) Right circular cone with apex at origin



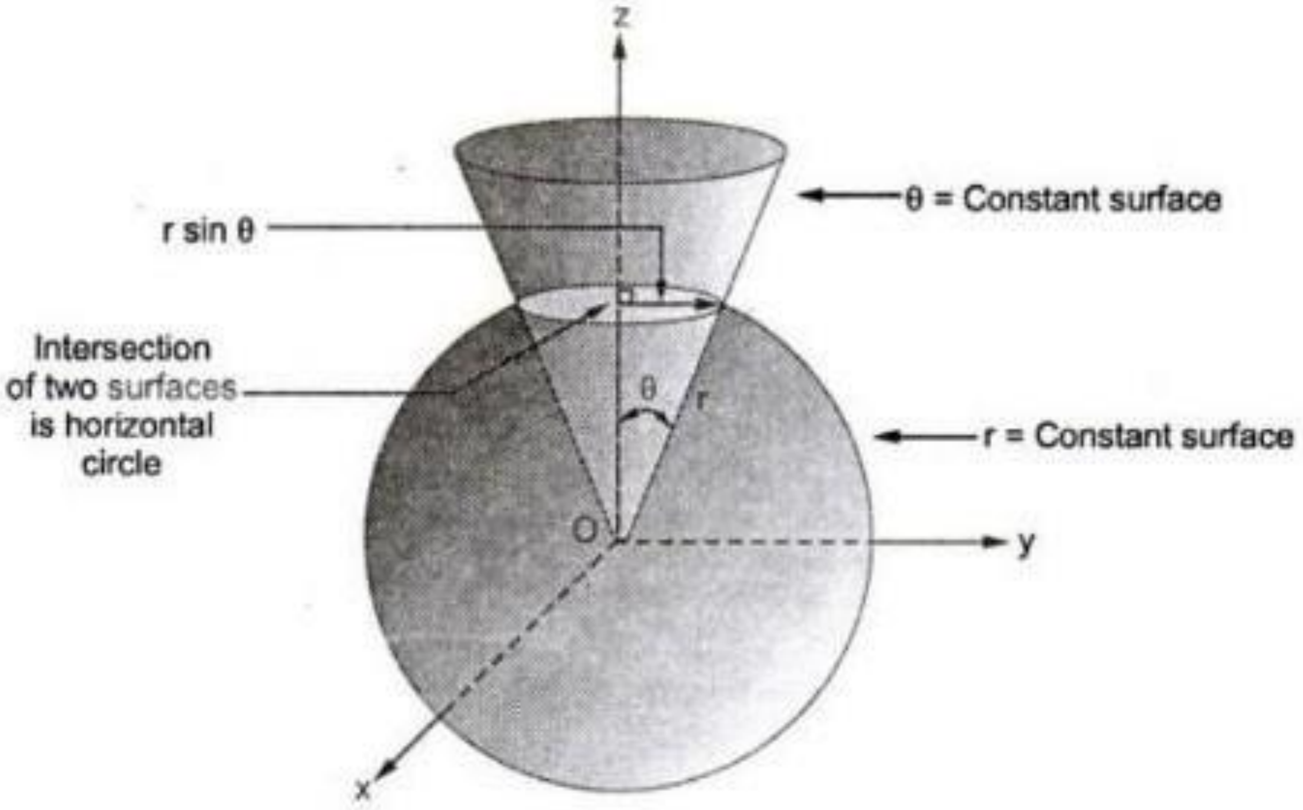
(c) Half plane perpendicular to xy plane

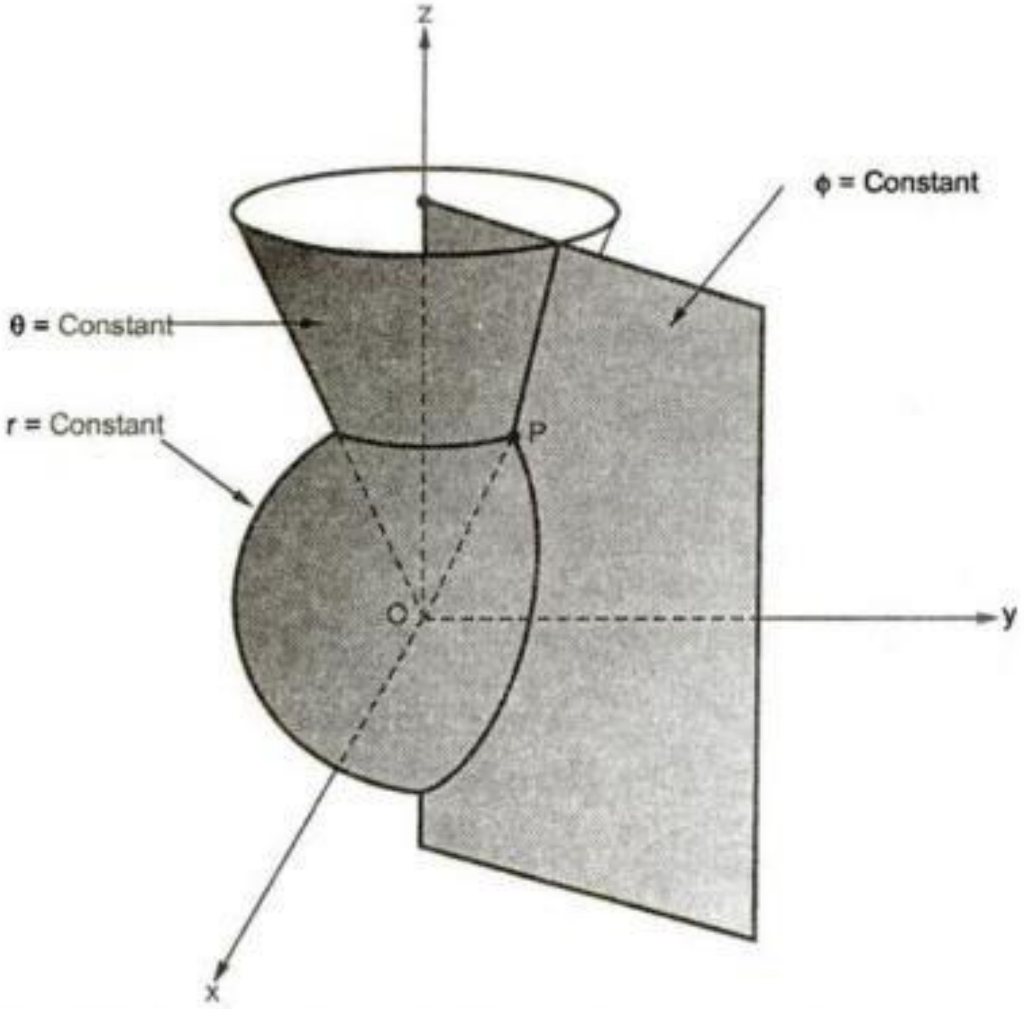
$0 \le r < \infty$

$0 \le \phi \le 2\pi$

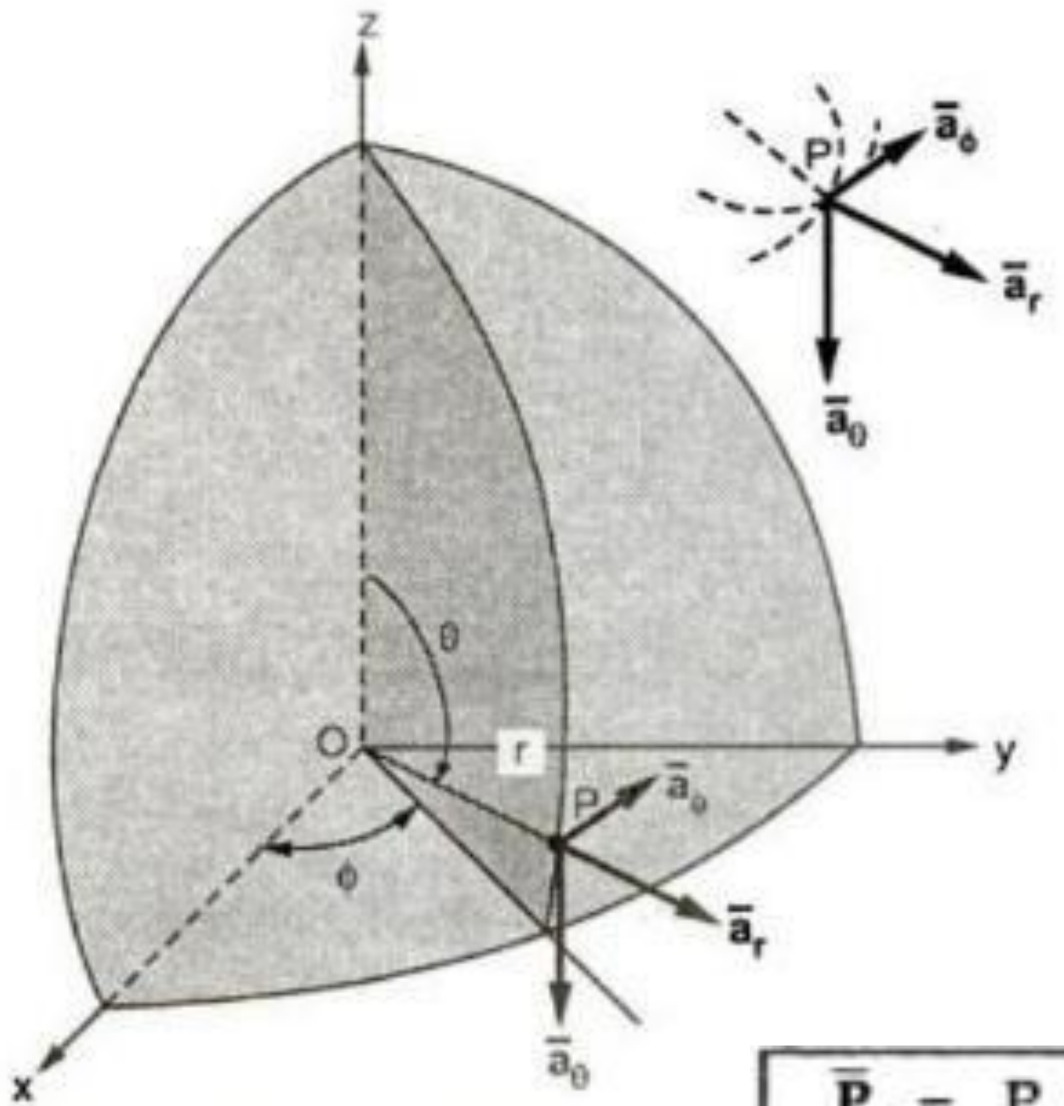
$0 \le \theta \le \pi$ as half angle

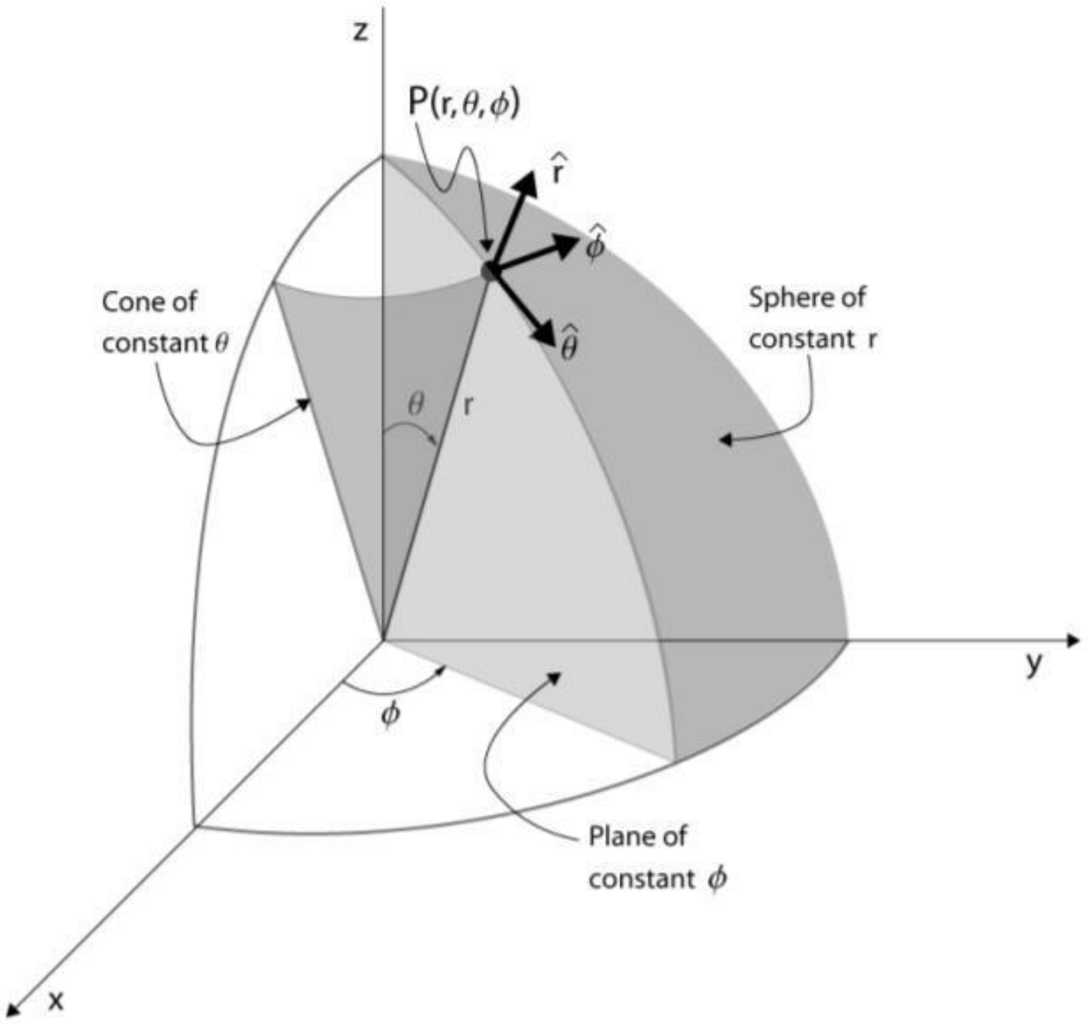
Spherical System





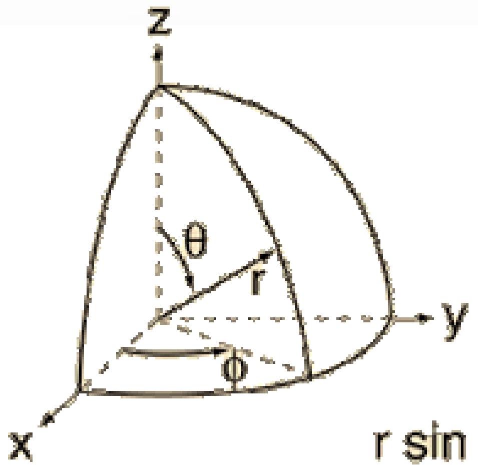
Spherical System

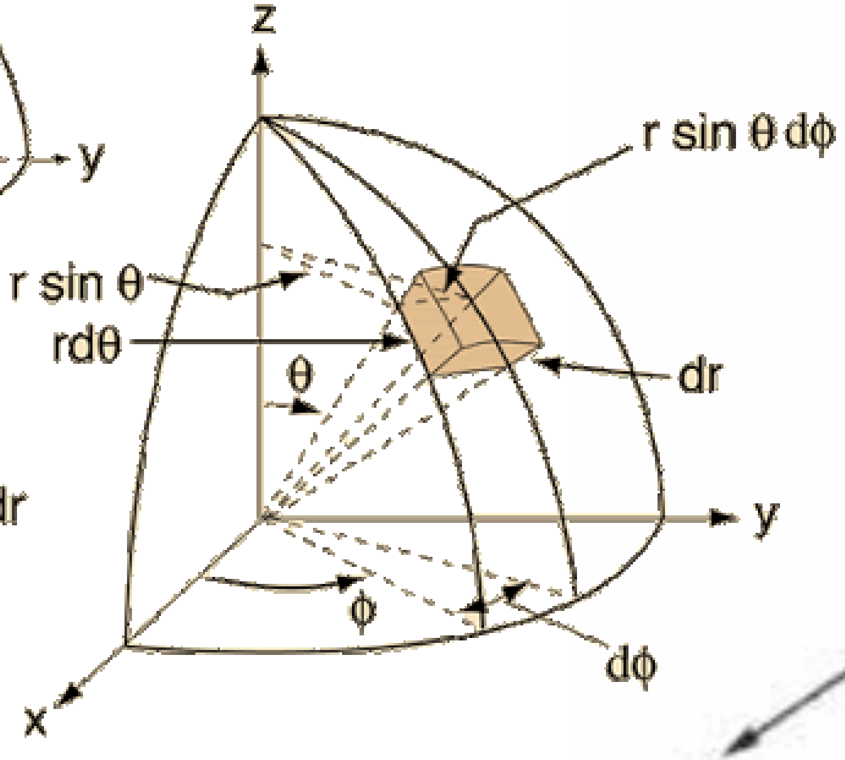




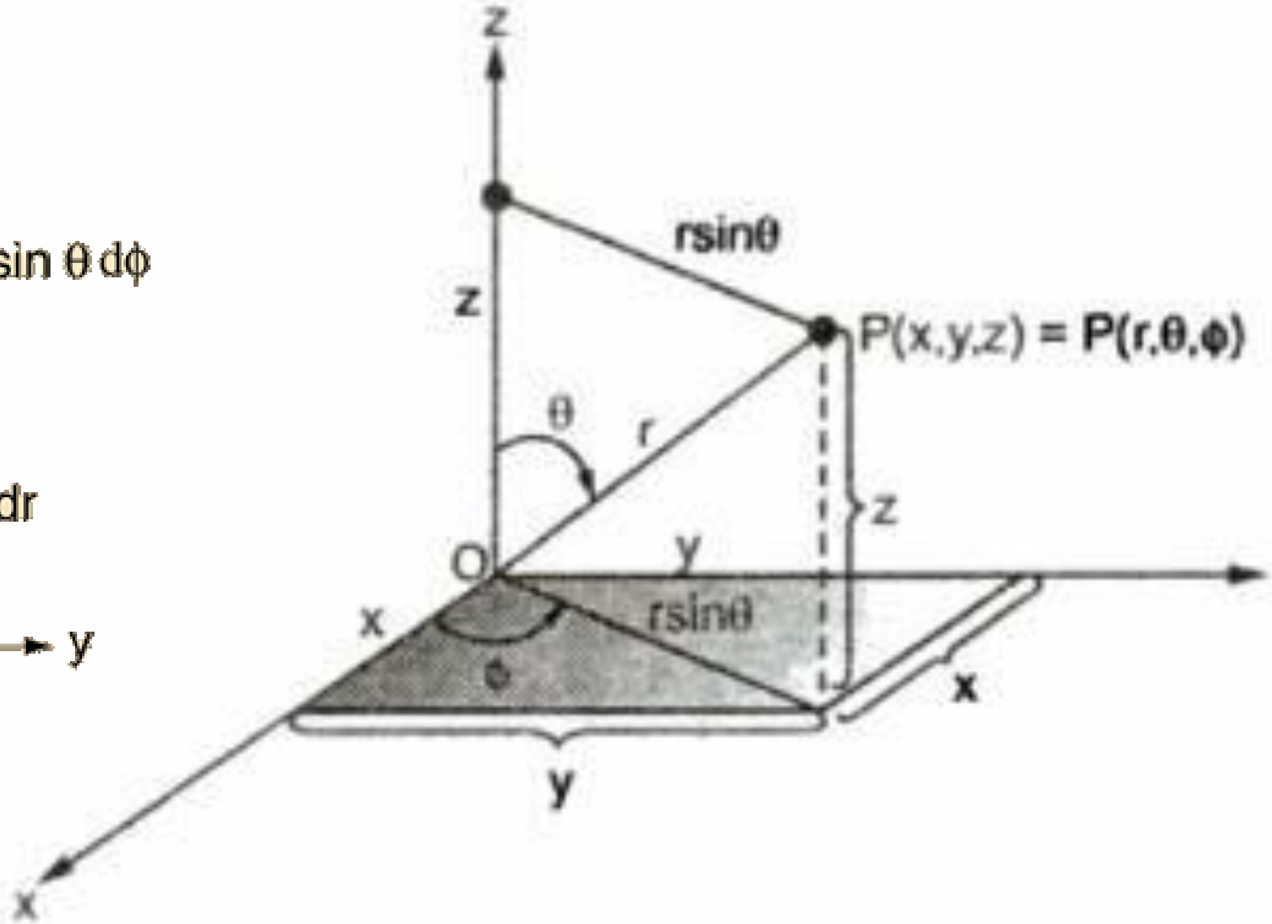
$$\boxed{\bar{P} = P_r \bar{a}_r + P_\theta \bar{a}_\theta + P_\phi \bar{a}_\phi}$$

Spherical System

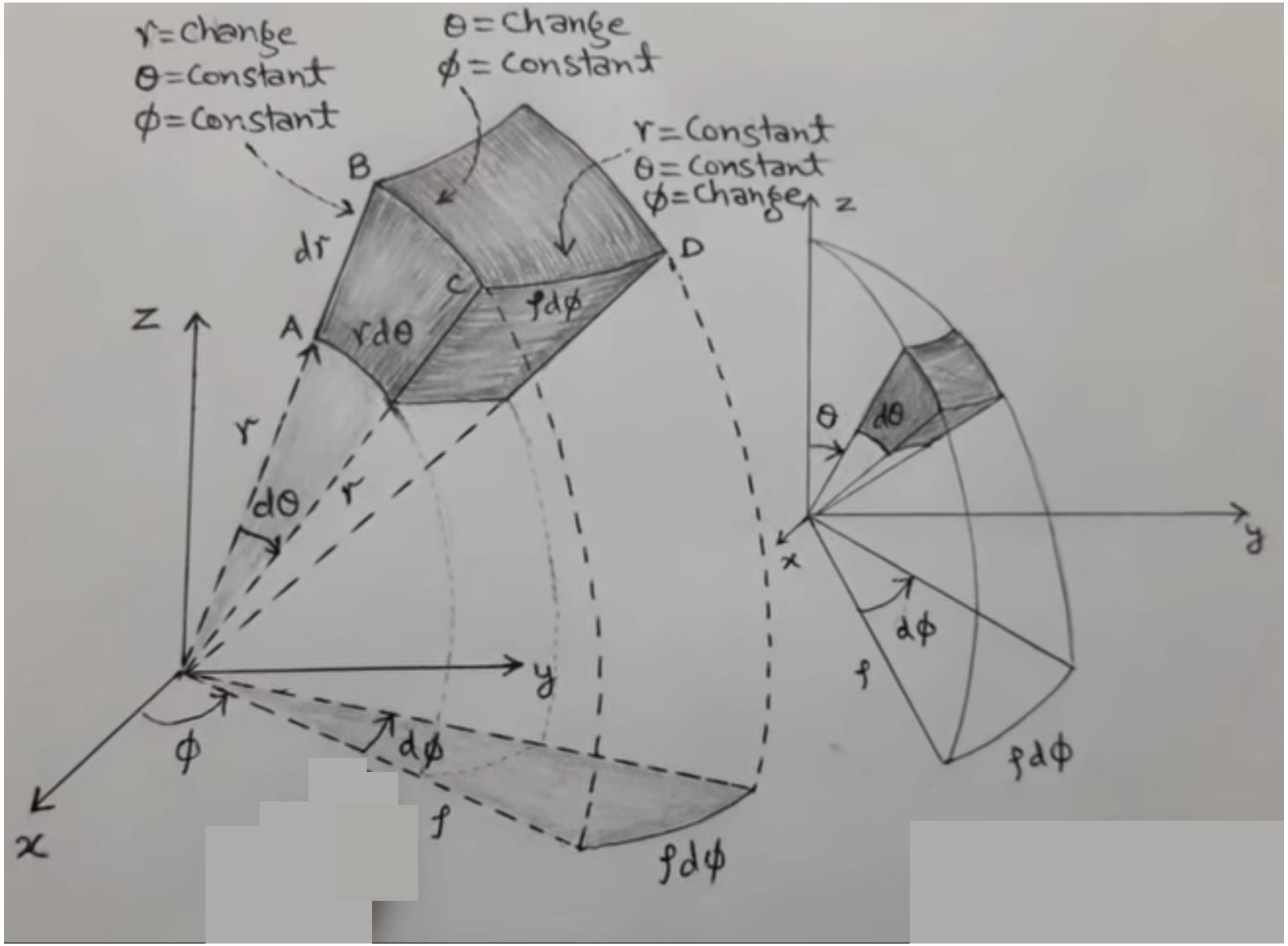




Volume element
$dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$



Spherical System



Spherical System

dr = Differential length in r direction

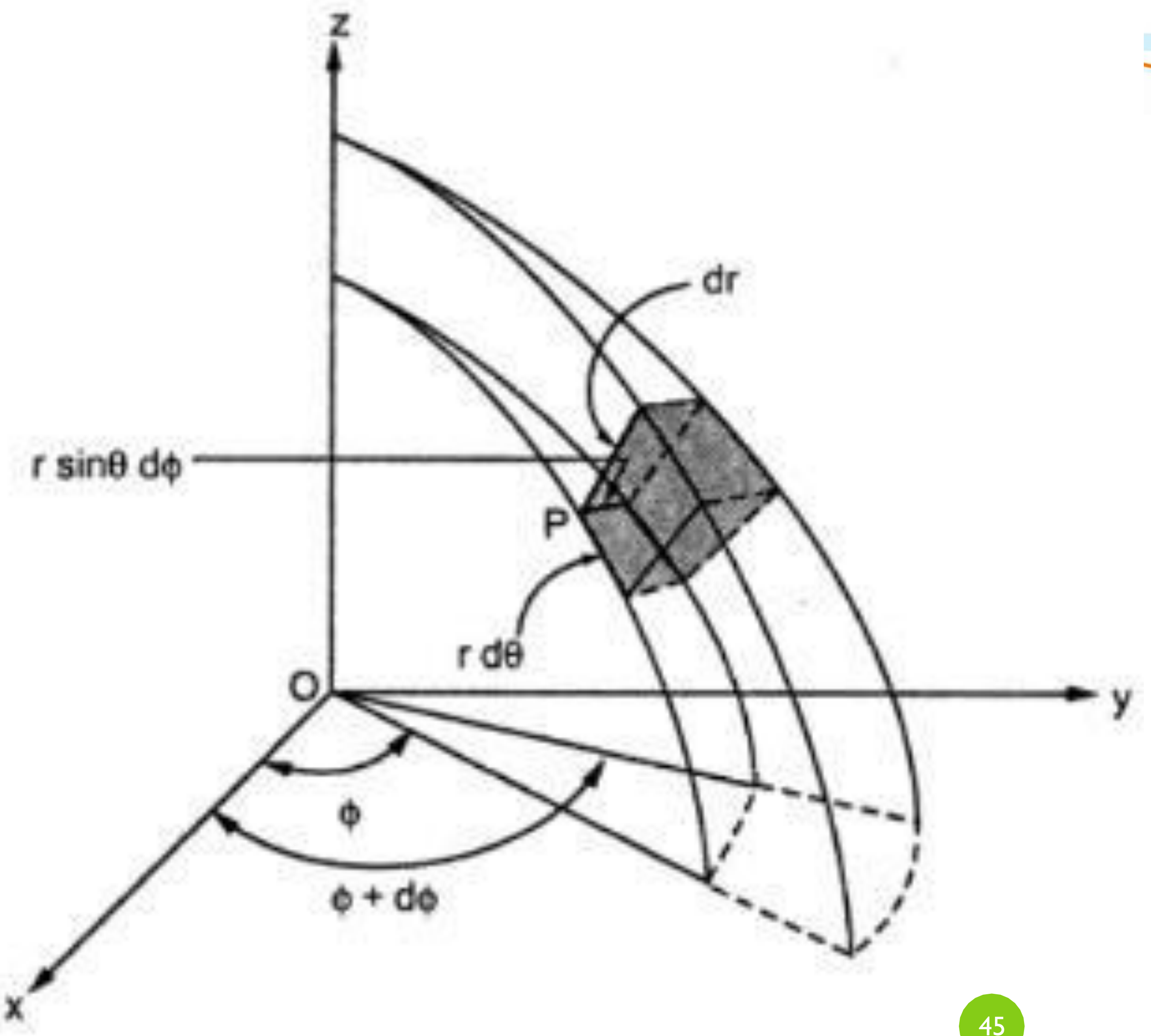
$r\,d\theta$ = Differential length in θ direction

$r\sin\theta\,d\phi$ = Differential length in ϕ direction

$$\boxed{\overline{dl} = dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi}$$

$$\boxed{\left|\overline{dl}\right| = \sqrt{(dr)^2 + (r\,d\theta)^2 + (r\sin\theta\,d\phi)^2}}$$

$$\boxed{dv = r^2\sin\theta\,dr\,d\theta\,d\phi}$$



Spherical System

$d\bar{S}_r$ = Differential vector surface area normal to r direction

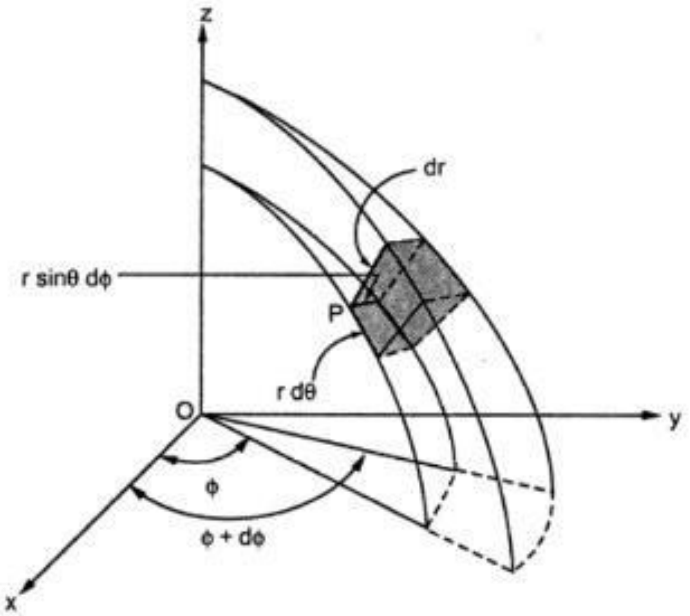
$= r^2 \sin\theta\, d\theta\, d\phi$

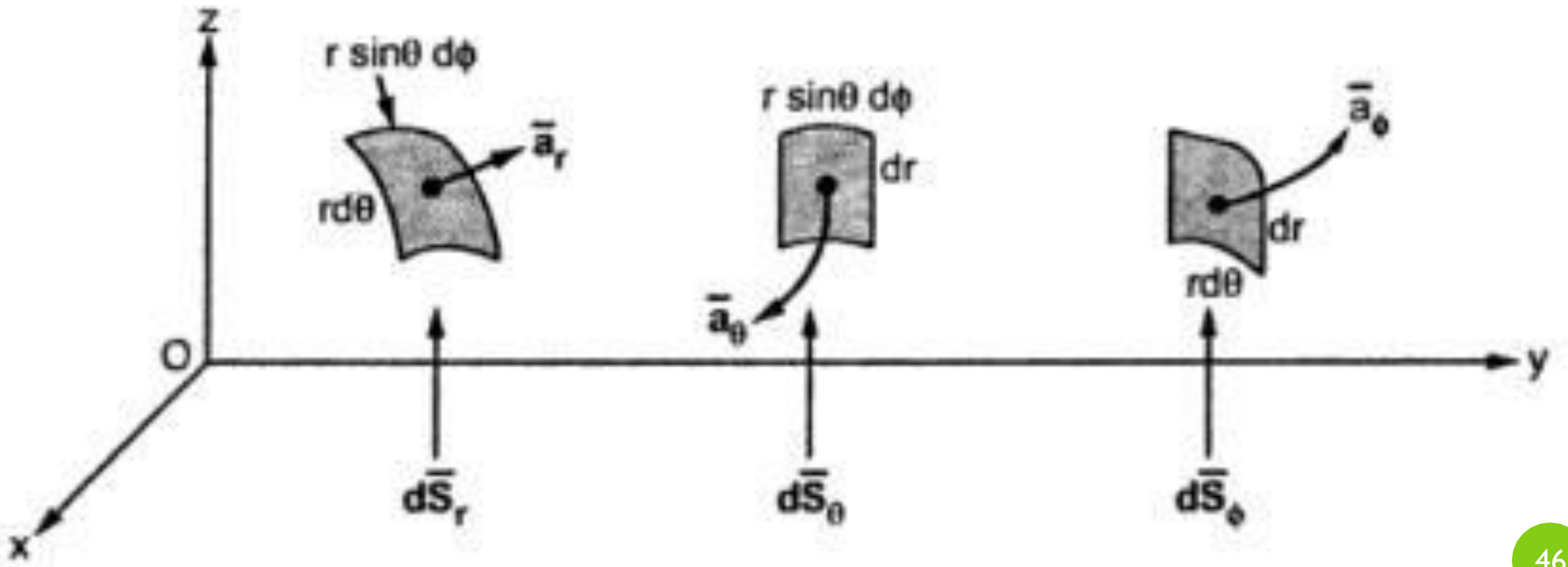
$d\bar{S}_\theta$ = Differential vector surface area normal to θ direction

$= r \sin\theta\, dr\, d\phi$

$d\bar{S}_\phi$ = Differential vector surface area normal to ϕ direction

$= r\, dr\, d\theta$

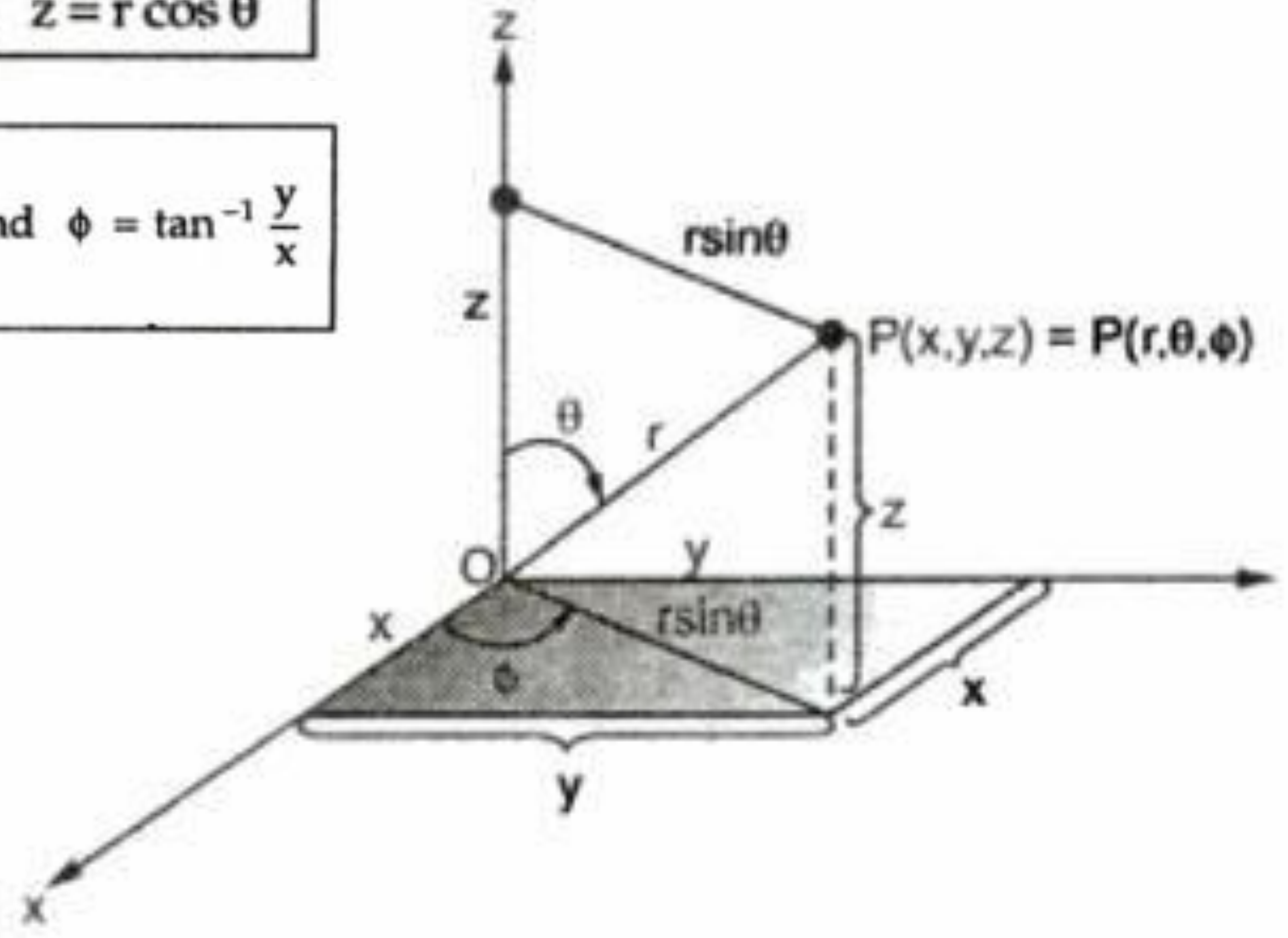




Relation between Cartesian and Spherical System

$$x = r\sin\theta\cos\phi,\quad y = r\sin\theta\sin\phi \quad \text{and} \quad z = r\cos\theta$$

$$r = \sqrt{x^2+y^2+z^2},\quad \theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right] \quad \text{and} \quad \phi = \tan^{-1}\frac{y}{x}$$



Vector Multiplication

The Knowledge of vector multiplication allows us to transform the vectors from one coordinate system to other.

1) Multiplication of a **vector** by a **scalar**. $c\vec{v}$
   (**Scalar Multiplication**)

2) Multiplication of one **vector** by a second **vector** so as to produce a **scalar**. $\vec{a} \cdot \vec{b}$
   (**Dot Product** or **Scalar Product**)

3) Multiplication of one **vector** by a second **vector** so as to produce another **vector**. $\vec{a} \times \vec{b}$
   (**Cross Product** or **Vector Product**)

Scalar Product or DOT Product

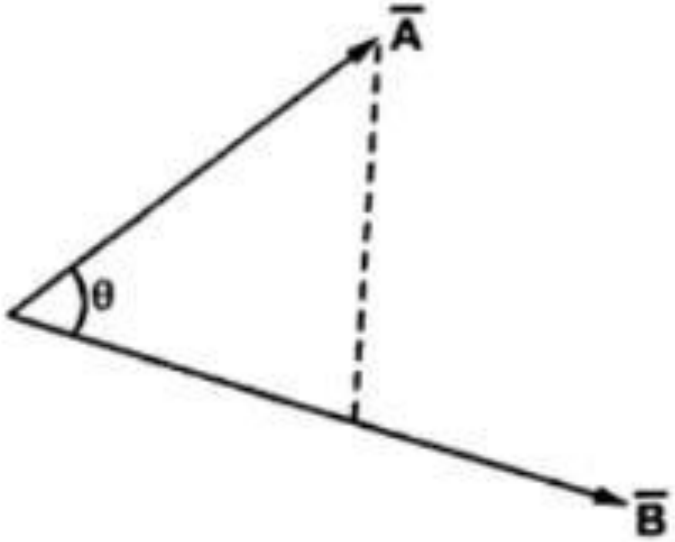


Fig. 1.33

$$\boxed{\bar{A} \cdot \bar{B} = |\bar{A}|\,|\bar{B}| \cos \theta_{AB}}$$

The scalar or dot of the two vectors $\bar{A}$ and $\bar{B}$ is denoted as $\bar{A} \cdot \bar{B}$ and defined as the product of the magnitude of A, the magnitude of B and the cosine of the smaller angle between them.

It also can be defined as the product of magnitude of $\bar{B}$ and the projection of $\bar{A}$ onto $\bar{B}$ or vice versa.

Mathematically it is expressed as,

The result of such a dot product is scalar hence it is also called scalar product.

Properties of DOT Product

$\bar{A} \cdot \bar{B} = |\bar{A}|\ |\bar{B}| \cos \theta_{AB}$ where θ_{AB} = smaller angle between $\bar{A}$ and $\bar{B}$

$\bar{A} \cdot \bar{B} = 0$ then $\bar{A}$ is perpendicular to $\bar{B}$

$\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$ ∴ Commutative

$\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{A} \cdot \bar{C}$ ... Distributive

The **dot product** of **perpendicular** unit vectors **is zero.**

$$\bar{a}_x \cdot \bar{a}_y = \bar{a}_y \cdot \bar{a}_z = \bar{a}_z \cdot \bar{a}_x = 0$$

$$\bar{A} \cdot \bar{A} = |\ \bar{A}\ |^2$$

$$\bar{a}_x \cdot \bar{a}_x = \bar{a}_y \cdot \bar{a}_y = \bar{a}_z \cdot \bar{a}_z = 1$$

$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$ ... In cartesian

The component of $\bar{P}$ in the direction of unit vector $\bar{a}$ is $\bar{P} \cdot \bar{a}$.

The component of $\bar{P}$ in the direction of $\bar{Q}$ is $\bar{P} \cdot \bar{a}_Q = \bar{P} \cdot \dfrac{\bar{Q}}{|\bar{Q}|}$

Applications of DOT Product

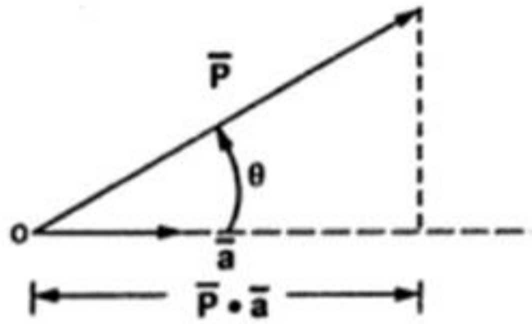
1. To determine the angle between the two vectors.

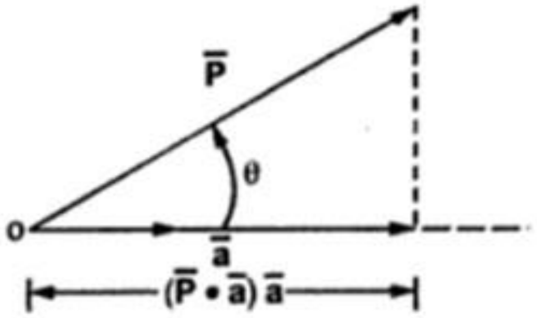
The angle can be determined as,

$$\boxed{\theta = \cos^{-1}\left\{\frac{\bar{A}\cdot\bar{B}}{|\bar{A}|\ |\bar{B}|}\right\}}$$

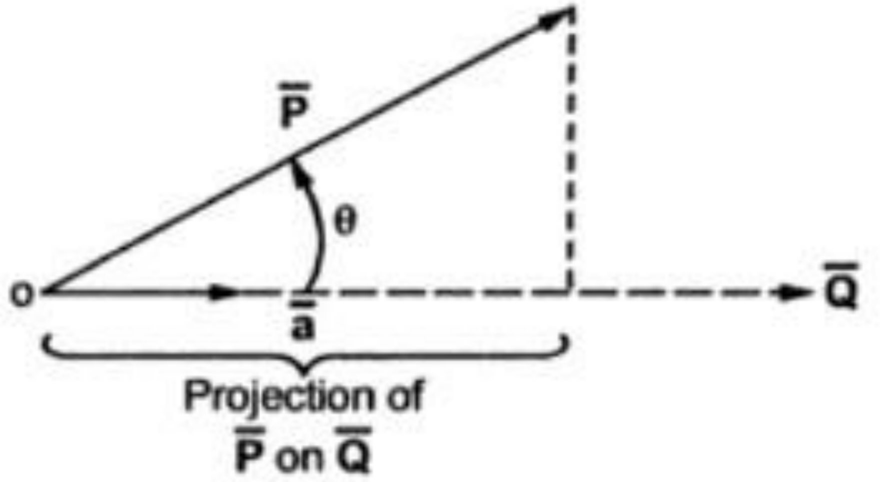
2. To find the component of a vector in a given direction.

Consider a vector $\bar{P}$ and a unit vector $\bar{a}$ The component of vector $\bar{P}$ in the direction of unit vector $\bar{a}$ is $\bar{P}\cdot\bar{a}$. This is a scalar quantity.





$$\boxed{\bar{P}\cdot\bar{a} = |\bar{P}|\ |\bar{a}|\cos\theta = |\bar{P}|\ \cos\theta}$$



Then the projection of $\bar{P}$ on $\bar{Q}$ is given by $\bar{P}\cdot\bar{a}_Q$.

As $\bar{a}_Q = \dfrac{\bar{Q}}{|\bar{Q}|}$ then the projection of $\bar{P}$ on $\bar{Q}$ can be expressed as,

$$\bar{P}\cdot\frac{\bar{Q}}{|\bar{Q}|} = \frac{\bar{P}\cdot\bar{Q}}{|\bar{Q}|}.$$

3. Physically, work done by a constant force can be expressed as a dot product of two vectors.

Consider a constant force $\bar{F}$ acting on a body and it causes the displacement $\bar{d}$ of that body. Then the work done W is the product of the force and the component of the displacement in the direction of force which can be expressed as,

$$W = |\bar{F}| d \cos\theta = \bar{F} \cdot \bar{d}$$

But if the force applied varies along the path then the total work done is to be calculated by the integration of a dot product as,

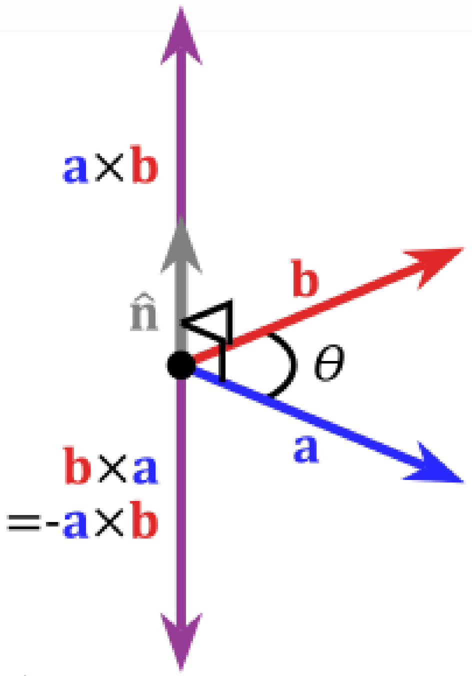
$$W = \int \bar{F} \cdot \overline{dl}$$

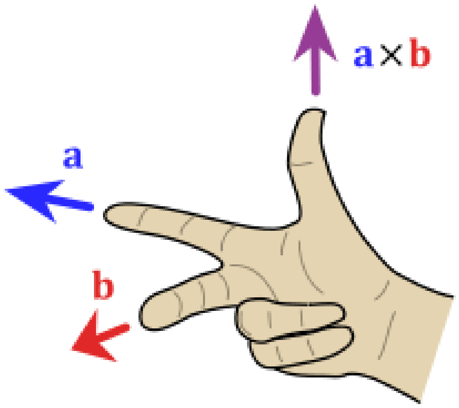
Vector Product or Cross Product

$$\vec{C} = \vec{A} \times \vec{B}$$

- The cross product of two vectors says something about how perpendicular they are.
- Magnitude: $\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = AB\sin\theta$
  - θ is smaller angle between the vectors
  - Cross product of any parallel vectors = zero
  - Cross product is maximum for perpendicular vectors
  - Cross products of Cartesian unit vectors:

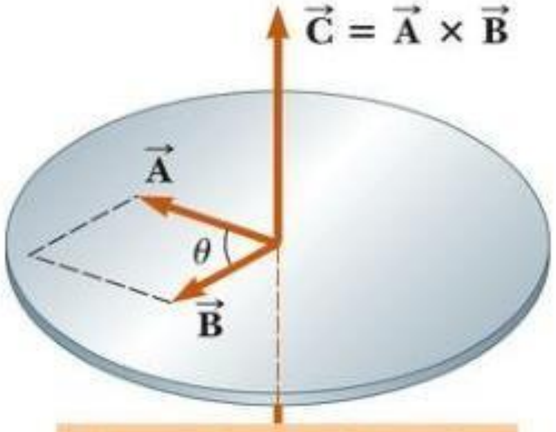
$$\hat{i} \times \hat{j} = \hat{k};\ \hat{i} \times \hat{k} = -\hat{j};\ \hat{j} \times \hat{k} = \hat{i}$$
$$\hat{i} \times \hat{i} = 0;\ \hat{j} \times \hat{j} = 0;\ \hat{k} \times \hat{k} = 0$$





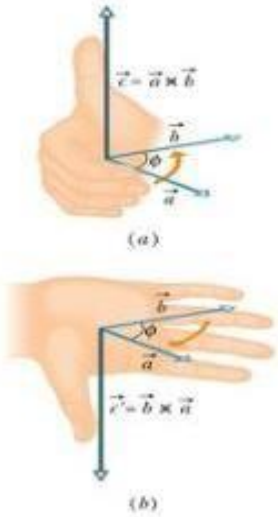
Vector Product or Cross Product

- Direction: C perpendicular to both A and B (right-hand rule)
  - Place A and B tail to tail
  - Right hand, not left hand
  - Four fingers are pointed along the first vector A
  - "sweep" from first vector A into second vector B through the smaller angle between them
  - Your outstretched thumb points the direction of C
- First practice



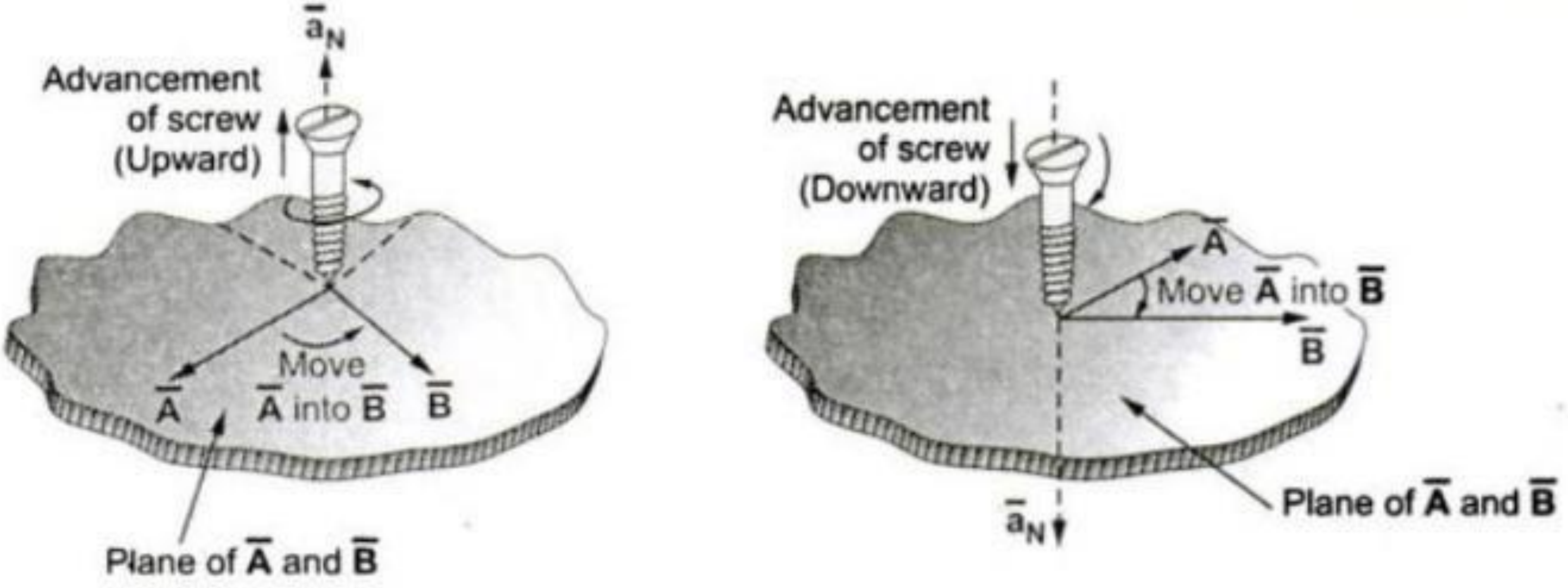
$$\vec{A} \times \vec{B} = \vec{B} \times \vec{A}\ ?$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$



$$\vec{A} \times \vec{B} = \vec{B} \times \vec{A}\ ?$$

Vector Product or Cross Product



$$\bar{A} \times \bar{B} = |\bar{A}|\,|\bar{B}|\sin\theta_{AB}\,\bar{a}_N$$

where $\bar{a}_N$ = Unit vector perpendicular to the plane of $\bar{A}$ and $\bar{B}$ in the direction decided by **right hand screw rule**

θ_{AB} = Smaller angle between $\bar{A}$ and $\bar{B}$

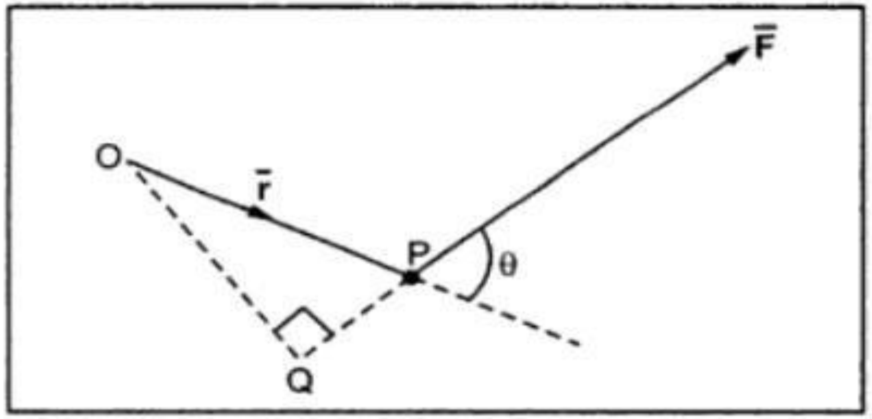
Applications of Cross Product

1. The cross product is the replacement to the right hand rule used in electrical engineering to determine the direction of force experienced by current carrying conductor placed in a magnetic field.

Thus if I is the current flowing through conductor while $\bar{L}$ is the vector length considered to indicate the direction of current through the conductor. The uniform magnetic flux density is denoted by vector $\bar{B}$. Then the force experienced by conductor is given by,

$$\bar{F} = I\,\bar{L} \times \bar{B}$$

2. Another physical quantity which can be represented by cross product is **moment of a force.** The moment of a force (or torque) acting on a rigid body, which can rotate about an axis perpendicular to a plane containing the force is defined to be the magnitude of the force multiplied by the perpendicular distance from the force to the axis.



The moment of force $\bar{F}$ about a point O is $\bar{M}$. Its magnitude is $|\bar{F}|\ |\bar{r}| \sin\theta$ where $|\bar{r}| \sin\theta$ is the perpendicular distance of $\bar{F}$ from O i.e. OQ.

$\therefore \bar{M} = \bar{r} \times \bar{F} = |\bar{r}|\ |\bar{F}| \sin\theta\ \bar{a}_N$ where $\bar{a}_N$ is the unit vector indicating direction of $\bar{M}$ which is perpendicular to the plane i.e. paper and coming out of paper according to right hand screw rule.

Vector Product or Cross Product

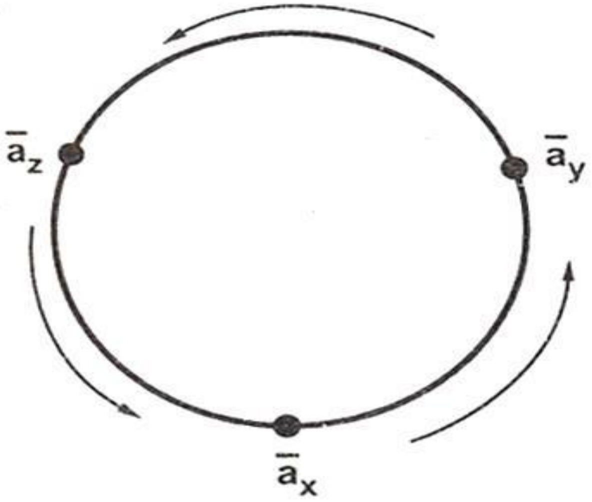
$$\bar{A} \times \bar{B} \neq \bar{B} \times \bar{A} \quad \text{but} \quad \bar{A} \times \bar{B} = -[\bar{B} \times \bar{A}]$$

$$\bar{A} \times (\bar{B} + \bar{C}) = \bar{A} \times \bar{B} + \bar{A} \times \bar{C}$$

For **parallel** vectors, **cross product is zero.**

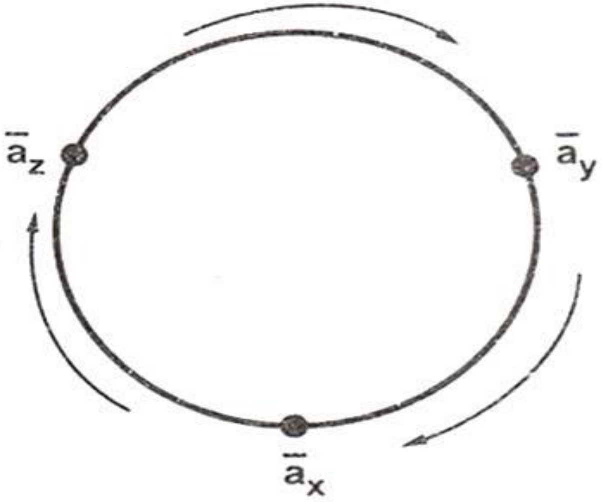
$$\bar{A} \times \bar{A} = 0$$

$$\bar{a}_x \times \bar{a}_x = \bar{a}_y \times \bar{a}_y = \bar{a}_z \times \bar{a}_z = 0$$



Anticlockwise positive result

$\bar{a}_x \times \bar{a}_y = \bar{a}_z$



Clockwise negative result

$\bar{a}_x \times \bar{a}_z = -\bar{a}_y$

Applicable in all the three coordinate systems

Vector Product or Cross Product

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \text{... In cartesian}$$

Note : The result can be used in all coordinate systems by proper replacement of unit vectors and components.

Product of three vectors

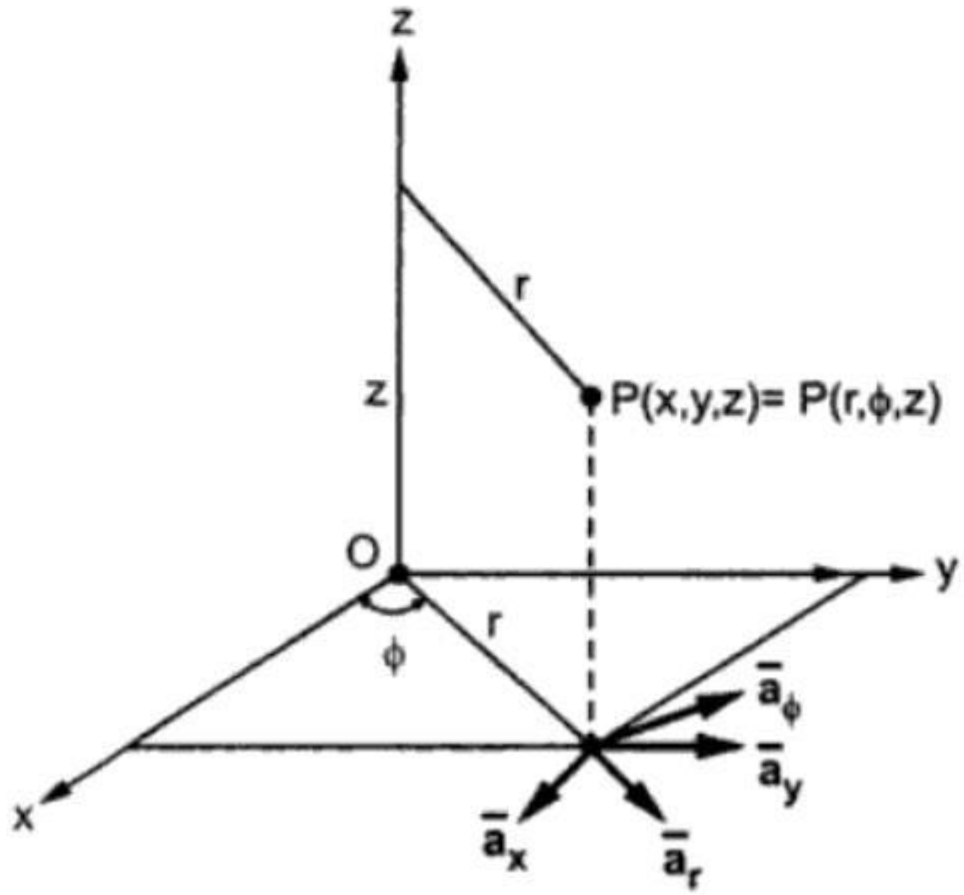
Scalar triple product is,

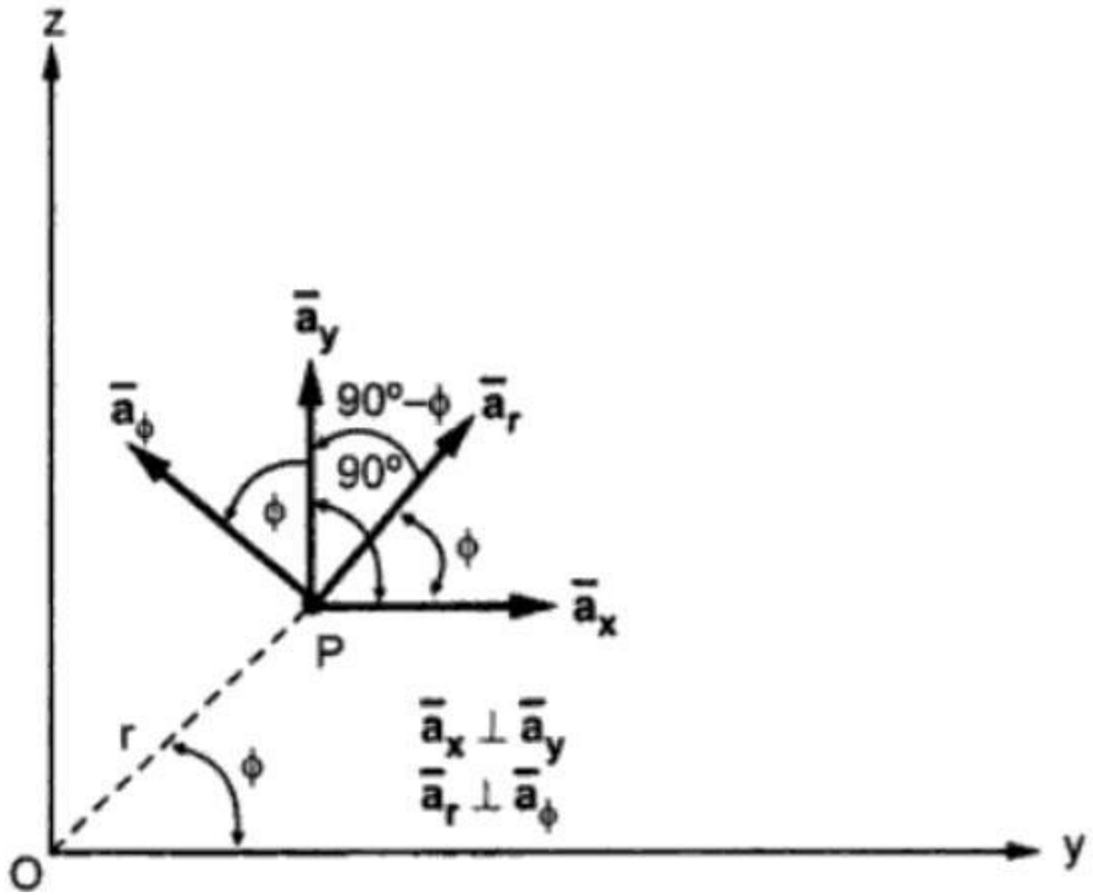
$$\bar{A} \bullet (\bar{B} \times \bar{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \quad \text{.... 'abc' rule}$$

Vector triple product is,

$$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \bullet \bar{C}) - \bar{C}(\bar{A} \bullet \bar{B}) \quad \text{... 'bac-cab' rule}$$

Cartesian to Cylindrical





Transformation of Vectors

The angle between $\bar{a}_x$ and $\bar{a}_r$ is ϕ

The angle between $\bar{a}_y$ and $\bar{a}_r$ is $90-\phi$

The angle between $\bar{a}_x$ and $\bar{a}_\phi$ is $90+\phi$

The angle between $\bar{a}_y$ and $\bar{a}_\phi$ is ϕ

$\therefore \quad \bar{a}_x \bullet \bar{a}_r = (1)(1)\cos(\phi) = \cos\phi$

$\therefore \quad \bar{a}_x \bullet \bar{a}_\phi = (1)(1)\cos(90+\phi) = -\sin\phi$

$\therefore \quad \bar{a}_y \bullet \bar{a}_r = (1)(1)\cos(90-\phi) = \sin\phi$

$\therefore \quad \bar{a}_y \bullet \bar{a}_\phi = (1)(1)\cos(\phi) = \cos\phi$

and $\bar{a}_z \bullet \bar{a}_r = \bar{a}_z \bullet \bar{a}_\phi = 0$ as $\bar{a}_z$ is perpendicular to $\bar{a}_r$ and $\bar{a}_\phi$

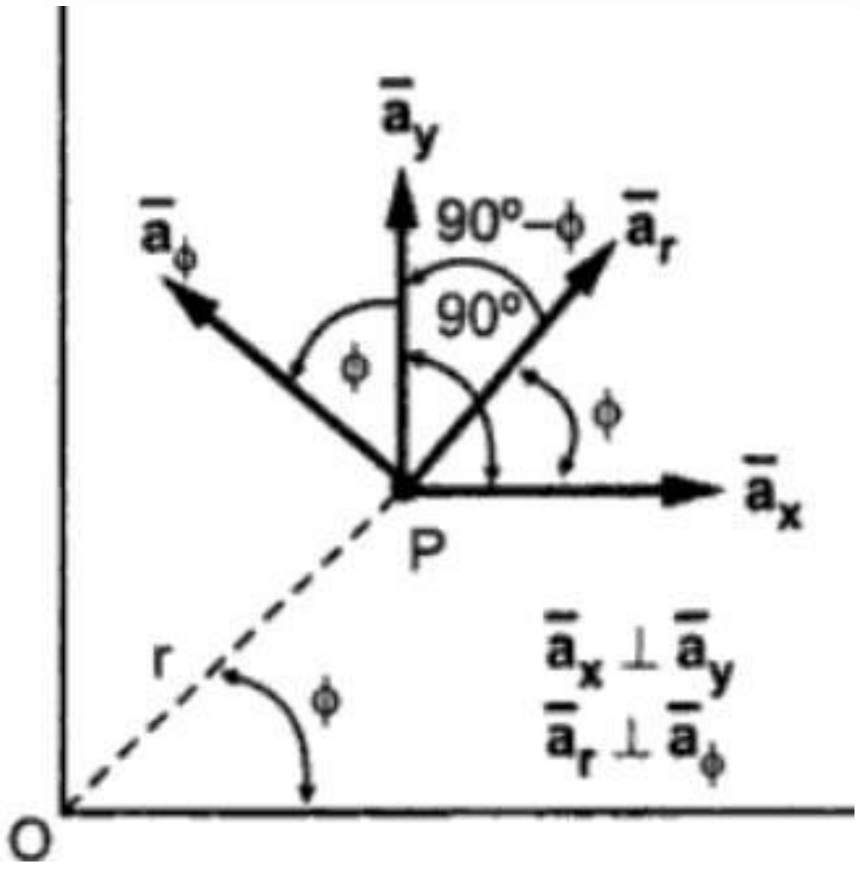
and $\quad \bar{a}_z \bullet \bar{a}_z = 1$

$$A_r = A_x \cos\phi + A_y \sin\phi$$

Similarly finding A_ϕ as $[\bar{A} \bullet \bar{a}_\phi]$ and A_z as $[\bar{A} \bullet \bar{a}_z]$ we get,

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

and $\quad A_z = A_z$



Transformation of Vectors

Cartesian to cylindrical, $\bar{A} = A_x\,\bar{a}_x + A_y\bar{a}_y + A_z\,\bar{a}_z$

$A_r = \bar{A} \bullet \bar{a}_r\ ,\quad A_\phi = \bar{A} \bullet \bar{a}_\phi,\quad A_z = \bar{A} \bullet \bar{a}_z$

Dot operator •	$\bar{a}_r$	$\bar{a}_\phi$	$\bar{a}_z$
$\bar{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\bar{a}_y$	$\sin\phi$	$\cos\phi$	0
$\bar{a}_z$	0	0	1

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Cylindrical to cartesian, $\bar{A} = A_r\,\bar{a}_r + A_\phi\,\bar{a}_\phi + A_z\,\bar{a}_z$

$A_x = \bar{A} \bullet \bar{a}_x\ ,\quad A_y = \bar{A} \bullet \bar{a}_y\ ,\quad A_z = \bar{A} \bullet \bar{a}_z$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix}$$

Transformation of Vectors

Cartesian to spherical, $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_y$

$A_r = \bar{A} \bullet \bar{a}_r$, $A_\theta = \bar{A} \bullet \bar{a}_\theta$, $A_\phi = \bar{A} \bullet \bar{a}_\phi$

Dot operator •	$\bar{a}_r$	$\bar{a}_\theta$	$\bar{a}_\phi$
$\bar{a}_x$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\bar{a}_y$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\bar{a}_z$	$\cos\theta$	$-\sin\theta$	0

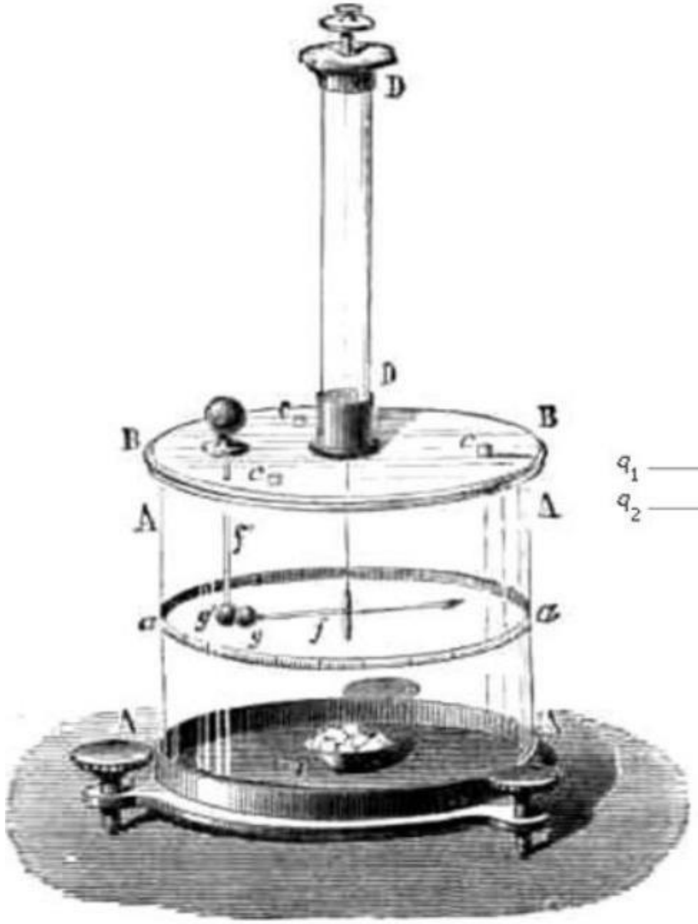
$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

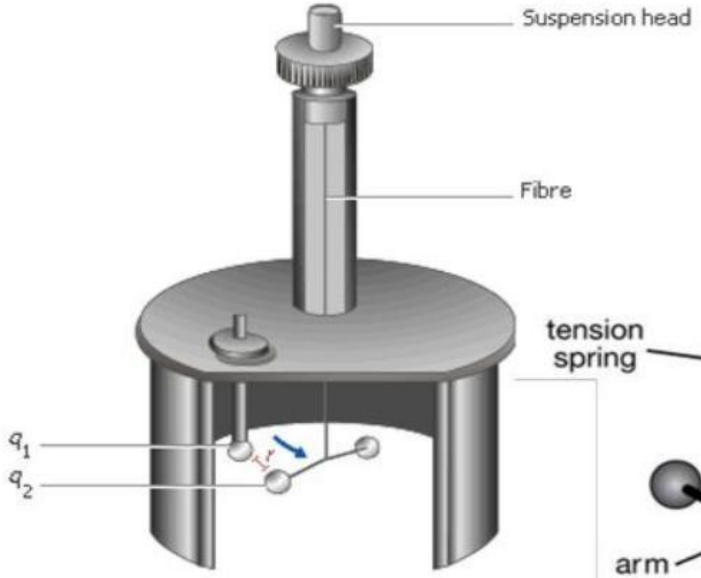
Spherical to cartesian, $\bar{A} = a_r \bar{a}_r + A_\theta \bar{a}_\theta + A_\phi \bar{a}_\phi$

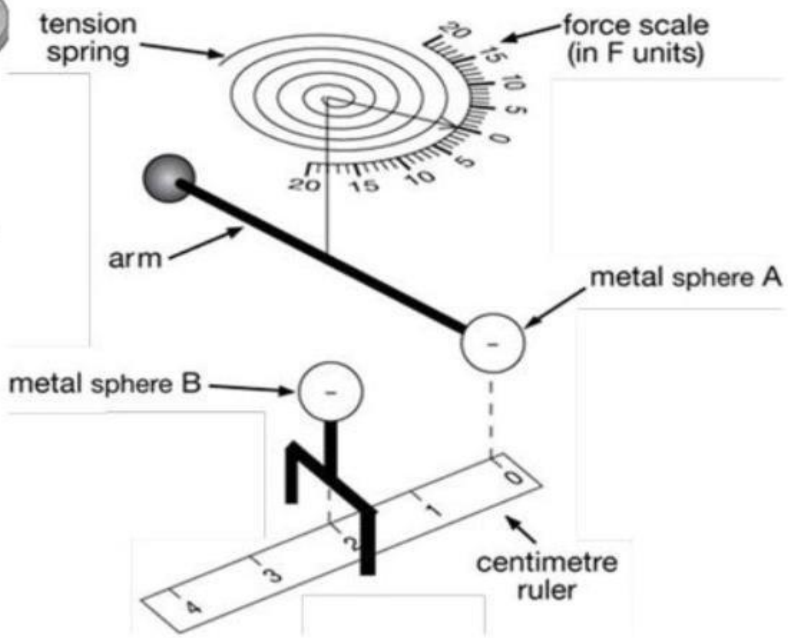
$A_x = \bar{A} \bullet \bar{a}_x$, $A_y = \bar{A} \bullet \bar{a}_\theta$, $A_z = \bar{A} \bullet \bar{a}_z$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

In 1785 **Charles Coulomb** used a **torsion balance** to m



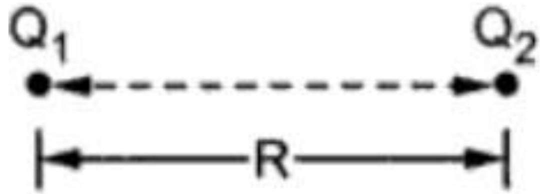




Coulomb's Law

The Coulomb's law states that force between the two point charges Q_1 and Q_2,

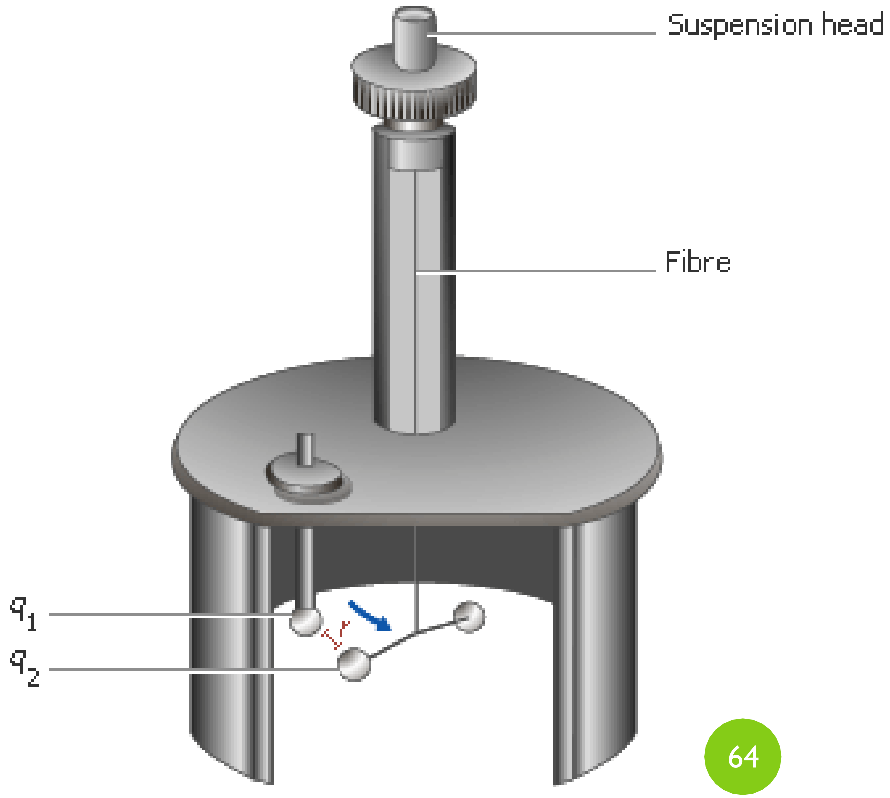
1. acts along the line joining the two point charges.
2. is directly proportional to the product (Q_1Q_2) of the two charges.
3. is inversely proportional to the square of the distance between them.



$$F \propto \frac{Q_1Q_2}{R^2}$$

Q_1Q_2 = Product of the two charges

R = Distance between the two charges



Coulomb's Law

The Coulomb's law also states that this force depends on the **medium** in which the point charges are located. The effect of medium is introduced in the equation of force as a constant of proportionality denoted as k.

$$\therefore \qquad \boxed{F = k\frac{Q_1 Q_2}{R^2}}$$

where k = Constant of proportionality

$$\boxed{k = \frac{1}{4\pi\varepsilon}}$$

where ε = Permittivity of the medium in which charges are located

The units of ε are farads/metre (F/m).

In general ε is expressed as,

$$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$$

where ε_0 = Permittivity of the free space or vacuum

ε_r = Relative permittivity or dielectric constant of the medium with respect to free space

ε = Absolue permittivity

For the **free space** or **vacuum**, the relative permittivity $\varepsilon_r = 1$, hence

$$\varepsilon = \varepsilon_0$$

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}$$

The value of permittivity of free space ε_0 is,

$$\boxed{\varepsilon_0 = \frac{1}{36\pi}\times 10^{-9} = 8.854\times 10^{-12} \text{ F/m}}$$

$$\therefore \quad k = \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi\times 8.854\times 10^{-12}} = 8.98\times 10^{9} \approx 9\times 10^{9} \text{ m/F}$$

Hence the Coulomb's law can be expressed as,

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

This is the force between the two point charges located in **free space** or **vacuum**.

$$\text{Unit of } \varepsilon_0 = \frac{(C)\,(C)}{(N)\ (m^2)} = \frac{C^2}{N-m^2} = \frac{C^2}{N\text{-}m}\times\frac{1}{m}$$

$$\frac{C^2}{N\text{-}m} = \text{Farad} \quad \text{which is practical unit of capacitance}$$

$$\boxed{\text{Unit of } \varepsilon_0 = \text{F/m}}$$

Vector form of Coulomb's Law

$$\bar{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\bar{a}_{12}$$

$$\bar{a}_{12} = \text{Unit vector along } \bar{R}_{12} = \frac{\text{Vector}}{\text{Magnitude of vector}}$$

$$\bar{a}_{12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{R}_{12}|} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|}$$

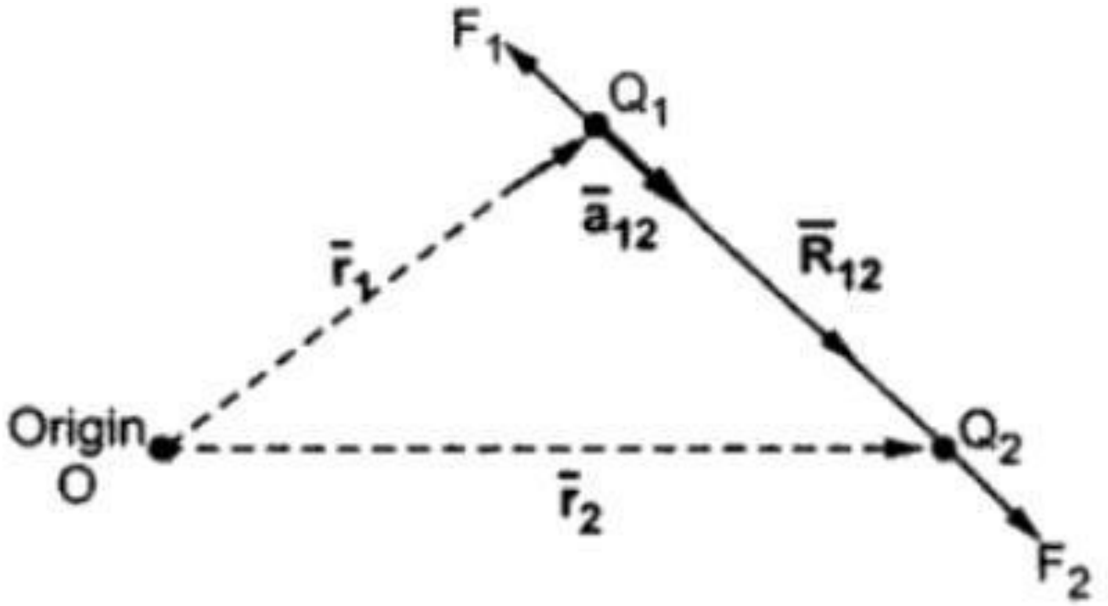
$$|\bar{R}_{12}| = R = \text{distance between the two charges}$$

$$\bar{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2}\bar{a}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2} = \frac{\bar{r}_1 - \bar{r}_2}{|\bar{r}_1 - \bar{r}_2|}$$

$$\bar{r}_1 - \bar{r}_2 = -[\bar{r}_2 - \bar{r}_1]$$

$$\bar{a}_{21} = -\bar{a}_{12}$$

$$\bar{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2}(-\bar{a}_{21}) = -\bar{F}_2$$



Vector form of Coulomb's Law

The like charges repel each other while the unlike charges attract each other.

It is necessary that the two charges are the point charges and stationary in nature.

The two point charges may be positive or negative. Hence their **signs must be considered** to calculate the force exerted.

The Coulomb's law is linear which shows that if any one charge is increased 'n' times then the force exerted also increase by n times.

$\therefore \qquad \bar{F}_2 = -\bar{F}_1$ then $n\bar{F}_2 = -n\bar{F}_1$

where $\qquad n =$ scalar

Force due to 'n' Charges

$$\bar{F}_{Q1Q} = \frac{Q_1 Q}{4\pi\varepsilon_0 R_{1Q}^2}\bar{a}_{1Q}$$

$$\bar{a}_{1Q} = \frac{\bar{r}-\bar{r}_1}{|\bar{r}-\bar{r}_1|}$$

$$\bar{F}_{Q2Q} = \frac{Q_2 Q}{4\pi\varepsilon_0 R_{2Q}^2}\bar{a}_{2Q}$$

where $$\bar{a}_{2Q} = \frac{\bar{r}-\bar{r}_2}{|\bar{r}-\bar{r}_2|}$$

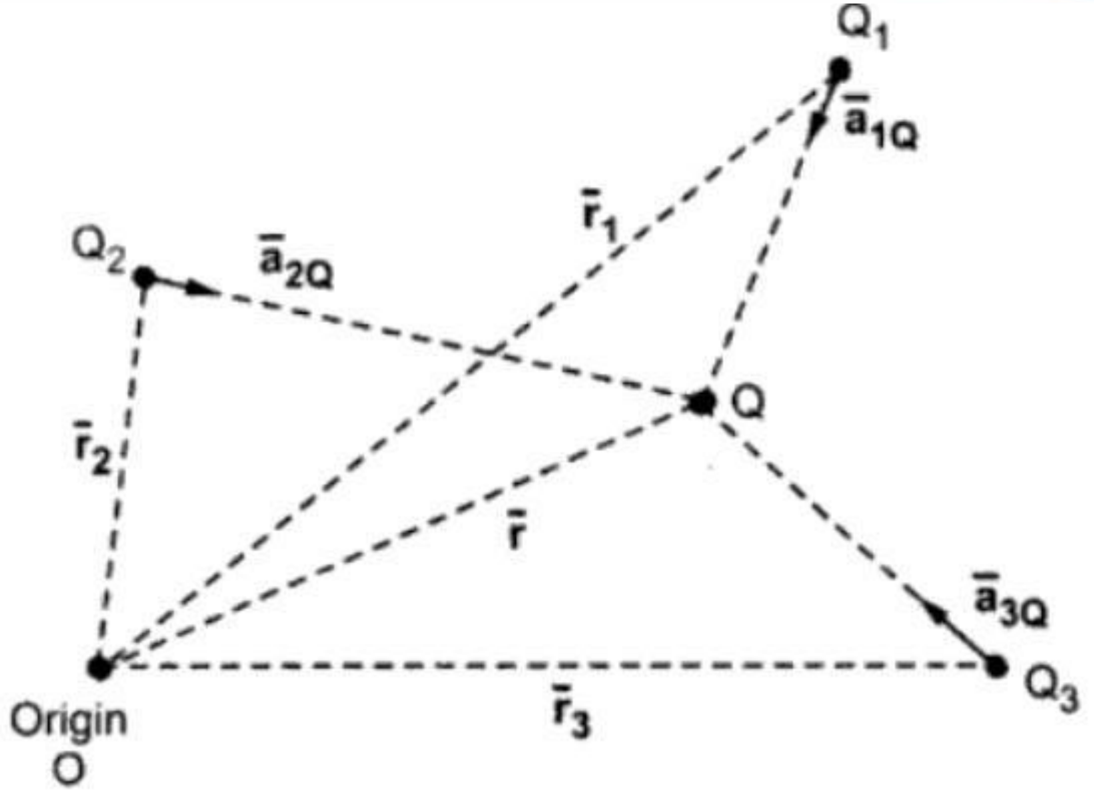
And force exerted due to Q_3 on Q is,

$$\bar{F}_{Q3Q} = \frac{Q_3 Q}{4\pi\varepsilon_0 R_{3Q}^2}\bar{a}_{3Q}$$

where $$\bar{a}_{3Q} = \frac{\bar{r}-\bar{r}_3}{|\bar{r}-\bar{r}_3|}$$

Hence the total force on Q is,

$$\bar{F}_Q = \bar{F}_{Q1Q} + \bar{F}_{Q2Q} + \bar{F}_{Q3Q}$$



$$\bar{F}_Q = \bar{F}_{Q1Q} + \bar{F}_{Q2Q} + \ldots\ldots + \bar{F}_{QnQ}$$

$$\boxed{\bar{F}_Q = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{Q_i\;\bar{r}-\bar{r}_i}{R_{iQ}^2\,|\bar{r}-\bar{r}_i|}}$$

Coulomb's Law : Steps to solve Problems

Step 1 : Obtain the position vectors of the points where the charges are located.

Step 2 : Obtain the unit vector along the straight line joining the charges. The direction is towards the charge on which the force exerted is to be calculated.

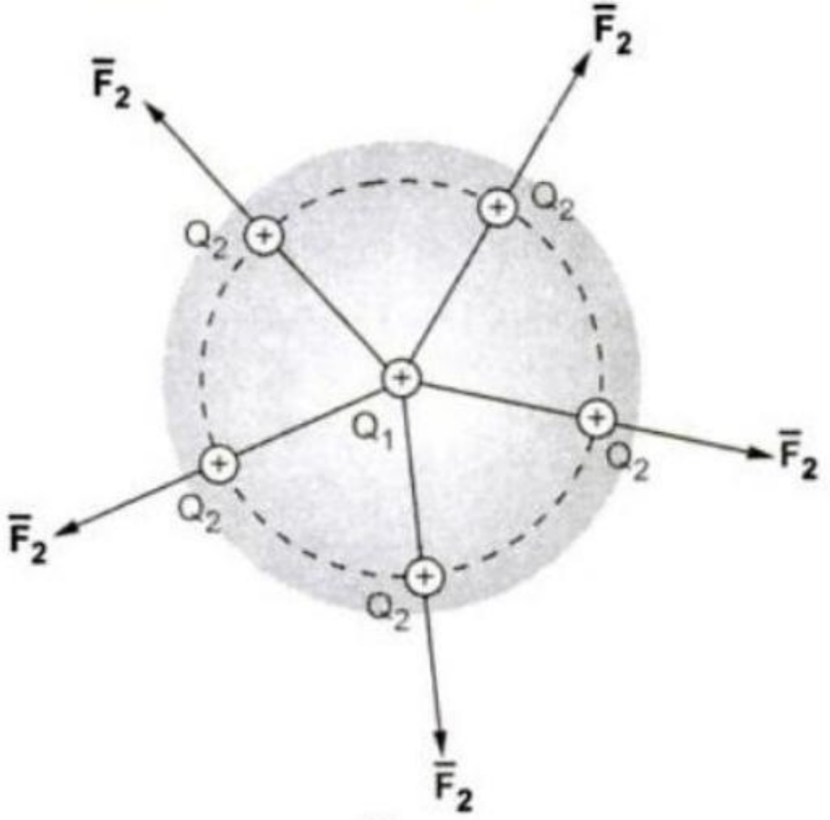
Step 3 : Using Coulomb's law, express the force exerted in the vector form.

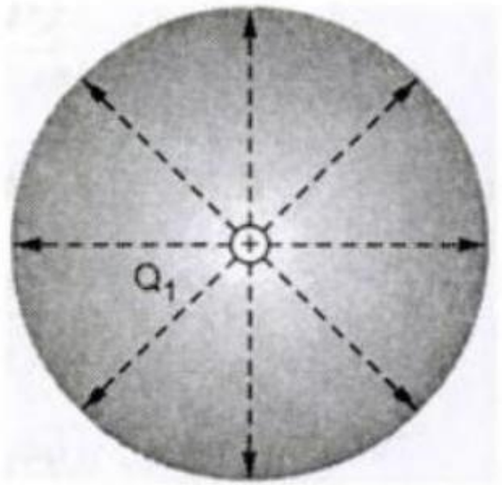
Step 4 : If there are more charges, repeat steps 1 to 3 for each charge exerting a force on the charge under consideration.

Step 5 : Using the principle of superposition, the vector sum of all the forces calculated earlier is the resultant force, exerted on the charge under consideration.

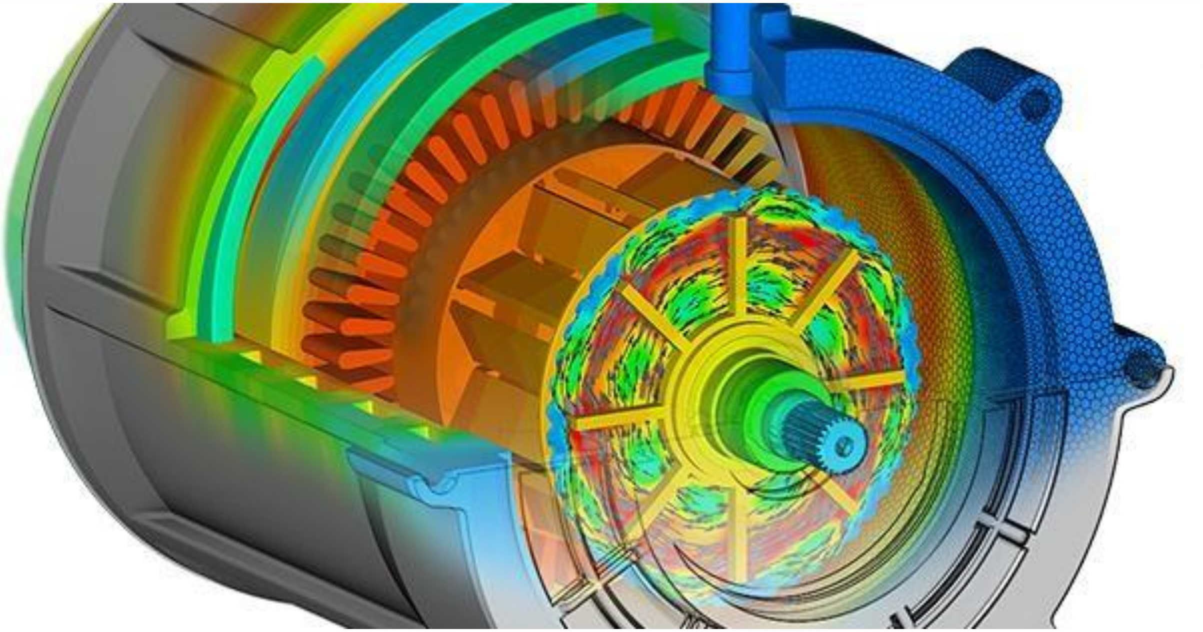
Electric Field Intensity

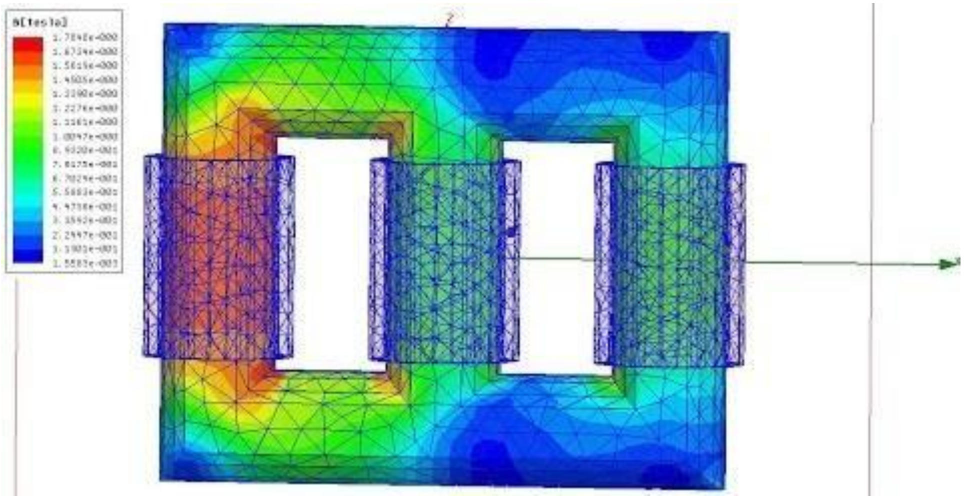
An electric field is an elegant way of characterising the electrical environment of a system of charges.

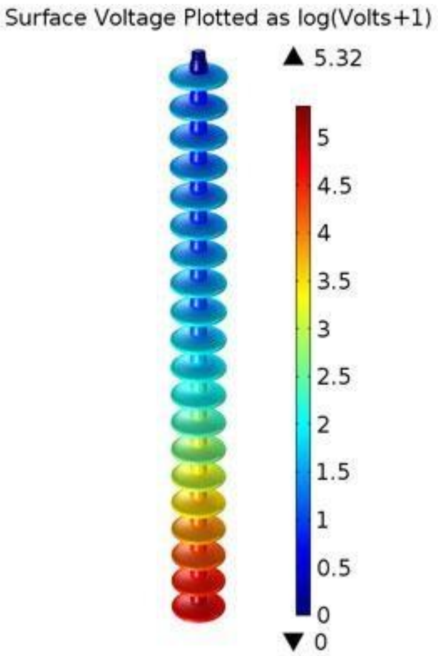




Electric Field Intensity







Electric Field Intensity

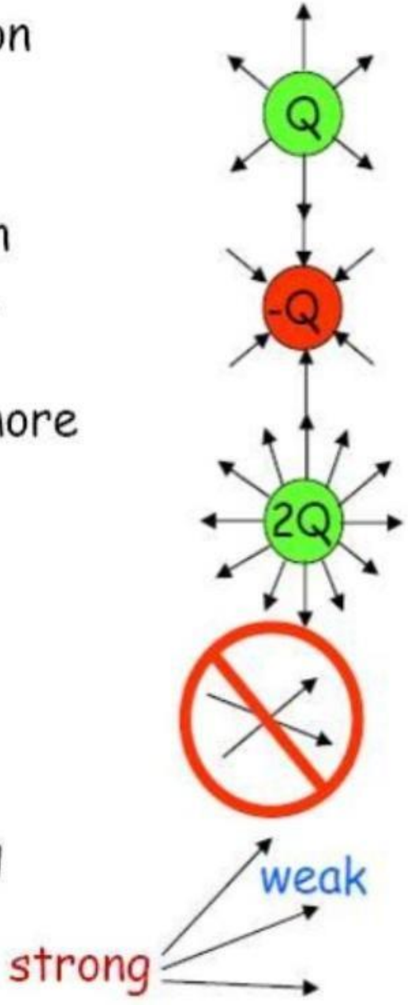
Field lines ***start*** on **positive** charges.

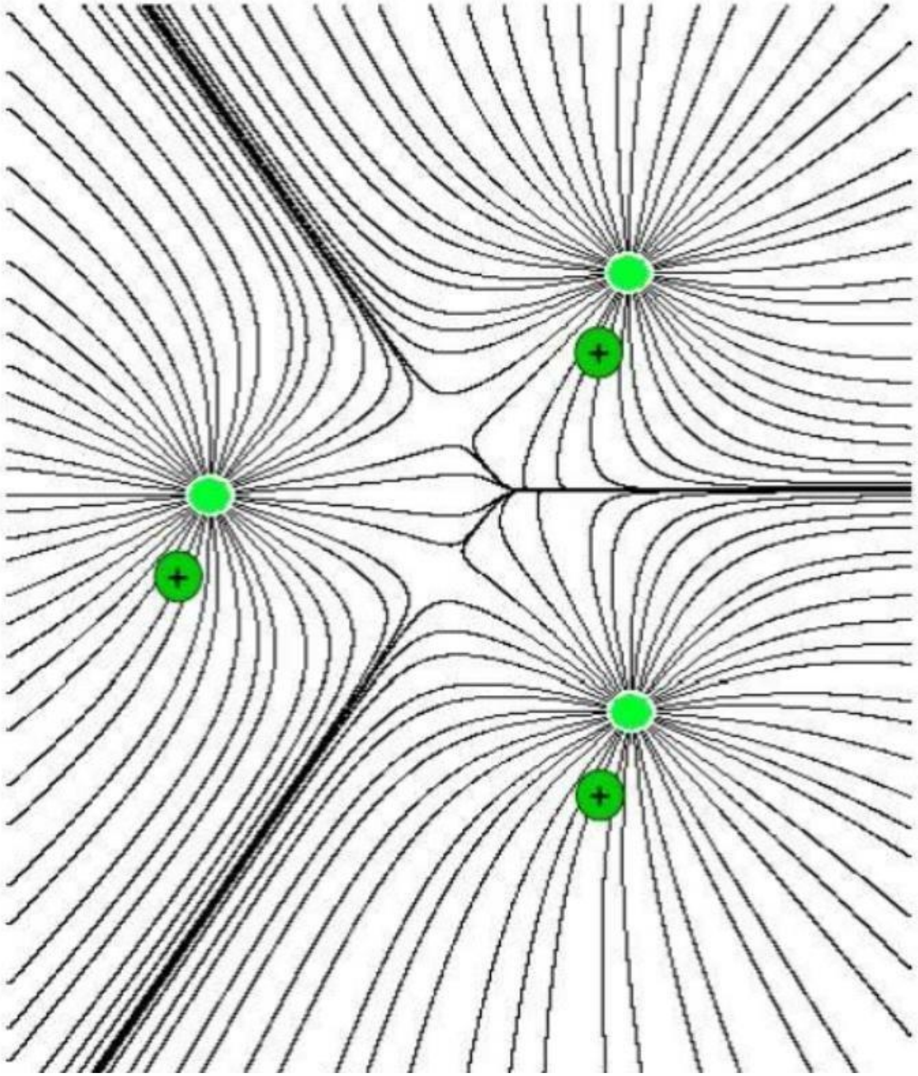
Field lines ***stop*** on **negative** charges.

More charge ⇒ more field lines.

Field lines never cross.

Field line spacing indicates field strength





Electric Field Intensity

The force experienced by the charge Q_2 due to Q_1 is given by Coulomb's law as,

$$\bar{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\bar{a}_{12}$$

Thus force per unit charge can be written as,

$$\frac{\bar{F}_2}{Q_2} = \frac{Q_1}{4\pi\varepsilon_0 R_{12}^2}\bar{a}_{12}$$

This force exerted per unit charge is called **electric field intensity** or **electric field strength**. It is a **vector quantity** and is directed along a segment from the charge Q_1 to the position of any other charge. It is denoted as $\bar{E}$.

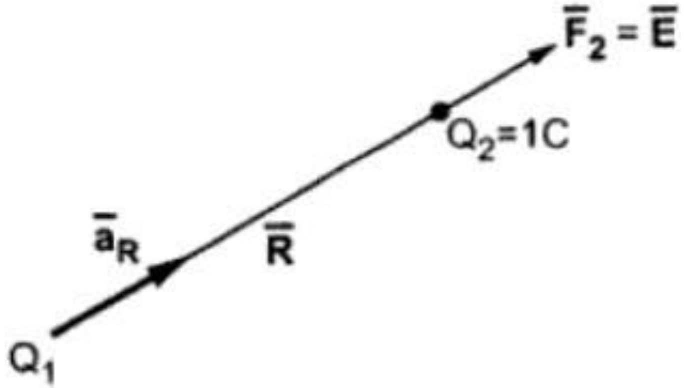
$$\therefore \quad \boxed{\bar{E} = \frac{Q_1}{4\pi\varepsilon_0 R_{1p}^2}\bar{a}_{1p}}$$

where $\quad p$ = position of any other charge around Q_1

Electric Field Intensity : Units

$$\bar{E} = \frac{Q_1}{4\pi\varepsilon_0 R^2}\bar{a}_R$$

$$\bar{E} = \frac{Q_1}{4\pi\varepsilon_0 r^2}\bar{a}_r \text{ in spherical system}$$



The definition of electric field intensity is,

$$\bar{E} = \frac{\text{Force}}{\text{Unit charge}} = \frac{\text{(N) Newtons}}{\text{(C) Coulomb}}$$

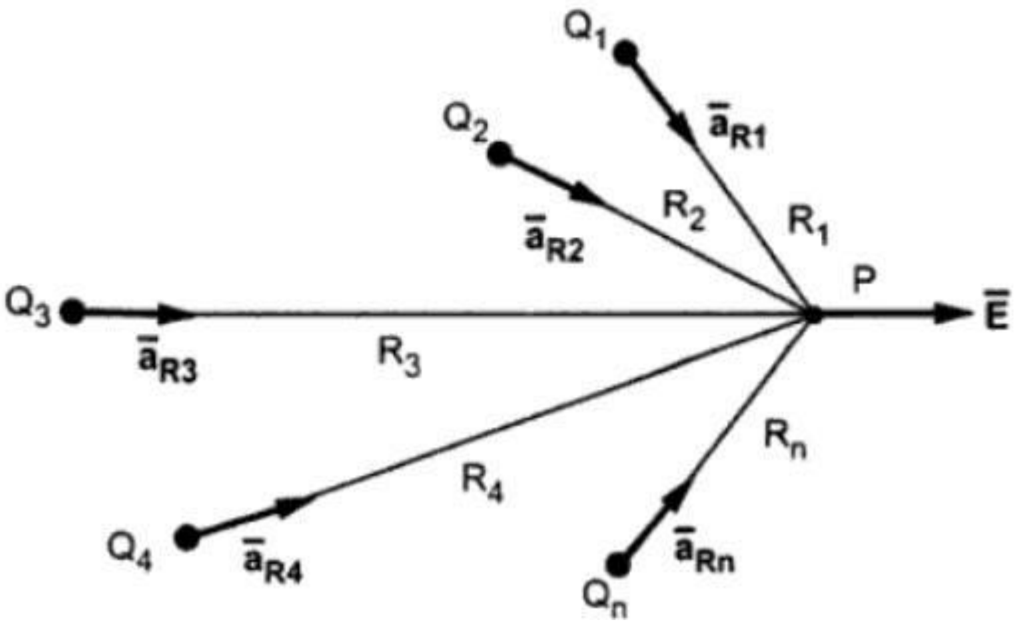
Hence units of $\bar{E}$ is N/C. But the electric potential has units J/C i.e. Nm/C and hence $\bar{E}$ is also measured in units **V/m (volts per metre)**. This unit is used practically to express $\bar{E}$.

Electric Field Intensity : n Charges

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \ldots + \bar{E}_n$$

$$= \frac{Q_1}{4\pi\varepsilon_0 R_1^2}\bar{a}_{R1} + \frac{Q_2}{4\pi\varepsilon_0 R_2^2}\bar{a}_{R2} + \ldots + \frac{Q_n}{4\pi\varepsilon_0 R_n^2}\bar{a}_{Rn}$$

$$\boxed{\bar{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{Q_i}{R_i^2}\bar{a}_{Ri}}$$



$$\bar{a}_{Ri} = \frac{\bar{r}_P - \bar{r}_i}{|\bar{r}_P - \bar{r}_i|}$$

$\bar{r}_P$ = Position vector of point P

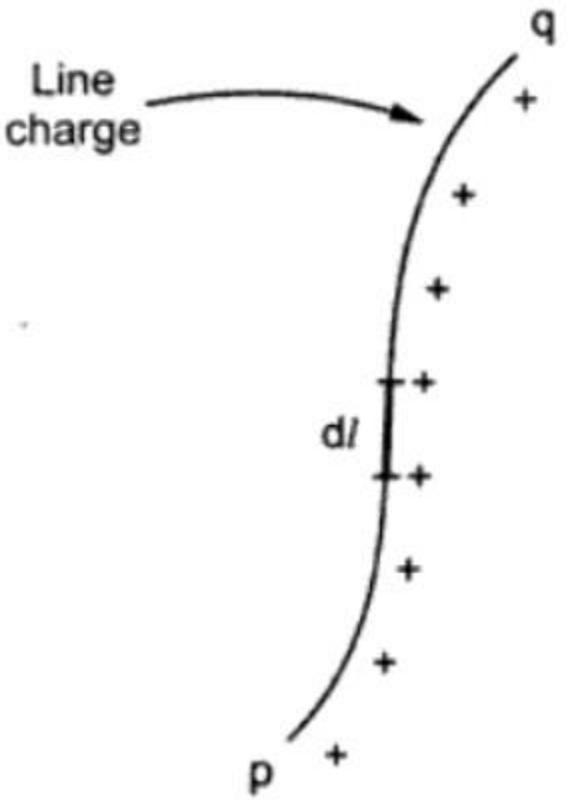
$\bar{r}_i$ = Position vector of point where charge Q_i is placed.

Electric Field Intensity : Observations

The important observations related to $\bar{E}$ are,

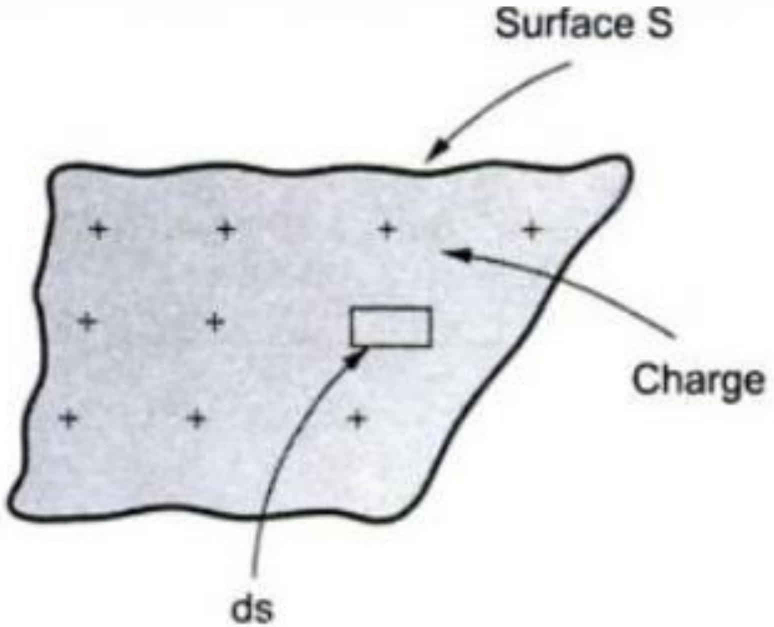
1. $\bar{E}$ around a charge Q_1 is directly proportional to the charge Q_1.
2. $\bar{E}$ around a charge Q_1 is inversely proportional to the distance between charge Q_1 and point at which $\bar{E}$ is to be calculated. More is the distance less is the electric field intensity and less will be the force experience by a unit charge placed at that point.
3. The field intensity $\bar{E}$ at any point and force $\bar{F}$ exerted on a charge placed at the same point are always in the same direction.
4. Placing unit charge is a method of detecting the presence of electric field around a charge. Without any unit test charge placed nearby, every charge has its electric field always existing, around it.
5. The test charge placed must be small enough so that the electric field intensity $\bar{E}$ to be measured should not get disturbed.

Charge Configurations



$$Q = \int_L \rho_L \, dl$$

$$\rho_L = \frac{\text{Total charge in coulomb}}{\text{Total length in metres}} \ (\text{C/m})$$

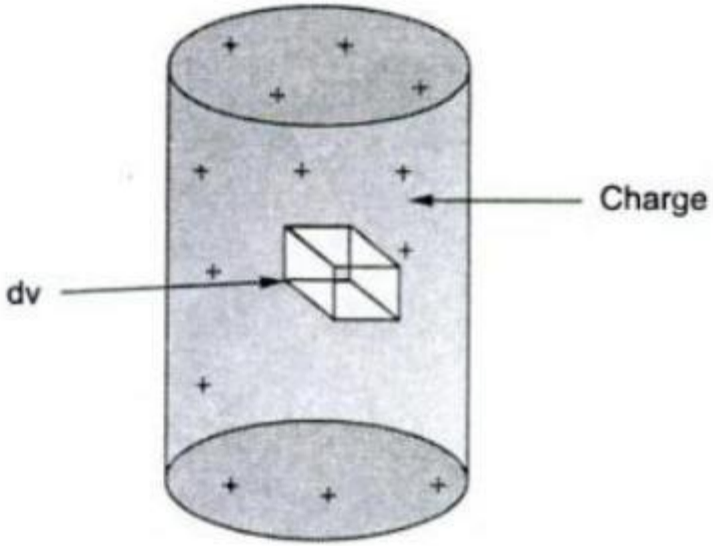


$$Q = \int_S \rho_s \, dS$$

ρ_s = Surface charge density in C/m^2

dS = Elementary surface

$$\rho_S = \frac{\text{Total charge in coulomb}}{\text{Total area in square metres}} \ (\text{C}/\text{m}^2)$$



$$Q = \int_v \rho_v \, dv$$

dv = Elementary volume

$$\rho_v = \frac{\text{Total charge in coulomb}}{\text{Total volume in cubic metres}} \left(\frac{\text{C}}{\text{m}^3}\right)$$

Charge Configurations

1. Point	2. Line	3. Surface	4. Volume

Line charge ρ_L = Line charge density in C/m

$$Q = \int_L \rho_L \, dl = \text{Line integral.}$$

Surface charge ρ_S = Surface charge density in C/m^2

$$Q = \int_S \rho_S \, dS = \text{Surface integral}$$

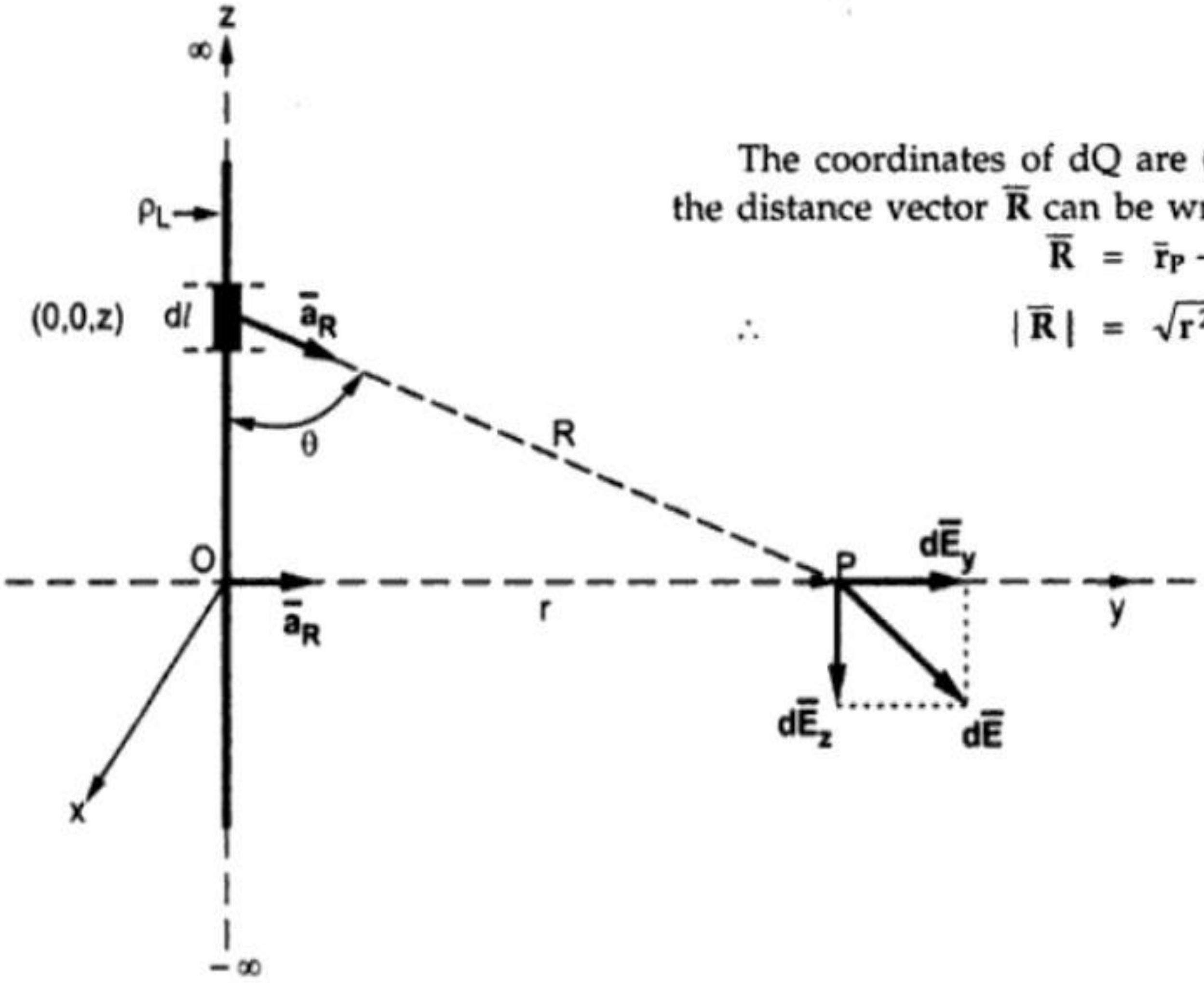
This is double integral.

Volume charge ρ_v = Volume charge density in C/m^3.

$$Q = \int_v \rho_v \, dv = \text{Volume integral}$$

This is triple integral.

E due to Infinite Line Charge



$$dQ = \rho_L \, dl = \rho_L \, dz$$

The coordinates of dQ are (0, 0, z) while the coordinates of point P are (0, r, 0). Hence the distance vector $\bar{R}$ can be written as,

$$\bar{R} = \bar{r}_P - \bar{r}_{dl} = [r\,\bar{a}_y - z\,\bar{a}_z]$$

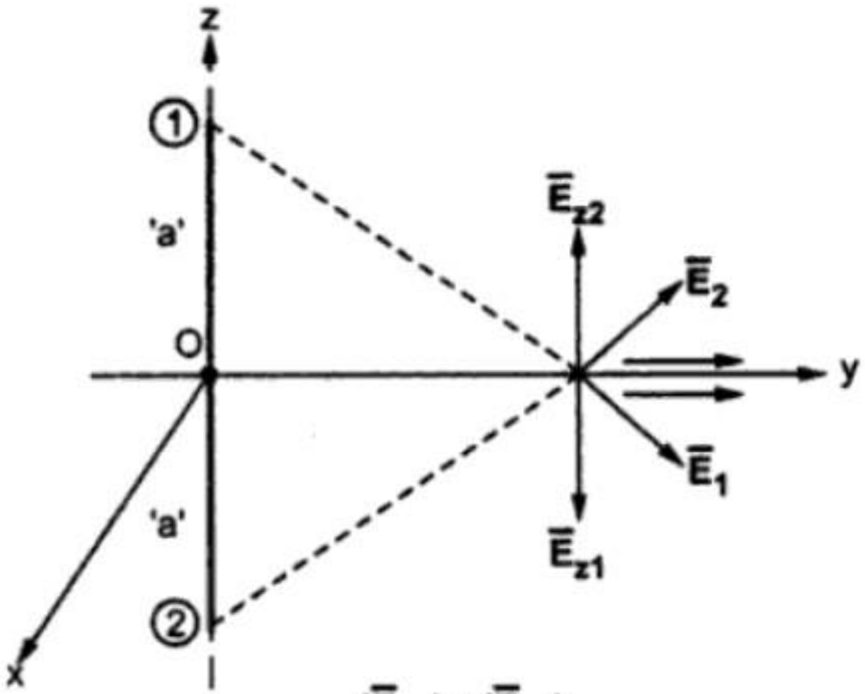
$$\therefore \quad |\bar{R}| = \sqrt{r^2 + z^2}$$

$$\therefore \quad \bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{r\,\bar{a}_y - z\,\bar{a}_z}{\sqrt{r^2 + z^2}}$$

$$\therefore \quad d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$$

$$= \frac{\rho_L \, dz}{4\pi\varepsilon_0 \left(\sqrt{r^2+z^2}\right)^2}\left[\frac{r\,\bar{a}_y - z\,\bar{a}_z}{\sqrt{r^2+z^2}}\right]$$

E due to Infinite Line Charge



$$\bar{E} = \int_{-\infty}^{\infty} \frac{\rho_L}{4\pi\varepsilon_0 (r^2 + z^2)^{3/2}} \, r \, dz \, \bar{a}_y$$

$$\left.\begin{array}{l} \text{For} \quad z = -\infty, \quad \theta = \tan^{-1}(-\infty) = -\pi/2 = -90^\circ \\ \text{For} \quad z = +\infty, \quad \theta = \tan^{-1}(\infty) = \pi/2 = +90^\circ \end{array}\right\} \text{changing the limits}$$

$$\therefore \quad \bar{E} = \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_L}{4\pi\varepsilon_0 \left[r^2 + r^2 \tan^2\theta\right]^{3/2}} \, r \times r \sec^2\theta \, d\theta \, \bar{a}_y$$

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \bar{a}_y \quad \text{V/m}}$$

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \bar{a}_r \quad \text{V/m}}$$

where r = Perpendicular distance of point P from the line charge

$\bar{a}_r$ = Unit vector in the direction of the perpendicular distance of point P from the line charge

E due to Charged Circular Ring

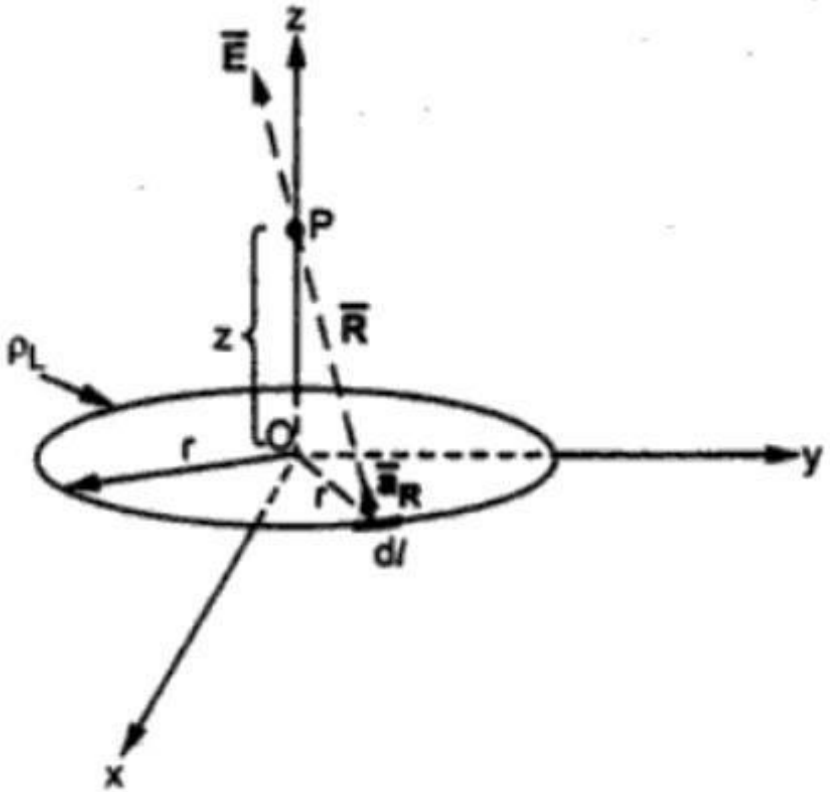
$$\therefore \quad dQ = \rho_L \, dl$$

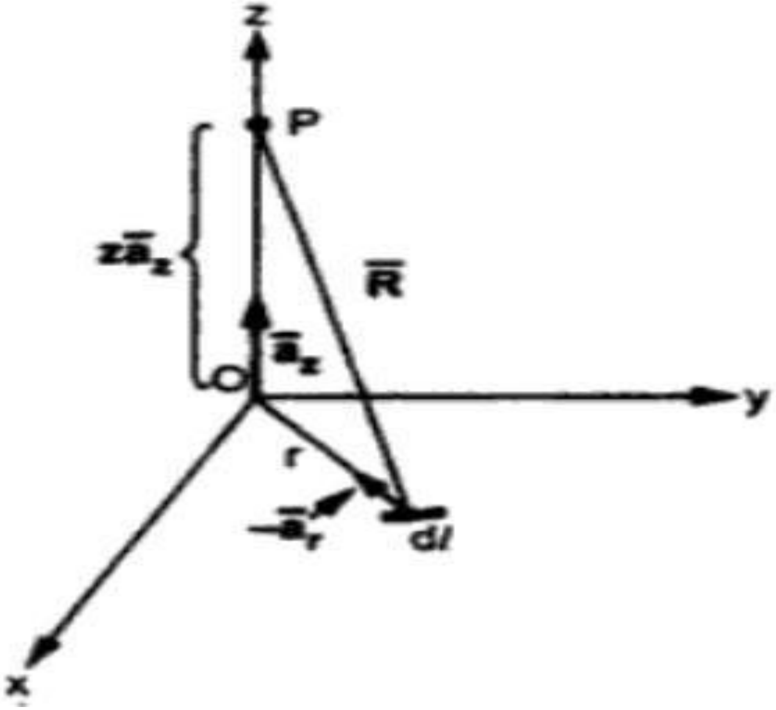
$$\therefore \quad d\bar{E} = \frac{\rho_L \, dl}{4\pi\varepsilon_0 R^2} \bar{a}_R$$

where R = Distance of point P from dl

$$dl = r \, d\phi$$

$$R^2 = r^2 + z^2$$





1) distance r in the direction of $-\bar{a}_r$, radially inwards i.e. $-r\bar{a}_r$.

2) distance z in the direction of $\bar{a}_z$ i.e. $z\,\bar{a}_z$

$$\therefore \quad \bar{R} = -r\bar{a}_r + z\bar{a}_z$$

E due to Charged Circular Ring

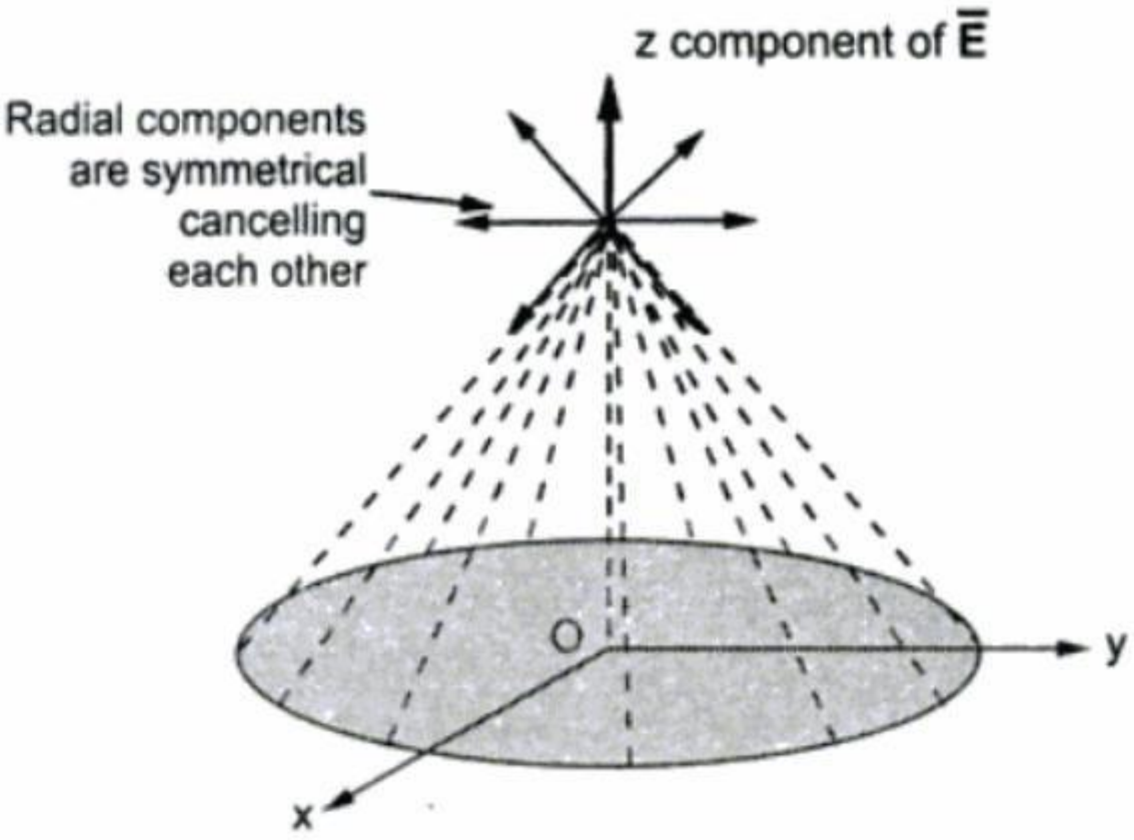
$$|\bar{R}| = \sqrt{(-r)^2 + (z)^2} = \sqrt{r^2 + z^2}$$

$$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-r\bar{a}_r + z\bar{a}_z}{\sqrt{r^2 + z^2}}$$

$$d\bar{E} = \frac{\rho_L \, dl}{4\pi\varepsilon_0 \left(\sqrt{r^2 + z^2}\right)^2} \times \frac{-r\bar{a}_r + z\bar{a}_z}{\sqrt{r^2 + z^2}}$$

$$d\bar{E} = \frac{\rho_L \, (r \, d\phi)}{4\pi\varepsilon_0 (r^2 + z^2)^{3/2}} z\bar{a}_z$$

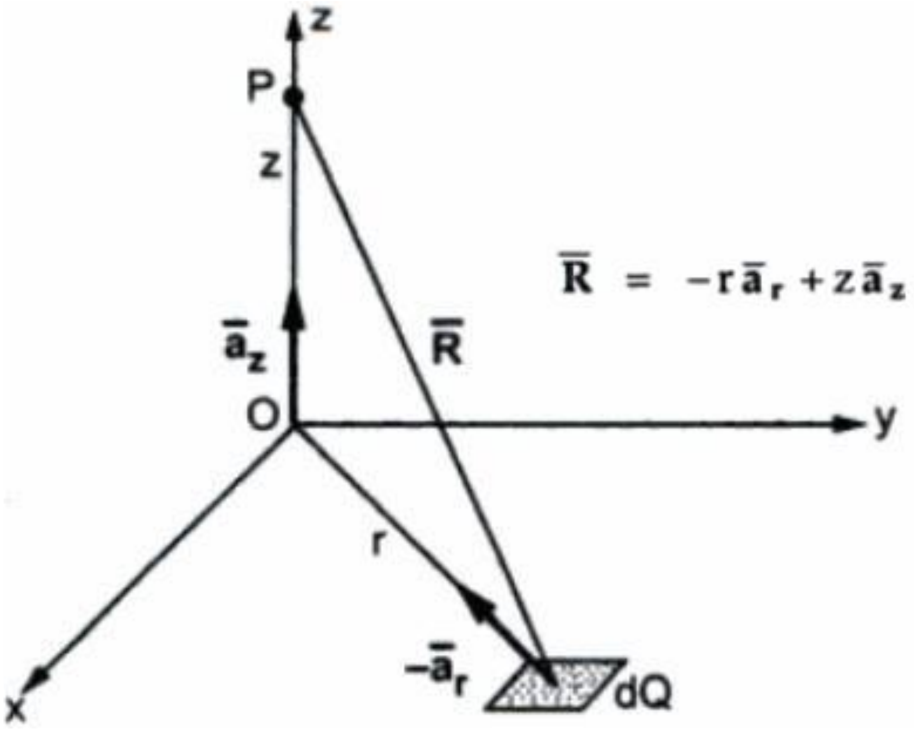
$$\therefore \quad \bar{E} = \frac{\rho_L \, r z}{2\varepsilon_0 (r^2 + z^2)^{3/2}} \bar{a}_z$$



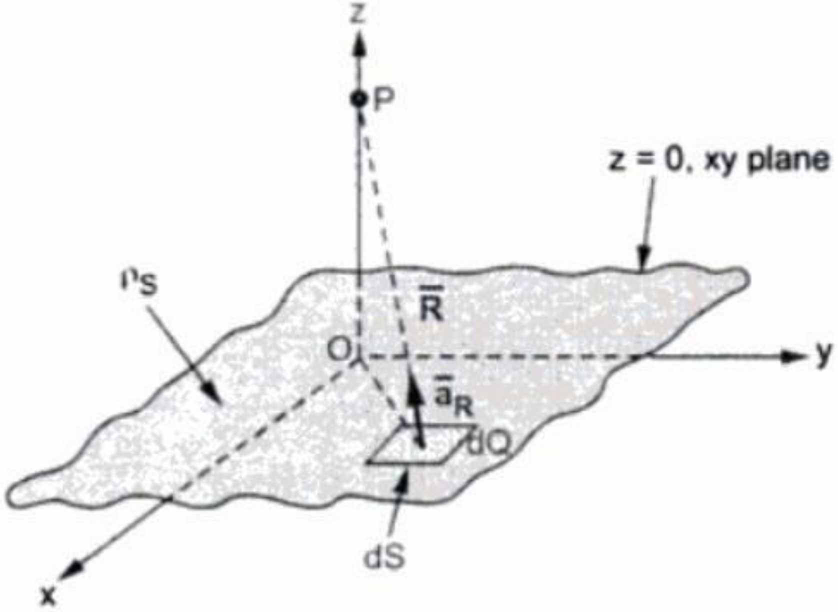
E due to Infinite Sheet of Charge

$$dQ = \rho_S\, dS = \rho_S\; r\, dr\, d\phi$$

$$d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\bar{a}_R = \frac{\rho_S\, r\, dr\, d\phi}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$$



$$\bar{R} = -r\,\bar{a}_r + z\,\bar{a}_z$$



$$d\bar{E} = \frac{\rho_S\, r\, dr\, d\phi}{4\pi\varepsilon_0\left(\sqrt{r^2+z^2}\right)^2}\left[\frac{-r\,\bar{a}_r + z\,\bar{a}_z}{\sqrt{r^2+z^2}}\right]$$

$$\bar{E} = \int_{\phi=0}^{2\pi}\int_{r=0}^{\infty} d\bar{E} = \int_0^{2\pi}\int_0^{\infty} \frac{\rho_S\, r\, dr\, d\phi}{4\pi\varepsilon_0\left(r^2+z^2\right)^{3/2}}\left(z\,\bar{a}_z\right)$$

$$\boxed{\bar{E} = \frac{\rho_S}{2\varepsilon_0}\,\bar{a}_n \quad \text{V/m}}$$

E due to various Configurations

Charge distribution	Field intensity $\bar{E}$ in V/m
Point charge Q C	$\dfrac{Q}{4\pi\varepsilon_0 r^2}\ \bar{a}_r$
Infinite line charge having density ρ_L C/m	$\dfrac{\rho_L}{4\pi\varepsilon_0 r}\ \bar{a}_r$
Infinite surface charge having density ρ_S C / m ^{2}	$\dfrac{\rho_S}{2\varepsilon_0}\ \bar{a}_n$
Vo ume charge having density ρ_v C / m ^{3}	$\dfrac{\int_v \rho_v\, dv}{4\pi\varepsilon_0 r^2}\ \bar{a}_r$

Thank you

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SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY

(Approved by A.I.C.T.E., New Delhi & Affiliated to J.N.T.U.A, Ananthapuramu)
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CHITTOOR DIST., A.P., INDIA

2.3.2 Teachers use ICT enabled tools including online resources for effective teaching and learning process.

S.No	Name of the teacher	Department	Name of the module developed	Platform on which module is developed	Date of launching e content	Name of the e-content development facility	Provide the link of the videos and media centre and recording facility
1	Dr.J.GOWRISHANKAR	EEE	ELECTRIC POTENTIAL,EQUIPOTENTIAL & POTENTIAL GRADIENT	YOU TUBE	12.20 PM	Zoom recording	https://www.youtube.com/watch?v=47ohMdTPPm4

N. Ravi
HOD

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Dept. of Electrical & Electronics Engineering
Siddharth Institute of Engineering & Technology
Siddarth Nagar, Narayanavanam Road
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Name : J. Jahnavi

Roll. No : 19F61A0210

Subject : ELECTRO MAGNETIC FIELD

Section : 01

Course : B.Tech

Sem & year : II & II

10/10

J. Jahnavi
B.Tech II Year
Section:-01

Electro Magnetic Flux Assignment

1) If $A = 10a_x - 4a_y + 6a_z$ and $B = 2a_x + a_y$ then find
(a) The Component of A along a_y.
(b) The magnitude of 3A−B.
(c) A unit vector along A+2B=C.

Sol:- Given,

$$A = 10a_x - 4a_y + 6a_z \quad , B = 2a_x + a_y$$

(a) If $A = 10a_x - 4a_y + 6a_z$, we have that $a_y = -4$; Hence −4 be the Component of A along a_y.

(b) The magnitude of 3A−B is,

$$3A - B = 3(10a_x - 4a_y + 6a_z) - (2a_x + a_y)$$

$$3A - B = 30a_x - 12a_y + 18a_z - 2a_x - a_y$$

$$3A - B = 28a_x - 13a_y + 18a_z$$

$$|3A-B| = \sqrt{(28)^2 + (13)^2 + (18)^2} = \sqrt{784 + 169 + 324} = \sqrt{1277} = 35.74$$

$$\therefore |(3A-B)| = 35.74.$$

Hence, the magnitude of 3A−B is 35.74.

(c) $C = A + 2B$

$$C = 10a_x - 4a_y + 6a_z + 2(2a_x + a_y)$$

$$C = 10a_x - 4a_y + 6a_z + 4a_x + 2a_y = 14a_x - 2a_y + 6a_z$$

Unit Vector is $\frac{C}{|C|} = \frac{14a_x - 2a_y + 6a_z}{\sqrt{(14)^2 + (-2)^2 + (6)^2}} = \frac{14a_x - 2a_y + 6a_z}{2\sqrt{59}}$

∴ A unit vector along A+2B=C is $0.911a_x + (-0.260)a_y + 0.390a_z$

2) Point P and Q are located at (0,2,4) and (-3,1,5)
2Q
(1) The position vector P.
(2) The distance vector from P to Q.
(3) The distance between P and Q.
(4) A vector parallel to PQ with magnitude of 10.

Sol: Given,

P and Q are located at (0,2,4) and (-3,1,5)

(1) The position of vector P is OP ie,

$$\Rightarrow (0-0)a_x + (2-0)a_y + (4-0)a_z$$

$$\Rightarrow 0 + 2a_y + 4a_z$$

$$\therefore r_P = 2a_y + 4a_z$$

(2) The distances vector from P to Q is r_{PQ} i.e,

$$r_{PQ} = r_Q - r_P = (-3-0)a_x + (1-2)a_y + (5-4)a_z$$

$$= -3a_x + (-a_y) + a_z$$

$$\therefore r_{PQ} = -3a_x - a_y + a_z$$

(3) The distance between P and Q is

$$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$PQ = \sqrt{(-3-0)^2 + (1-2)^2 + (5-4)^2} = \sqrt{9+1+1} = \sqrt{11}$$

$$PQ = 3.3166$$

(4) A vector parallel to PQ with magnitude of 10.

Let the required be A, then $A = A_x a_A$.
where $A_x = 10$ is the magnitude of A. since A is parallel to PQ it must have the same unit vector as r_{PQ} (or) r_{QP}

Hence, $a_A = \dfrac{\pm r_{PQ}}{|r_{PQ}|} = \dfrac{\pm(-3,-1,1)}{\sqrt{(-3)^2+(-1)^2+(1)^2}} = \dfrac{\pm(-3,-1,1)}{3.317}$

Hence the required vector A be

$$A = \frac{\pm 10(-3,-1,1)}{3.317}$$

$$A = \pm(-9.045a_x - 3.015a_y + 3.015a_z).$$

Find the angle θ between the two vectors i.e, $A = 3a_x + 4a_y + a_z$ and $B = 2a_y - 5a_z$.

Given vectors,

$$\bar{A} = 3a_x + 4a_y + a_z \;\&\; \bar{B} = 2a_y - 5a_z$$

$$\bar{A}\cdot\bar{B} = (3a_x + 4a_y + a_z)\cdot(2a_y - 5a_z)$$

$$\bar{A}\cdot\bar{B} = (0+8-5)$$

$$\bar{A}\cdot\bar{B} = 3$$

$$|\bar{A}| = \sqrt{(3)^2+(4)^2+(1)^2} = \sqrt{9+16+1} = \sqrt{26}$$

$$|\bar{B}| = \sqrt{(2)^2+(5)^2} = \sqrt{4+25} = \sqrt{29}$$

$$\therefore \cos\theta = \frac{\bar{A}\cdot\bar{B}}{|\bar{A}|\cdot|\bar{B}|}$$

$$\cos\theta = \frac{3}{\sqrt{26}\cdot\sqrt{29}}$$

$$\cos\theta = \frac{3}{\sqrt{754}}$$

$$\theta = \cos^{-1}\left[\frac{3}{\sqrt{754}}\right]$$

$$\theta = 83.7277.$$

∴ The angle between A & B is $\theta = 83.7277$

4) (a) Given point $P(-2, 6, 3)$ and vector $A = y a_x + (x+z) a_y$. Express P & A is in cylindrical & spherical Co-ordinates. Evaluate A at P in the cartesian, cylindrical and spherical systems.

Sol- At point, P ; $x=-2$, $y=6$, $z=3$

Hence $\rho=\sqrt{x^2+y^2}=\sqrt{(-2)^2+(6)^2}=\sqrt{4+36}=\sqrt{40}=6.32$

$\phi=\tan^{-1}(y/x)=\tan^{-1}(6/-2)=\tan^{-1}(-3)=-71.5650+180$

$\phi=108.435°$

$z=3$

$r=\sqrt{x^2+y^2+z^2}=\sqrt{4+36+9}=\sqrt{49}=7$

$\theta=\tan^{-1}\frac{\sqrt{x^2+y^2}}{z}=\tan^{-1}\frac{\sqrt{40}}{3}=64.62°$.

Thus, $P(-2,6,3)=P(6.32,\ 108.435)=P(7,64.62°,108.435°)$

In the cartesian system A at P is $A=6a_x+a_y$

For vector A, $A_x=y$, $A_y=x+z$, $A_z=0$.

Hence it is in cylindrical system.

$$\begin{bmatrix}A_\rho\\A_\phi\\A_z\end{bmatrix}=\begin{bmatrix}\cos\phi & \sin\phi & 0\\-\sin\phi & \cos\phi & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}y\\x+z\\0\end{bmatrix}$$

(or)

$\Rightarrow A_\rho=y(\cos\phi)+(x+z)\sin\phi$

$\Rightarrow A_\phi=-y\sin\phi+(x+z)\sin\phi$

$\Rightarrow A_z=0$

But $x=\rho\cos\phi$, $y=\rho\sin\phi$ and substituting these fields,

$$A=(A_\rho,A_\phi,A_z)=[\rho\cos\phi\sin\phi+(\rho\cos\phi+z)\sin\phi]a_\rho+[-\rho\sin^2\phi+(\rho\cos\phi+z)\cos\phi]a_\phi$$

At P, we have $\rho=\sqrt{40}$, $\tan^{-1}(6/-2)=\phi \Rightarrow \tan\phi=(6/-2)$

If $\tan\phi=6/-2 \Rightarrow \cos\phi=\frac{-2}{\sqrt{40}}$, $\sin\phi=\frac{6}{\sqrt{40}}$

$$A = \left[\sqrt{40}\cdot\frac{-2}{\sqrt{40}}\cdot\frac{6}{\sqrt{40}} + \left(\sqrt{40}\cdot\frac{-2}{\sqrt{40}} + 3\right)\cdot\frac{6}{\sqrt{40}}\right]a_\rho$$

$$+\left[\sqrt{40}\cdot\frac{36}{40} + \sqrt{40}\cdot\frac{-2}{\sqrt{40}} + 3\right)\cdot\frac{-2}{\sqrt{40}}\left]a_\phi\right.$$

$$A = \frac{-6}{\sqrt{40}}a_\rho - \frac{38}{\sqrt{40}}a_\phi$$

$$\therefore A = -0.9487a_\rho - 6.008a_\phi$$

Similarly in the spherical system,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}\begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

(or)

$$\Rightarrow A_r = y\sin\theta\cos\phi + (x+z)\sin\theta\sin\phi$$

$$\Rightarrow A_\theta = y\cos\theta\cos\phi + (x+z)\cos\theta\sin\phi$$

$$\Rightarrow A_\phi = -y\sin\phi + (x+z)\cos\phi$$

But $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

$$A = (A_r, A_\theta, A_\phi)$$

$$A = r\left[\sin^2\theta\cos\phi\sin\phi + (\sin\phi\sin\theta)\cdot(\sin\theta\cos\phi + \cos\theta)\right]a_r +$$

$$r\left[\sin\theta\cos\theta\sin\phi\cos\phi + (\sin\theta\cos\phi + \cos\theta)\cos\theta\sin\phi\right]a_\theta +$$

$$r\left[-\sin\theta\sin^2\phi + (\sin\theta\cos\phi + \cos\theta)\cos\phi\right]a_\phi$$

At P, $r = 7$, $\tan\phi = 6/-2$, $\tan\theta = \sqrt{40}/3$

If $\tan\phi = 6/-2$, $\tan\theta = \sqrt{40}/3$

$\Rightarrow \cos\phi = -2/\sqrt{40}$ $\qquad \cos\theta = 3/7$

$\sin\phi = 6/\sqrt{40}$ $\qquad \sin\theta = \sqrt{40}/7$

$$A = 7\left[\frac{40}{49}\cdot\frac{-2}{\sqrt{40}}\cdot\frac{6}{\sqrt{40}} + \left(\frac{\sqrt{40}}{7}\cdot\frac{-2}{\sqrt{40}} + \frac{3}{7}\right)\cdot\frac{\sqrt{40}}{7}\cdot\frac{6}{\sqrt{40}}\right]a_r$$

$$+ 7\left[\frac{\sqrt{40}}{7}\cdot 3/7\cdot 6/\sqrt{40}\cdot -2/\sqrt{40} + \left(\sqrt{40}/7\cdot -2/\sqrt{40} + 3/7\right)\cdot\frac{3}{7}\cdot\frac{6}{\sqrt{40}}\right]a_\theta +$$

$$7\left[\frac{\sqrt{40}}{7}\cdot\frac{36}{40}+\left(\frac{\sqrt{40}}{7}\cdot\frac{-2}{\sqrt{40}}+\frac{3}{7}\right)\cdot\frac{-2}{\sqrt{40}}\right]a_\phi$$

$$A=\frac{-6}{7}a_r-\frac{18}{7\sqrt{40}}a_\theta-\frac{38}{\sqrt{40}}a_\phi$$

$$A=-0.8571a_r-0.4066a_\theta-6.008a_\phi$$

Hence, $|A|$ is the same in all the three systems, tha

$$|A(x,y,z)|=|A(\rho,\phi,z)|=|A(r,\theta,\phi)|=6.0843.$$

---

5) Express vector $B=\frac{10}{r}a_r+r\cos\theta\, a_\theta+a_\phi$ in Cartesian and cylindrical form. Find $B(-3,4,0)$ & $B(5,\pi/2,-2)$.

3@)

Sol:- Given Vector,

$$B=\frac{10}{r}a_r+r\cos\theta\, a_\theta+a_\phi$$

We have

$$B_x=\frac{10}{r}\sin\theta\cos\phi+r\cos^2\theta\cos\phi-\sin\phi$$

$$B_y=\frac{10}{r}\sin\theta\sin\phi+r\cos^2\theta\sin\phi+\cos\phi$$

$$B_z=\frac{10}{r}\cos\theta-r\cos\theta\sin\theta$$

(or)

$$\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}=\begin{bmatrix}\sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi\\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi\\ \cos\theta & -\sin\theta & 0\end{bmatrix}\begin{bmatrix}10/r\\ r\cos\theta\\ 1\end{bmatrix}$$

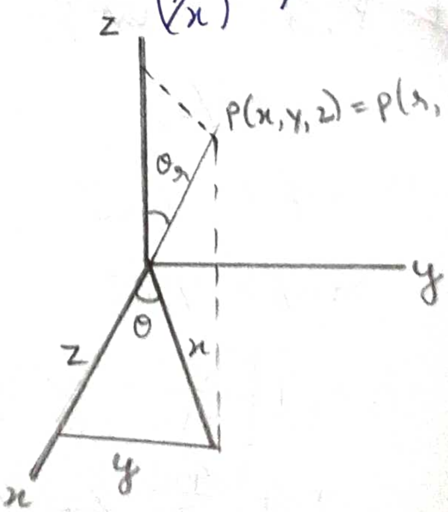
But $r=\sqrt{x^2+y^2+z^2}$, $\theta=\tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$ and $\tan^{-1}(y/x)=\phi$

Hence, $\sin\theta=\rho/r=\dfrac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$

$$\cos\theta=\frac{z}{r}=\frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\sin\phi=\frac{y}{\rho}=\frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\phi=x/\rho=x/\sqrt{x^2+y^2}$$



Substituting all these given values we have,

$$B_x = \frac{10\sqrt{x^2+y^2}}{(x^2+y^2+z^2)} \cdot \frac{x}{\sqrt{x^2+y^2}} + \frac{\sqrt{x^2+y^2+z^2}}{(x^2+y^2+z^2)} \cdot \frac{z^2 x}{\sqrt{x^2+y^2}} \cdot \frac{y}{\sqrt{x^2+y^2}}$$

$$B_x = \frac{10x}{(x^2+y^2+z^2)} + \frac{xz^2}{\sqrt{(x^2+y^2)}\,(x^2+y^2+z^2)} - \frac{y}{\sqrt{x^2+y^2}}$$

$$B_y = \frac{10\sqrt{x^2+y^2}}{(x^2+y^2+z^2)} \cdot \frac{y}{\sqrt{x^2+y^2}} + \frac{\sqrt{x^2+y^2+z^2}}{(x^2+y^2+z^2)} \cdot \frac{z^2 y}{\sqrt{x^2+y^2}} + \frac{x}{\sqrt{x^2+y^2}}$$

$$B_y = \frac{10y}{(x^2+y^2+z^2)} + \frac{yz^2}{(x^2+y^2)(x^2+y^2+z^2)} + \frac{x}{\sqrt{x^2+y^2}}$$

$$B_z = \frac{10z}{x^2+y^2+z^2} - \frac{z\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}.$$

$B = B_x a_x + B_y a_y + B_z a_z$ where B_x, B_y, B_z are given above at

$(-3,4,0)$, $x=-3$, $y=-4$ & $z=0$, so

$$B_x = \frac{-30}{25} + 0 - \frac{4}{5} = \frac{-30-20}{25} = \frac{-50}{25} = -2$$

$$B_y = \frac{40}{25} + 0 - \frac{3}{5} = \frac{40-15}{25} = \frac{25}{25} = 1$$

$$B_z = 0 - 0 = 0$$

Thus, $\therefore B = -2a_x + a_y$.

For spherical to cylindrical vector transformer,

we have,

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r\cos\theta \\ 1 \end{bmatrix}$$

(or)

$$B_\rho = 10/r\,(\sin\theta) + r\cos^2\theta$$

$$B_\phi = 1$$

$$B_z = 10/r\,(\cos\theta) - r\sin\theta\cos\theta$$

But, $r = \sqrt{p^2+z^2}$ & $\theta = \tan^{-1}(P/z)$

Thus, $\sin\theta = \dfrac{P}{\sqrt{p^2+z^2}}$, $\cos\theta = \dfrac{z}{\sqrt{p^2+z^2}}$

$$\Rightarrow B_p = \frac{10p}{p^2+z^2} + \sqrt{p+z^2}\cdot\frac{z^2}{p^2+z^2}$$

$$B_z = \frac{10z}{p^2+z^2} - \sqrt{p^2+z^2}\cdot\frac{pz}{p^2+z^2}$$

Hence, $B = \left(\dfrac{10P}{p^2+z^2} + \sqrt{p^2+z^2}\cdot\dfrac{z^2}{p^2+z^2}\right)a_p + a_\phi + \left(\dfrac{10z}{p^2+z^2} - \sqrt{p^2+z^2}\cdot\dfrac{\text{-}}{p}\right.$

At, $(5, \pi/2, -2)$, $P=5$, $\phi = \pi/2$ & $z=-2$ so,

$$B = \left(\frac{50}{29} + \frac{4}{\sqrt{29}}\right)a_p + a_\phi + \left(\frac{-20}{29} + \frac{10}{\sqrt{29}}\right)a_z$$

$$B = 2.467a_p + a_\phi + 1.167a_z.$$

Q2 6) Find the gradient of the following vector fields.

(a) $V = e^{-z}\sin 2x\cosh y$ (b) $U = p^2 z\cos 2\phi$ (c) $W = 10 r\sin^2\theta\cos\phi$.

sol:- (a) $V = e^{-z}\sin 2x\cosh y$

we have $\nabla V = \dfrac{\partial V}{\partial x}a_x + \dfrac{\partial V}{\partial y}a_y + \dfrac{\partial V}{\partial z}a_z$

$$\nabla V = \frac{\partial}{\partial x}(e^{-z}\sin 2x\cosh y)a_x + \frac{\partial}{\partial y}(e^{-z}\sin 2x\cosh y)a_y + \frac{\partial}{\partial z}(e^{-z}\sin 2x\cosh y)a_z$$

$$\nabla V = 2e^{-z}\cos 2x\cosh y\, a_x + e^{-z}\sin 2x\sinh y\, a_y - e^{-z}\sin 2x\cosh y\, a_z$$

(b) $U = p^2 z\cos 2\phi$

we have $\nabla U = \dfrac{\partial U}{\partial p}a_p + \dfrac{\partial U}{\partial \phi}a_\phi + \dfrac{\partial U}{\partial z}a_z$

$$\nabla U = \frac{\partial}{\partial p}(p^2 z\cos 2\phi)a_p + \frac{\partial}{\partial \phi}(p^2 z\cos 2\phi) + \frac{\partial}{\partial z}(p^2 z\cos 2\phi)$$

$$\nabla U = 2pz\cos 2\phi\, a_p + (-2pz\sin 2\phi)a_\phi + p^2\cos 2\phi\, a_z$$

$$\nabla U = 2pz\cos 2\phi\, a_p + (-2pz\sin 2\phi)a_\phi + p^2\cos 2\phi\, a_z$$

$$\nabla U = 2pz\cos 2\phi\, a_p - 2pz\sin 2\phi\, a_\phi + p^2\cos 2\phi\, a_z.$$

(c) $W = 10r\sin^2\theta\cos\phi$

we have $\nabla W = \frac{\partial W}{\partial r} a_r + \frac{1}{r}\frac{\partial W}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial W}{\partial \phi} a_\phi$

$$\nabla W = \frac{\partial}{\partial r}\left(10r\sin^2\theta\cos\phi\right)a_r + \frac{1}{r}\frac{\partial}{\partial\theta}\left(10r\sin^2\theta\cos\phi\right)a_\theta + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\phi}\left(10r\sin^2\theta\cos\phi\right)a_\phi$$

$$\nabla W = 10\sin^2\theta\cos\phi\, a_r + 10\sin2\theta\cos\phi\, a_\theta - 10\sin\theta\sin\phi\, a_\phi$$

Determine the divergence of these vector fields

(a) $P = (x^2yz)a_x + (xz)a_z$ (b) $Q = \rho\sin\phi\, a_\rho + \rho^2 z a_\phi + z\cos\phi\, a_z$

(c) $T = \frac{1}{r^2}\cos\theta\, a_r + r\sin\theta\cos\phi\, a_\theta + \cos\theta\, a_\phi$

(a) $P = (x^2yz)a_x + (xz)a_z$

we have $\nabla P = \frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}$

$$\nabla P = \frac{\partial}{\partial x}(x^2yz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(xz)$$

$$\nabla P = 2xyz + x$$

(b) $Q = \rho\sin\phi\, a_\rho + \rho^2 z a_\phi + z\cos\phi\, a_z$

we have $\nabla Q = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho Q_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(Q_\phi) + \frac{\partial}{\partial z}Q_z$

$$\nabla Q = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^2\sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho^2 z) + \frac{\partial}{\partial z}(z\cos\phi)$$

$$\nabla Q = 2\sin\phi + \cos\phi.$$

(c) $T = \frac{1}{r^2}\cos\theta\, a_r + r\sin\theta\cos\phi\, a_\theta + \cos\theta\, a_\phi$

we have $\nabla T = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(T_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(T_\phi)$

$$\nabla T = \frac{1}{r^2}\frac{\partial}{\partial r}(\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin^2\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\cos\theta)$$

$$\nabla T = 0 + \frac{1}{r\sin\theta}2r\sin\theta\cos\theta\cos\phi + 0$$

$$\therefore \nabla T = \frac{1}{r\sin\theta}2r\sin\theta\cos\theta\cos\phi$$

$$\nabla T = 2\cancel{\sin\theta}\cos\theta\cos\phi \times \frac{1}{\cancel{\sin\theta}} = 2\cos\theta\cos\phi.$$

$$\nabla T = 2\cos\theta\cos\phi$$

8) Determine the curl of the vector fields.

10A

(a) $P = x^2yz\,a_x + xz\,a_z$ (b) $Q = \rho\sin\phi\, a_\rho + \rho^2 z\, a_\phi + z\cos\phi\, a_z$

(c) $T = \frac{1}{r^2}\cos\theta\, a_r + r\sin\theta\cos\phi\, a_\theta + \cos\theta\, a_\phi$.

Sol: (a) $P = x^2yz\,a_x + xz\,a_z$

we have $\nabla \times P = \left(\frac{\partial P_z}{\partial y} - \frac{\partial P_y}{\partial z}\right)a_x + \left(\frac{\partial P_x}{\partial z} - \frac{\partial P_z}{\partial x}\right)a_y + \left(\frac{\partial P_y}{\partial x} - \frac{\partial P_x}{\partial y}\right)$

$$\nabla \times P = (0-0)a_x + (x^2y - z)a_y + (0 - x^2z)a_z$$

$$\therefore \nabla \times P = (x^2y - z)a_y - x^2z\,a_z$$

(b) $Q = \rho\sin\phi\, a_\rho + \rho^2 z\, a_\phi + z\cos\phi\, a_z$

we have,

$$\nabla \times Q = \left[\frac{1}{\rho}\frac{\partial Q_z}{\partial \phi} - \frac{\partial Q_\phi}{\partial z}\right]a_\rho + \left[\frac{\partial Q_\rho}{\partial z} - \frac{\partial Q_z}{\partial \rho}\right]a_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho} \cdots - \frac{\partial Q_\rho}{\partial \phi}\right]a$$

$$\nabla \times Q = \left(\frac{-z}{\rho}\sin\phi - \rho^2\right)a_\rho + (0-0)a_\phi + \frac{1}{\rho}\left(3\rho^2 z - \rho(\cos\phi)\right)a_z$$

$$\nabla \times Q = \frac{-1}{\rho}\left(z\sin\phi + \rho^3\right)a_\rho + \left(3\rho z - \cos\phi\right)a_z$$

$$\nabla \times Q = \frac{-1}{\rho}\left(z\sin\phi + \rho^3\right)a_\rho + \left(3\rho z - \cos\phi\right)a_z$$

$\sin\theta$ $\frac{1}{r\sin\theta}$

(c) $T = \frac{1}{r^2}\cos\theta\, a_r + r\sin\theta\cos\phi\, a_\theta + \cos\theta\, a_\phi$

we have,

$$\nabla \times T = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(T_\phi\sin\theta) - \frac{\partial}{\partial\phi}T_\theta\right]a_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}T_r \cdots\right.$$
$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rT_\theta) - \frac{\partial}{\partial\theta}T_r\right]a_\phi$$

$$\nabla \times T = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\cos\theta\sin\theta) - \frac{\partial}{\partial\phi}(r\sin\theta\cos\phi)\right]a_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\cdot(\cos\theta)/r^2 - \frac{\partial}{\partial r}(r\cos\theta)\right]a_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(r\sin\theta\cos\phi) - \frac{\partial}{\partial\theta}\frac{(\cos\theta)}{r^2}\right]a_\phi$$

$$\nabla \times T = \frac{1}{r\sin\theta}(\cos 2\theta + r\sin\theta\sin\phi)a_r + \frac{1}{r}(0 - \cos\theta)a_\theta + \frac{1}{r}\left(2r\sin\theta\cos \cdots \frac{\sin\theta}{r^2}\right)a_\phi$$

$$\therefore \nabla \times T = \left(\frac{\cos 2\theta}{r\sin\theta} + \sin\phi\right)a_r - \frac{\cos\theta}{r}a_\theta + \left(2\cos\phi + \frac{1}{r^3}\right)\sin\theta\, a_\phi$$

If $G(r) = 10e^{-2z}(\rho a_\rho + a_z)$, determine the flux of G out of the entire cylindrical surface at $\rho = 1, 0 \le z \le 1$. Confirm the result Using the divergence theorem.

If ψ is the flux of G through the given surface, as shown in the figure, then

$$\psi = \oint G \cdot ds = \psi_t + \psi_b + \psi_s$$

where ψ_t, ψ_b & ψ_s are the fluxes through the top, bottom and sides (curved surface) of the cylinder as shown in figure.

for ψ_t, $z = 1$, $ds = \rho\, d\rho\, d\phi\, a_z$. Hence,

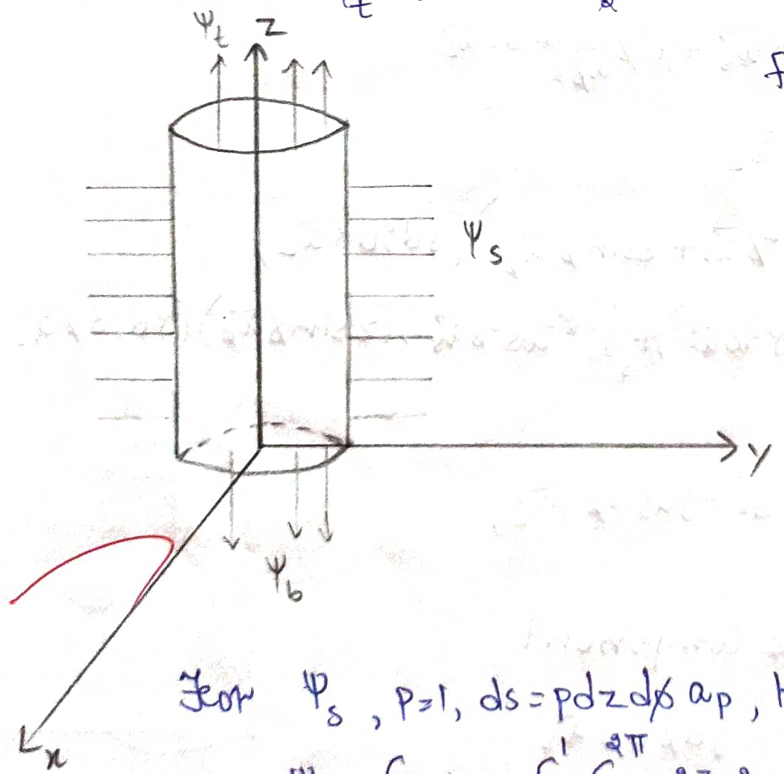
$$\psi_t = \int G ds = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{1} 10\, e^{-2}\rho\, d\rho\, d\phi$$

$$\psi_t = 10e^{-2}\int_{\phi=0}^{2\pi} d\phi \int_{\rho=0}^{1} \rho\, d\rho$$

$$\psi_t = 10e^{-2}[\phi]_0^{2\pi}[\rho^2/2]_0^1$$

$$\therefore \psi_t = 10e^{-2}\,\cancel{2}\pi \cdot \frac{1}{\cancel{2}} = 10\pi e^{-2}$$

$$\therefore \psi_t = 10\pi e^{-2}$$



for ψ_b, $z = 0$ & $ds = \rho\, d\rho\, d\phi(-a_z)$.

$$\text{Hence, } \psi_b = \int_b G \cdot ds = \int_{\rho=0}^{1}\int_{\phi=0}^{2\pi} 10e^{0}\rho\, d\rho\, d\phi\ (-a_z)$$

$$\psi_b = -10\int_{\rho=0}^{1}\rho\, d\rho \int_{\phi=0}^{2\pi} d\phi$$

$$\psi_b = -10[\phi]_0^{2\pi}[\rho^2/2]_0^1$$

$$\psi_b = -10(2\pi)\cdot \tfrac{1}{2}$$

$$\therefore \psi_b = -10\pi$$

For ψ_s, $\rho = 1$, $ds = \rho\, dz\, d\phi\, a_\rho$, Hence,

$$\psi_s = \int_s G ds = \int_{z=0}^{1}\int_{\phi=0}^{2\pi} 10e^{-2z}\rho^2\, dz\, d\phi = 10(1)^2(2\pi)\left[\frac{e^{-2z}}{-2}\Big|_0^1\right]$$

$$= 10\pi(1 - e^{-2})$$

$$\therefore \psi_s = 10\pi(1 - e^{-2})$$

Thus,

$\therefore \psi = \psi_t + \psi_b + \psi_s = 10\pi e^{-2} - 10\pi + 10\pi(1-e^{-2}) = 0 \quad \therefore \psi = 0$

Alternatively, since 's' is a closed surface, we can apply divergence theorem:

$$\psi = \oint_S G\,ds = \int_V (\nabla \cdot G)\,dv$$

But

$$\nabla \cdot G = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho G_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \phi}G_\phi + \frac{\partial}{\partial z}G_z$$

$$\nabla \cdot G = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^2 10 e^{-2z}) - 20e^{-2z}$$

$$\therefore \nabla G = 0$$

This showing that G has no source, Hence $\psi = \int_V (\nabla \cdot G)\,dv =$

---

10) Determine the flux of $D = \rho^2 \cos^2\phi\, \vec{a}_\rho + z\sin\phi\, \vec{a}_\phi$ over the close surface of the cylinder $0 \le z \le 1$, $\rho = 4$. Verify the divergence theorem for this case.

sol:- If ψ is the flux of $\vec{D}$ through the given surface.

$$\therefore \psi = \oint_S \bar{D} \cdot \bar{ds} = \psi_t + \psi_b + \psi_s$$

$$= d\rho\,\vec{a_\rho} + \rho\, d\phi\, \vec{a_\phi} + dz\,\vec{a_z}$$

for ψ_t; $z = 1$, $\bar{ds} = \rho\, d\rho\, d\phi\, \bar{a}_z$

$$\psi_t = \int_S \bar{D} \cdot \bar{ds} = \int_S (\rho^2 \cos^2\phi\, \vec{a_\rho} + z\sin\phi\, \vec{a_\phi})(\rho\, d\rho\, d\phi\, \vec{a_z})$$

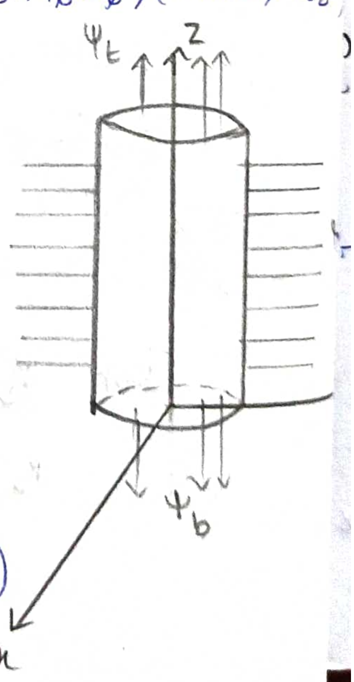
$$\psi_t = \int_S (\rho^2 \cos^2\phi\, \vec{a_\rho})(\rho\, d\rho\, d\phi\, \vec{a_z}) + \int_S (\rho^2\cos^2\phi\,\vec{a_\rho})(z\sin\phi\,\vec{a_\phi})(\rho\, d\rho\, d\phi\, \vec{a_z})$$

$\psi_t = 0$

~~At~~ for ψ_b; $z = 0$, $\bar{ds} = \rho\, d\rho\, d\phi\, (-\bar{a}_z)$

$\psi_b = 0$

As $\bar{D}$ has no z component



for ψ_s i.e $\vec{a_\rho}$ $\vec{ds} = \rho\, d\phi\, dz\, \vec{a_\rho}$

$$\psi_s = \int \vec{D} \cdot \vec{ds} = \int \rho\, d\phi\, dz\, \vec{a}_\rho$$

$$\psi_s = \int \vec{D} \cdot \vec{ds} = \int_{\phi=0}^{2\pi} \int_{z=0}^{1} (\rho^2 \cos^2\phi)(\rho\, d\phi\, dz)$$

$$\psi_s = \rho^3 \int_0^{2\pi} \cos^2\phi\, d\phi \int_0^1 dz$$

$$\Psi_s = (4)^2(\pi)(1)$$

$$\Psi_s = 64\pi$$

$$\therefore \Psi = \Psi_t + \Psi_b + \Psi_s = 64\pi \quad \therefore \Psi = 64\pi \text{ Amp}$$

$$D = \rho^2 \cos^2\phi \, \bar{a}_\rho + z\sin\phi \, \bar{a}_\phi$$

By the divergences theorem, $\oint_s \bar{D} \cdot \bar{ds} = \int_v \nabla \cdot \bar{D} \, dv$

solve RHS

$$\nabla \cdot \bar{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^3 \cos^2\phi) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(z\sin\phi)$$

$$= \cos^2\phi\left(\frac{3\rho^2}{\rho}\right) + \frac{1}{\rho} z\cos\phi$$

$$\therefore \nabla \cdot \bar{D} = 3\rho\cos^2\phi + \frac{1}{\rho} z\cos\phi$$

$$\int_v \nabla \cdot \bar{D} \, dv = \int_v \left(3\rho\cos^2\phi + \frac{1}{\rho} z\cos\phi\right) dv$$

$$\int_v \nabla \cdot \bar{D} \, dv = \int_v \left(3\rho\cos^2\phi + \frac{1}{\rho} z\cos\phi\right) \rho \, d\rho \, d\phi \, dz$$

$$\int_v \nabla \cdot \bar{D} \, dv = \int_v \left(3\rho\cos^2\phi + \frac{1}{\rho} z\cos\phi\right) \rho \, d\rho \, d\phi \, dz$$

$$\int_v \nabla \cdot \bar{D} \, dv = \int_{\rho=0}^{4}\int_{z=0}^{1}\int_{\phi=0}^{2\pi} 3\rho\cos^2\phi \, \rho \, d\rho \, d\phi \, dz + \int_{\rho=0}^{4}\int_{z=0}^{1}\int_{\phi=0}^{2\pi} \frac{1}{\rho} z\cos\phi \, \rho \, d\rho \, d\phi \, dz$$

$$\int_v \nabla \cdot \bar{D} \, dv = 3\int_{\rho=0}^{4} \rho^2 d\rho \int_{\phi=0}^{1} \cos^2\phi \, d\phi \int_{z=0}^{1} d\phi + \int_{\rho=0}^{4} \rho \, d\rho \int_{\phi=0}^{2\pi} \cos\phi \, d\phi \int_{z=0}^{1} dz$$

$$\int_v \nabla \cdot \bar{D} \, dv = 3\left[\frac{\rho^3}{3}\right]_0^4 \times \pi \times 1 + 0$$

$$\int_v \nabla \cdot \bar{D} \, dv = 64\pi \text{ Amp.}$$

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1) If $B = y\bar{a}_x + (x+z)\bar{a}_y$ and a point Q is located at $(-2,6,3)$. Express

(1) The point Q is in cylindrical & spherical Co-ordinate.

(2) B is in spherical co-ordinates.

Sol:- Given, $B = y\bar{a}_x + (x+z)\bar{a}_y$ and point $Q(-2,6,3)$

At point $Q \rightarrow x = -2, y = 6, z = 3$

$$\rho = \sqrt{x^2+y^2} = \sqrt{(-2)^2+(6)^2} = \sqrt{4+36} = \sqrt{40} = 6.32$$

$$\phi = \tan^{-1}(y/x) = \tan^{-1}(6/-2) = \tan^{-1}(-3) = -71.5650 + 180$$

$$\phi = +180.435^\circ$$

$$z = 3$$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{4+36+9} = \sqrt{49} = 7$$

$\theta = \tan^{-1}\dfrac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\dfrac{\sqrt{4+36}}{3} = \tan^{-1}\dfrac{\sqrt{40}}{3} = 64.62^\circ$

cylindrical co-ordinates Q $(6.32, 108.435^\circ, 3)$

spherical co-ordinates Q $(7, 64.62^\circ, 108.435^\circ)$

Hence $Q(-2,6,3) = Q(6.32, 108.435^\circ) = Q(7, 64.62^\circ, 108.435^\circ)$

(ii) for a vector B is said to be in spherical co-ordinates

we have,
$$\begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

$B_r = y\sin\theta\cos\phi + (x+z)\sin\theta\sin\phi$

$B_\theta = y\cos\theta\cos\phi + (x+z)\cos\theta\sin\phi$

$B_\phi = -y\sin\phi + (x+z)\cos\phi$

But $x = r\sin\theta\sin\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

$B = (B_r, B_\theta, B_\phi)$

$= r[\sin^2\theta\cos\phi\sin\phi + (\sin\phi\sin\theta)\cdot(\sin\theta\cos\phi + \cos\theta)]\, b_r +$

$r[\sin\theta\cos\theta\sin\phi\cos\phi + (\sin\theta\cos\phi + \cos\phi)\cos\theta\sin\phi]\, b_\theta$

$r[-\sin\phi\sin\theta + (\sin\theta\cos\phi + \cos\theta)\cos\phi]\, b_\phi$

At Q, $r = 7$, $\tan\phi = 6/_{-2} \Rightarrow \tan\phi = -3$, $\tan\theta = \sqrt{40}/3$

If $\tan\phi = 6/-2$; $\tan\theta = \sqrt{40}/3$

$\Rightarrow \cos\phi = -2/\sqrt{40}$; $\Rightarrow \cos\theta = 3/7$

$\Rightarrow \sin\phi = 6/\sqrt{40}$; $\Rightarrow \sin\theta = \sqrt{40}/7$

$\therefore B = 7\left[\dfrac{40}{49}\cdot\dfrac{-2}{\sqrt{40}}\cdot\dfrac{6}{\sqrt{40}} + \left(\dfrac{\sqrt{40}}{7}\cdot\dfrac{-2}{\sqrt{40}} + \dfrac{3}{7}\right)\dfrac{\sqrt{40}}{7}\cdot\dfrac{6}{\sqrt{40}}\right] b_r +$

$7\left[\dfrac{\sqrt{40}}{7}\cdot\dfrac{3}{7}\cdot\dfrac{6}{\sqrt{40}} + \left(\dfrac{\sqrt{40}}{7}\cdot\dfrac{-2}{\sqrt{40}} + \dfrac{3}{7}\right)\cdot\dfrac{3}{7}\cdot\dfrac{6}{\sqrt{40}}\right] b_\theta +$

$7\left[\dfrac{-\sqrt{40}}{7}\cdot\dfrac{36}{40} + \left(\dfrac{\sqrt{40}}{7}\cdot\dfrac{-2}{\sqrt{40}} + \dfrac{3}{7}\right)\cdot\dfrac{-2}{\sqrt{40}}\right] b_\phi$

$B = \dfrac{-6}{7} b_r - \dfrac{18}{7\sqrt{40}} b_\theta - \dfrac{38}{\sqrt{40}} b_\phi \Rightarrow -\dfrac{6}{7} b_r - \dfrac{18}{7\sqrt{40}} b_\theta - \dfrac{38}{\sqrt{4}}$

$B = -0.8571 b_r - 0.4066 b_\theta - 6.008 b_\phi$

1) Three Concentrated charges of 0.25 μc are located at the vertices of an equivalent triangle of 10cm side. Find the magnitude and direction of the forces on the charge at one side is due to other two charges.

Sol:- Given,

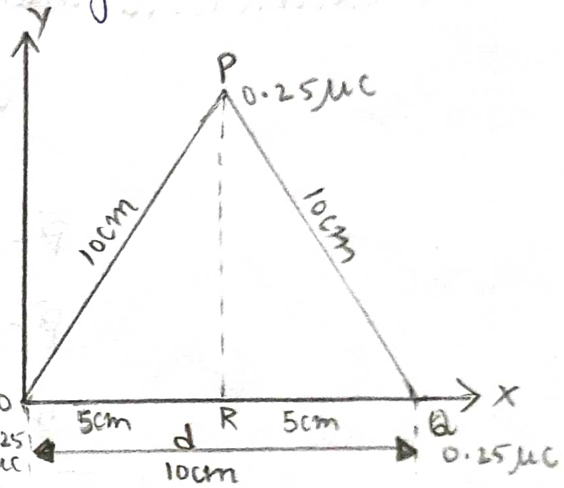
three Concentrated charges of 0.25μc are located at the vertices of an equivalent triangle of 10cm side. The arrangement of the system is as shown in the figure.

The Co-ordinates of the vertices of triangle are

point O → (0,0,0)

point Q → (0.1,0,0)

point P → (0.05, 0.0866, 0)



Let us find the force on ~~plane~~ due to the changes at O and Q.

$$\therefore \bar{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{op}^2}\bar{a}_{op} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{op}^2} \times \frac{\bar{R}_{op}}{|\bar{R}_{op}|}$$

$$\therefore \bar{R}_{op} = 0.05\bar{a}_x + 0.0866\bar{a}_y \;;\; |\bar{R}_{op}| = \sqrt{(0.05)^2 + (0.0866)^2} = 0.1$$

$$\bar{F}_1 = \frac{0.25\times10^{-6}\times0.25\times10^{-6}}{4\pi\times8.854\times10^{-12}\times0.1^2} \times \frac{[0.05\bar{a}_x + 0.866\bar{a}_y]}{0.1} = 0.02808\bar{a}_x + 0.0486\bar{a}_y$$

$$\therefore \bar{F}_1 = 0.02808\bar{a}_x + 0.0486\bar{a}_y \text{ N.}$$

$$\bar{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{Qp}^2}\bar{a}_{Qp} = \frac{Q_1 \cdot Q_2}{4\pi\varepsilon_0 R_{Qp}^2} \times \frac{\bar{R}_{Qp}}{|\bar{R}_{Qp}|}$$

$$\therefore \bar{R}_{Qp} = (0.05 - 0.1)\bar{a}_x + 0.0866\bar{a}_y = -0.05\bar{a}_x + 0.0866\bar{a}_y$$

$$|\bar{R}_{Qp}| = \sqrt{(-0.05)^2 + (0.0866)^2} = 0.1$$

$$\bar{F}_2 = \frac{0.025\times10^{-6}\times0.025\times10^{-6}}{4\pi\times8.854\times10^{-12}\times0.1^2} \times \left[\frac{-0.05\bar{a}_x + 0.0866\bar{a}_y}{0.1}\right]$$

$$\bar{F}_2 = -0.02808\bar{a}_x + 0.0486\bar{a}_y \text{ N}$$

$\therefore \bar{F} = \bar{F}_1 + \bar{F}_2 = 0.02808\bar{a}_x + 0.0486\,\bar{a}_y - 0.002808\,\bar{a}_x + 0.0486\,\bar{a}_y$

$\bar{F} = 0.09729\,\bar{a}_y\,N$ (Direction $\bar{a}_y$)

$|\bar{F}| = 0.09729N$ (Magnitude).

---

2) Determine the electric field intensity at $P(-0.2, 0, -2.3)m$

3a) to a point change of 5nc at $Q(0.2, 0.1, -2.5)m$ in air.

Sol:- Given,

To find the electric field intensity at $P(-0.2, 0, -2.$

due to a point charge of 5nc at $Q(0.2, 0.1, -2.5)m$ in air

The electricfield intensity $\bar{E}$ is given by $\bar{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$

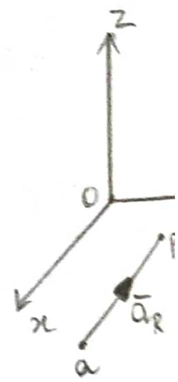
$$\bar{a}_R = \frac{\bar{R}_{QP}}{|\bar{R}_{QP}|} = \frac{\bar{P}-\bar{Q}}{|\bar{P}-\bar{Q}|}$$

$$\bar{P}-\bar{Q} = (-0.2-0.2)\bar{a}_x + (0-0.1)\bar{a}_y + [-2.3-(-2$$

$$\bar{P}-\bar{Q} = -0.4\bar{a}_x - 0.1\,\bar{a}_y + 0.2\,\bar{a}_z$$

$$|\bar{P}-\bar{Q}| = \sqrt{(-0.4)^2 + (0.1)^2 + (0.2)^2} = \sqrt{0.21} = 0.45$$

$$\therefore \bar{a}_R = \frac{-0.4\bar{a}_x - 0.1\bar{a}_y + 0.2\bar{a}_z}{0.45825}$$



$$\bar{R} = |\bar{P}-\bar{Q}| = 0.45825$$

$$\therefore \bar{E} = \frac{5\times10^{-9}}{4\pi \times 8.854\times10^{-12} \times (0.45825)^2}[\bar{a}_R]$$

$$\bar{E} = 214\,\bar{a}_R$$

$$\bar{E} = \frac{5\times10^{-9}}{4\pi \times 8.854\times10^{-12} \times (0.45825)^2}[\bar{a}_R] = 214$$

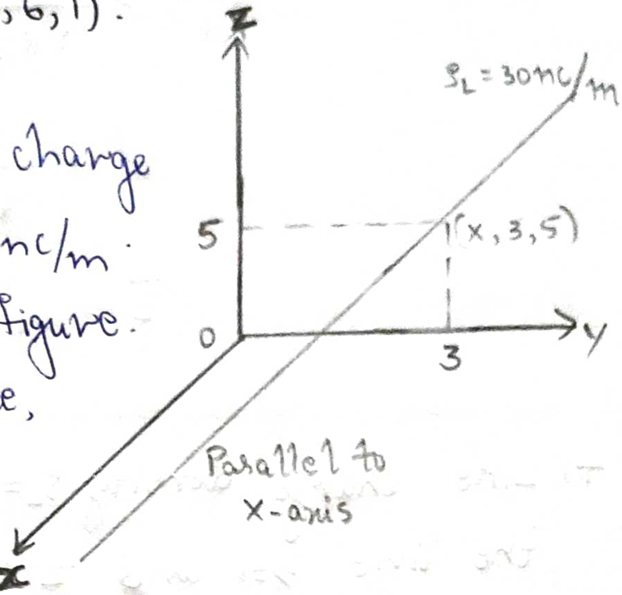
Substitue value of $\bar{a}_R$

$$\therefore \bar{E} = 214\left[-0.8728\,\bar{a}_x - 0.2182\,\bar{a}_y + 0.4364\,\bar{a}_z\right]$$

$$\bar{E} = -186.779\bar{a}_x - 46.694\bar{a}_y + 93.389\,\bar{a}_z\ V/m$$

6) An infinitely long uniform the line charge is located at $y=3$, $z=5$. If $\rho_L = 30\,nC/m$, find the field intensity E at (i) Origin, (ii) $P(0,6,1)$ and (iii) $P(5,6,1)$.

Sol.- Given,



A infinitely long uniform line charge is located at $(y=3, z=5)$ and $\rho_L = 30\,nC/m$.
The arrangement is shown in the figure.
The charge is parallel to x-axis. Hence, E cannot have ~~any cannot have~~ any component in x-direction hence donot consider x while calculating $\bar{E}$.

(i) $\bar{E}$ at $P(0,0,0)$

$$\bar{r} = (0-3)\bar{a}_y + (0-5)\bar{a}_z = -3\bar{a}_y - 5\bar{a}_z$$

$$r = |\bar{r}| = \sqrt{3^2+5^2} = \sqrt{34}$$

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r = \frac{\rho_L}{2\pi\varepsilon_0 r}\cdot\frac{\bar{r}}{|\bar{r}|}$$

$$\bar{E} = \frac{30\times10^{-9}}{2\pi\times8.854\times10^{-12}\times\sqrt{34}}\left(\frac{-3\bar{a}_y-5\bar{a}_z}{\sqrt{34}}\right)$$

$$\bar{E} = -47.58\bar{a}_y - 79.3\bar{a}_z$$

$\therefore$ $\bar{E} = -47.58\bar{a}_y - 79.3\bar{a}_z$ V/m be the $\bar{E}$ at origin.

(ii) $\bar{E}$ at $P(0,6,1)$

$$\bar{r} = (6-3)\bar{a}_y + (1-5)\bar{a}_z = 3a_y - 4a_z,\quad |\bar{r}| = \sqrt{3^2+(4)^2} = \sqrt{25} = 5 = r$$

$$\therefore \bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r = \frac{\rho_L}{2\pi\varepsilon_0 r}\cdot\frac{\bar{r}}{|\bar{r}|}$$

$$\bar{E} = \frac{30\times10^{-9}}{2\pi\times8.854\times10^{-12}\times5}\left(\frac{3\bar{a}_y-4\bar{a}_z}{5}\right)$$

$$\bar{E} = 64.71\bar{a}_y - 86.28\bar{a}_z \text{ V/m}$$

(iii) $\bar{E}$ at $P(5,6,1)$

$\bar{r} = (6-3)\bar{a}_y + (1-5)\bar{a}_z = 3\bar{a}_y - 4\bar{a}_z$, $|\bar{r}| = \sqrt{(3)^2+(4)^2} = \sqrt{9+16}$

$|\bar{r}| = \sqrt{25} = 5$

$$\therefore \bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r = \frac{\rho_L}{2\pi\varepsilon_0 r}\cdot\frac{\bar{r}}{|\bar{r}|}$$

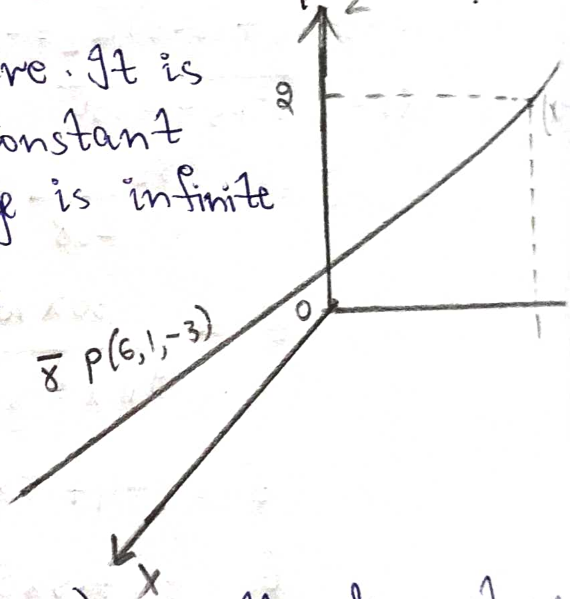
$$\bar{E} = \frac{30\times10^{-9}}{2\pi\times 8.854\times10^{-12}\times 5}\cdot\left(\frac{3\bar{a}_y - 4\bar{a}_z}{5}\right)$$

$$\bar{E} = 64.71\bar{a}_y - 86.28\bar{a}_z \text{ V/m}$$

---

4) Line charge density $\rho_L = 24\text{nC/m}$ is located in free space on the line $y=1$ and $z=2$m. (a) find E at the point P(6,-1,3) (b) what point charge on Q_a should be located at A(-3,4,...) to make y component of total E zero at point P?

Sol:- The line charge is shown in figure. It is parallel to the X-axis as $y=1$ constant and $z=2$ constant. The line charge is infinite hence using the standard result,



$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r$$

To find $\bar{a}_r$, Consider a point on the line charge (x,1,2) while P(6,-1,3). As the line charge parallel to X-axis, do not consider X Co-ordinates while finding $\bar{a}_r$.

$$\bar{r} = (-1-1)\bar{a}_y + (3-2)\bar{a}_z$$

$$\bar{r} = -2\bar{a}_y + \bar{a}_z.$$

$$|\bar{r}| = \sqrt{(-2)^2+(1)^2} = \sqrt{4+1} = \sqrt{5} = r$$

$$\bar{a}_r = \frac{\bar{r}}{|\bar{r}|} = \frac{-2\bar{a}_y + \bar{a}_z}{\sqrt{5}}$$

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0\sqrt{5}}\cdot\frac{-2\bar{a}_y+\bar{a}_z}{\sqrt{5}} = \frac{24\times10^{-9}(-2\bar{a}_y+\bar{a}_z)}{2\pi\times8.854\times10^{-12}\times5}$$

$$\bar{E} = -172.56\bar{a}_y + 86.282\bar{a}_z \text{ V/m}$$

$$\therefore \bar{E} = -172.56\bar{a}_y + 86.282\bar{a}_z \text{ V/m}$$

(b) Consider a point charge Q_A at $A(-3,4,1)$
The electric field due to Q_A at $P(6,-1,3)$ is

$$\bar{E}_A = \frac{Q_A}{4\pi\varepsilon_0 R_{AP}^2}\bar{a}_{AP}$$

$$\bar{R}_{AP} = [6-(-3)]\bar{a}_x + [-1-4]\bar{a}_y + [3-1]\bar{a}_z = 9\bar{a}_x - 5\bar{a}_y + 2\bar{a}_z$$

$$|\bar{R}_{AP}| = \sqrt{(9)^2+(-5)^2+(2)^2} = \sqrt{81+25+4} = \sqrt{110} = 10.4888.$$

$$\bar{a}_{AP} = \frac{\bar{R}_{AP}}{|\bar{R}_{AP}|} = \frac{9\bar{a}_x - 5\bar{a}_y + 2\bar{a}_z}{10.4888}$$

$$\bar{E}_A = \frac{Q_A}{4\pi\varepsilon_0(10.4888)^2}\left[\frac{9\bar{a}_x - 5\bar{a}_y + 2\bar{a}_z}{10.4888}\right]$$

The total field at P is now, $\bar{E}_t = \bar{E} + \bar{E}_A$

* The y Component of total $\bar{E}_t$ is to be made zero.

$$\left[-172.56 - \frac{5Q_A}{4\pi\varepsilon_0(10.4888)^2}\right]\bar{a}_y = 0$$

$$\frac{5Q_A}{4\pi\varepsilon_0(10.4888)^2} = -172.56$$

$$Q_A = \frac{-172.564 \times 4\pi \times 8.854 \times 10^{-12} \times (10.4888)^2}{5}$$

$$Q_A = -4.4311\ \mu C.$$

5, Find $\bar{E}$ at $(0,0,2)$m due to charged circular disc in xy plane with $\rho_s = 20$n C/m^2 and radius 1m.

Sol: Given, $h = 2$ m, $a = 1$m, $\rho_s = 20$nC/m^2

$$\therefore \bar{E} = \frac{\rho_s h}{2\varepsilon_0}\left[\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right]\bar{a}_z \text{ V/m}$$

$$\bar{E} = \frac{20\times10^{-9}\times2}{2\times8.854\times10^{-12}}\left[\frac{1}{2} - \frac{1}{\sqrt{1^2+2^2}}\right]\bar{a}_z$$

$$\bar{E} = 119.237\,\bar{a}_z \text{ V/m}$$

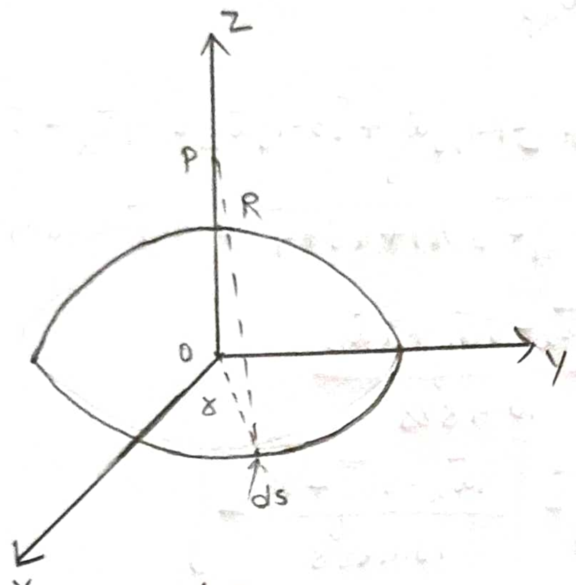
6) A circular disc of 10cm radius is charged uniformly ... total charge of 100μc. find E at a point 20cm on its ...
(5b)

Sol:- Given,

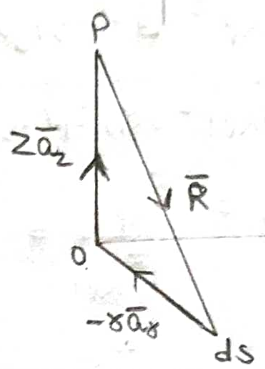
$Q = 100\mu C$, $r = 10cm = 0.1m$, area $= \pi r^2 = \pi(0.1)^2 = 0.0314$

$$\rho_s = \frac{Q}{Area} = \frac{100\times10^{-6}}{0.03141} = 3.1831\times10^{-3}\ c/m^2$$

$$\bar{E} = \frac{Qh}{4\pi\varepsilon_0 r}\left(\frac{1}{h} - \dots\right.$$
$\bar{a}_z$



Disc



Consider differential surface area ds using cylindrical co-
nates system, $ds = r\,dr\,d\phi$, $\bar{R} = -r\bar{a}_r + z\bar{a}_z$

$$\bar{a}_R = \frac{-r\bar{a}_r + z\bar{a}_z}{\sqrt{r^2+z^2}}$$

Note:- All radical components of $\bar{e}$ at p will cancel each other due to symmetry.

$$\bar{e} = \int_S \frac{dQ}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$$

$$\bar{e} = \int_{\phi=0}^{2\pi}\int_{r=0}^{0.1} \frac{\rho_s\,[r\,dr\,d\phi]}{4\pi\varepsilon_0\,[r^2+z^2]}\;\frac{z\bar{a}_z}{\sqrt{r^2+z^2}} = \frac{\rho_s z}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi}\int_{r=0}^{0.1}\dots$$

$$\bar{e} = \frac{\rho_s z}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi}\int_{r=0}^{0.1}\frac{r\,dr\,d\phi}{[r^2+z^2]^{3/2}}\cdot\bar{a}_z$$

use $r^2+z^2 = u^2$ i.e, $r\,dr = du\cdot u \Rightarrow r\,dr = u\,du$

limits :- $r=0$, $u_1 = z$ & $r = 0.1$ $u_2 = \sqrt{0.1^2+z^2}$

$$\bar{e} = \frac{\rho_s z}{4\pi\varepsilon_0}\int_{\phi=0}\int_{r=0}\frac{u\,du\,d\phi}{u^2}\,\bar{a}_z = \frac{\rho_s z}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi}\int_{r=0}^{0.1}\frac{r\,dr\,d\phi}{[r^2+z^2]^{3/2}}\,\bar{a}_z$$

$$= \frac{\rho_s z}{4\pi\varepsilon_0}[\phi]_0^{2\pi}\left[\frac{-1}{4}\right]_{u_1}^{u_2}\bar{a}_z$$

$$\bar{e} = \frac{\rho_s z}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi}\int_{r=0}^{0.1}\frac{u\,du\,d\phi}{u^2}\,\bar{a}_z = \frac{\rho_s z}{4\pi\varepsilon_0}[\phi]_0^{2\pi}\left[\frac{-1}{4}\right]_{u_1}^{u_2}\bar{a}_z$$

$$\bar{\varepsilon} = \frac{\rho_s z}{4\pi\varepsilon_0}\times 2\pi\times\left[\frac{-1}{u_2}+\frac{1}{u_1}\right]\bar{a}_z$$

using $z = 20\,cm = 0.2\,m$, $u_1 = 0.2$ & $u_2 = \sqrt{0.05} = 0.2236$

$$\therefore \bar{e} = \frac{3.1831\times10^{-3}\times0.2}{4\pi\times8.854\times10^{-12}}\times2\pi\times\left[\frac{1}{0.2}-\frac{1}{0.2236}\right]\bar{a}_z$$

$\therefore \bar{e} = 18.97236\bar{a}_z\ \mu V/m$.

---

7) The electric flux density is given by as $D = \left(\frac{r}{4}\right)a_r\ nC/m^2$ in freespace. (1) Calculate the electricity field intensity at $r = 0.25m$. (2) The total charge with in a sphere of $r = 0.25m$.

Sol:- Given,

$$\bar{D} = \left(\frac{r}{4}\right)a_r\ nC/m^2$$

(1) The electric field intensity at $r = 0.25m$

$$\bar{E} = \frac{\bar{D}}{\varepsilon_0} = \frac{\left(r/4\,\bar{a}_r\right)\times10^{-9}}{8.854\times10^{-12}}$$

where, $r = 0.25m$

$$\bar{E} = \frac{0.25\times10^{-9}}{4\times8.854\times10^{-12}}\,\bar{a}_r$$

$\therefore \bar{E} = 7.0589\,\bar{a}_r\ V/m$

(2) The total charge with in a sphere of $r = 0.25m$

$$Q = \oint_s \bar{D}\cdot\bar{ds}$$

$\bar{ds} = r^2\sin\theta\,d\theta\,d\phi$ (Normal to $\bar{a}_r$)

$$Q = \oint_s\left(\frac{r}{4}\bar{a}_r\right)\times10^{-9}\times r^2\sin\theta\,d\theta\,d\phi\,(\bar{a}_r)$$

$$= \oint_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left(\frac{r^3}{4} \sin\theta d\theta d\phi\right) \times 10^{-9}$$

$$= \frac{r^3}{4} \times 10^{-9} \times [-\cos\theta]_0^{\pi} [\phi]_0^{2\pi}$$

$$= \frac{r^3}{4} \times 10^{-9} \times [1+1][2\pi] = \frac{r^3}{\cancel{4}} \times 10^{-9} \times \cancel{4}\pi$$

$$Q = r^3 \times 10^{-9} \times \pi$$

for a sphere of radius $r = 0.25m$

$$\therefore Q = (0.25)^3 \times 10^{-9} \times \pi$$

$$Q = 49.087pC.$$

---

8) An electric potential is given by $V = (60 \sin\theta / r^2) V$. Find V of $P(3, 60°, 25°)$.

8(a)

Sol:- Given, $V = \frac{60 \sin\theta}{r^2}$ & $P(3, 60°, 25°)$

At $P(3, 60°, 25°)$, $r = 3, \theta = 60°, \phi = 25°$

$$V = \frac{60 \sin 60}{(3)^2} = 5.7735 V$$

$$\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial r} \bar{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \bar{a}_\theta + \frac{1}{r\sin\theta} \frac{\partial V}{\partial \phi} \bar{a}_\phi\right]$$

$$\frac{\partial V}{\partial r} = 60 \sin\theta (-2) r^{-3} = \frac{-120 \sin\theta}{r^3} \quad \text{---- } \theta \text{ constant}$$

$$\frac{\partial V}{\partial \theta} = 60/r^2 \cos\theta \quad \text{---- } r \text{ constant}$$

$$\frac{\partial V}{\partial \phi} = 0$$

$$\therefore \bar{E} = -\left[\frac{-120 \sin\theta}{r^3} \bar{a}_r + \frac{1}{r} \cdot \frac{60}{r^2} \cos\theta \bar{a}_\theta\right]$$

$$\text{At } P, \bar{e} = -\left[-\frac{120 \sin(60)}{(3)^2} \bar{a}_r + \frac{1}{3} \frac{60}{(3)^2} \cos(60) \bar{a}_\theta\right]$$

$$\therefore \bar{E} = 3.849 \bar{a}_r - 1.111 \bar{a}_\theta \, V/m$$

1) In cylindrical Co-ordinates $J = 10e^{-100\rho}\,\bar{a}_\phi\ A/m^2$. find the current crossing through the region $0.01 < \rho < 0.02m$ and $0 < z < 1m$ and intersection of those region with the Constant plane(ϕ).

Sol:- The current is given by integral form of the Continuity equation as, $I = \int_S \bar{J}\cdot\bar{ds}$

Now, $\bar{ds} = d\rho\, dz\, \bar{a}_\phi$ $\quad\because$ Normal to $\bar{a}_\phi$ direction as $\bar{J}$ is in $\bar{a}_\phi$ direction

$$\therefore \bar{J}\cdot\bar{ds} = \left[10e^{-100\rho}\,\bar{a}_\phi\right]\cdot\left[d\rho\, dz\, \bar{a}_\phi\right]$$

$$= 10e^{-100\rho}\, d\rho\, dz$$

$$I = \int_S 10e^{-100\rho}\, d\rho\, dz$$

$$I = \int_{z=0}^{1}\int_{\rho=0.01}^{0.02} 10e^{-100\rho}\, d\rho\, dz$$

$$I = 10\left[\frac{e^{-100\rho}}{-100}\right]_{0.01}^{0.02}[z]_0^1$$

$$I = 10\left[\frac{e^{-2}}{-100} - \frac{e^{-1}}{-100}\right](1-0)$$

$$I = 10\left[-1.353\times10^{-3} + 3.678\times10^{-3}\right]$$

$$I = 10\left[2.325\times10^{-3}\right]$$

$$\therefore I = 2.325\times10^{-2} A$$

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2) An aluminum Conductor is 2000ft long and has a circular cross section with a diameter of 20mm. If there is a DC voltage of 1.2V between the ends. find (a) The current density, (b) The current, (c) power dissipated from the I = Knowledge of Circuit theory. Assume ~~σ = 3.82*~~ (30×10⁻²)m ~~= 600'~~ $\sigma = 3.82 * 10^7$ mho/m for aluminum.

Sol:- Given, $L = 2000ft = 2000\,(30\times10^{-2})m = 600m$

$$E = \frac{V}{L} = \frac{1.2}{600} = 2\times10^{-3}\ V/m$$ where, $V = 1.2V$

(a) $J = \sigma E = 3.82 \times 10^{7} \times 2\times10^{-3} = 76.4 KA/m^2$

(b) $I = JS = J \times \frac{\pi}{4} d^2 = \left[(3.82 \times 10^{7}) \times \left(\frac{\pi}{4}(20\times10^{-3})^2\right)\right]$

$$I = 76.4\times10^{3} \times \frac{\pi}{4} \times (20\times10^{-3})^2$$

$$I = 24 A$$

(c) P = power dissipated $= VI = \frac{V^2}{R} = I^2 R\ W$

$$P = 1.24 \times 24 = 28.802 W$$

$\therefore$ (a) The current density $J = 76.4 KA/m^2$

(b) The current $I = 24A$

(c) power dissipated $P = 28.802 W$.,

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3) Find the polarization in dielectric material with $\epsilon_r = 2.8$

5M if $D = 3*10^{-7} c/m^2$.

Sol:- Given $\epsilon_r = 2.8$, $\bar{D} = 3*10^{-7} c/m^2$

for the dielectric

$$\bar{P} = X_e \epsilon_0 \bar{e}$$

Now, $\epsilon_r = X_e + 1$

$$X_e = \epsilon_r - 1 = 2.8 - 1 = 1.8 \Rightarrow X_e = 1.8 \text{ and } \bar{D} = \epsilon_0 \epsilon_r \bar{e}$$

$$\bar{e} = \frac{\bar{D}}{\epsilon_0 \epsilon_r} = \frac{3\times10^{7}}{8.854\times10^{-12}\times 2.8} = 12.101\times10^{3} V/m.$$

$$\therefore \bar{P} = 1.8 \times 8.854 \times 10^{-12} \times 12.101 \times 10^{3}$$

$$\bar{P} = 1.9285 \times 10^{-7} c/m^2.$$

$\therefore$ polarization in dielectric material with ϵ_r & D is

$$\bar{P} = 1.9285 \times 10^{-7} c/m^2 .,$$

---

4) A parallel plate Capacitor has an area of $0.8m^2$ separa[tion]

7M of 0.1mm with a dielectric for which $\epsilon_r = 1000$ and a field

of $10^6 V/m$. Calculate C and V.

Sol:- Given, $A = 0.8m^2$ $d = 0.1mm$

$E_r = 1000 \qquad E = 10^6\, V/m$

$$C = \frac{\epsilon A}{d} = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{8.854\times10^{-12}\times1000\times0.8}{0.1\times10^{-3}} = 8.854\times 8\times10^{-6}$$

$C = 70.732\ \mu f$

$V = Ed = 10^6\times0.1\times10^{-3} = 100\,V.$

Hence, the value of Capacitances (c) is 70.732 μf.

the value of Voltage (v) is 100V

∴ The values of Capacitance and voltage across the two plates were determined.

---

5) Find the magnitude of D and P for a dielectric material in which $E = 0.15\ mV/m$ and $x = 4.25$.

sol: Given, $E = 0.15\ mV/m$ & $x = 4.25$

For a dielectric medium $\bar{D} = \epsilon_0 \epsilon_R \bar{e}$

where, $\epsilon_R = x_e + 1 = 4.25 + 1 = 5.25$

$|D| = 8.854\times10^{-12}\times5.25\times0.15\times10^{-3} = 6.9725\times10^{-15}\ C/m^2$

and $\bar{P} = x_e \epsilon_0 \bar{e}$

$|\bar{P}| = 4.25\times8.854\times10^{-12}\times0.15\times10^{-3} = 5.644\times10^{-15}\ C/m^2$

∴ The magnitude of D & P for a dielectric material is of the values are $|D| = 6.9725\times10^{-15}\ C/m^2$, $|\bar{P}| = 5.644\times10^{-15}\ C/m^2$.

---

6) Find V at P(2,1,3) for the field of two coaxial conducting cones, with $V = 50V$ at $\theta = 30°$ and $V = 20V$ at $\theta = 50°$

sol:- V is the function of θ only and not the function of r & ϕ. ~~bop~~

$$\nabla^2 V = \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = 0 \quad \text{....laplaces equation}$$

$$\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] = 0$$

Integrating, $\sin\theta \frac{\partial V}{\partial\theta} = \int 0\, d\theta + C_1 = C_1$

$$\therefore \frac{\partial V}{\partial\theta} = \frac{C_1}{\sin\theta} = C_1 \operatorname{cosec}\theta$$

Integrating, $V = \int C_1 \operatorname{cosec}\theta \, d\theta + C_2$

$$V = C_1 \ln\left[\tan \theta/2\right] + C_2$$

At $\theta = 30°$, $V = 50V$ and at $\theta = 50°$; $V = 20V$

$$50 = C_1(-1.3169) + C_2 \rightarrow ① \quad \text{and} \quad 20 = C_1(-0.7629) + C_2$$

By solving ① & ②:-

we have $C_1 = -54.152$ & $C_2 = -21.3125$

$$\therefore V = -54.152 \ln\left[\tan\left(\theta/2\right)\right] - 21.3125 V$$

For $P(2,1,3)$; $x = 2$, $y = 1$, $z = 3$

$$\therefore \theta = \cos^{-1}\frac{z}{r} = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}} = \cos^{-1}\frac{3}{\sqrt{4+1+9}} = 36.69$$

$$\therefore V_p = -54.152 \ln\left[\tan\left(\frac{36.6992°}{2}\right)\right] - 21.3125$$

$$\therefore V_p = 38.4489 V$$

---

7) Two parallel conducting disc are separated by distances at $z=0$ & $z=5mm$. If $V=0$ and $V=100V$ at $z=5mm$, find the charge densities on the disc.

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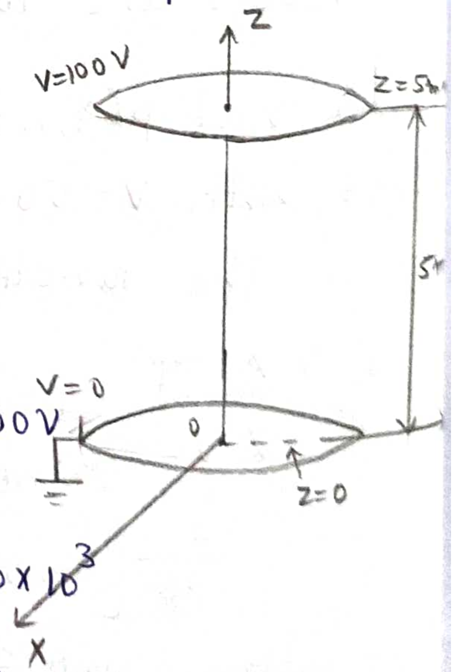
Sol: Consider cylindrical co-ordinates. The potential V is the func of z alone ~~densities on the~~ and is independent of r and ϕ.



$$\therefore \nabla^2 V = \frac{\partial^2 V}{\partial z^2} = 0 \text{ (laplace's equation)}$$

Integrating, $\frac{\partial V}{\partial z} = \int 0 \, dz + C_1 = C_1$

Integrating, $V = \int C_1 dz + C_2 = C_1 z + C_2$

At $z=0$, $V=0V$ and at $z = 0.005m$, $V = 100V$

$\therefore 0 = C_1(0) + C_2$ then $C_2 = 0$

and $100 = C_1 \times 0.005 + C_2$ thus $C_1 = 20 \times 10^3$

$$V = 20 \times 10^3 z V$$

Now, $E = -\nabla V = -\frac{\partial V}{\partial z}\bar{a}_z = \frac{-\partial}{\partial z}\left[20 \times 10^3 z\right]\bar{a}_z = 20 \times 10^3 \bar{a}_z \, V/m$

$D = \epsilon_0 E = -8.854\times10^{-12} \times 20\times10^{3}\ \bar{a}_z = -1.77\times10^{-7}\bar{a}_z\ C/m^2$

The $\bar{D}$ acts in the normal direction as per the boundary Conditions. Thus $\bar{D} = \bar{D_N}$

$$\therefore \bar{D}_N = -1.7708\times10^{-7} a_z$$

$$P_s = |\bar{D}_N| = 1.7708\times10^{-7}\ C/m^2 = 177.08\ nC/m^2$$

This is the magnitude of surface charge densities on the disc so,

$$P_s = \pm 177.08\ nC/m^2$$

which shows positive on Upper plate and Negative on lower plate.

8, Determine the whether or not the following potential fields satisfy the laplace's equation.

(a) $V = x^2 - y^2 + z^2$ (b) $V = r\cos\phi + z$ ~~(c) V=~~

Sol: (a) $V = x^2 - y^2 + z^2$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2}{\partial x^2}\left[x^2 - y^2 + z^2\right] + \frac{\partial^2}{\partial y^2}\left[x^2 - y^2 + z^2\right] + \frac{\partial^2}{\partial z^2}\left[x^2 - y^2 + z^2\right]$$

$$\nabla^2 V = 2x + (-2y) + 2z$$

$$\nabla^2 V = 2x - 2y + 2z$$

$$\nabla^2 V = 2\frac{\partial x}{\partial x} - 2\frac{\partial y}{\partial y} + 2\frac{\partial z}{\partial z}$$

$$\nabla^2 V = 2 - 2 + 2$$

$$\nabla^2 V = 2 \neq 0$$

$$\therefore \nabla^2 V \neq 0$$

Hence field (v) is not supported (or) satisfy the ~~H~~ laplace's equation.

(b) $V = r\cos\phi + z$

In cylindrical Co-ordinate system,

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(r\cos\phi + z)\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}(r\cos\phi + z) + \frac{\partial^2}{\partial z^2}(\ldots$$

$$\nabla^2 V = r\cos\phi + z$$

$$\frac{\partial V}{\partial r} = \cos\phi \quad \text{and} \quad \frac{\partial V}{\partial \phi} = -r\sin\phi \ \ \& \ \ \frac{\partial V}{\partial z} = 1$$

$$\therefore \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = \frac{1}{r}\cdot\frac{\partial}{\partial r}[r\cos\phi] = \frac{1}{r}\cos\phi$$

$$\frac{1}{r^2}\left[\frac{\partial^2 V}{\partial \phi^2}\right] = \frac{1}{r^2}\left[\frac{\partial}{\partial \phi}(-r\sin\phi)\right] = \frac{-r\cos\phi}{r^2} = \frac{-\cos\phi}{r}$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{\partial}{\partial z}[1] = 0$$

$$\nabla^2 V = \frac{1}{r}\cos\phi - \frac{\cos\phi}{r} = 0$$

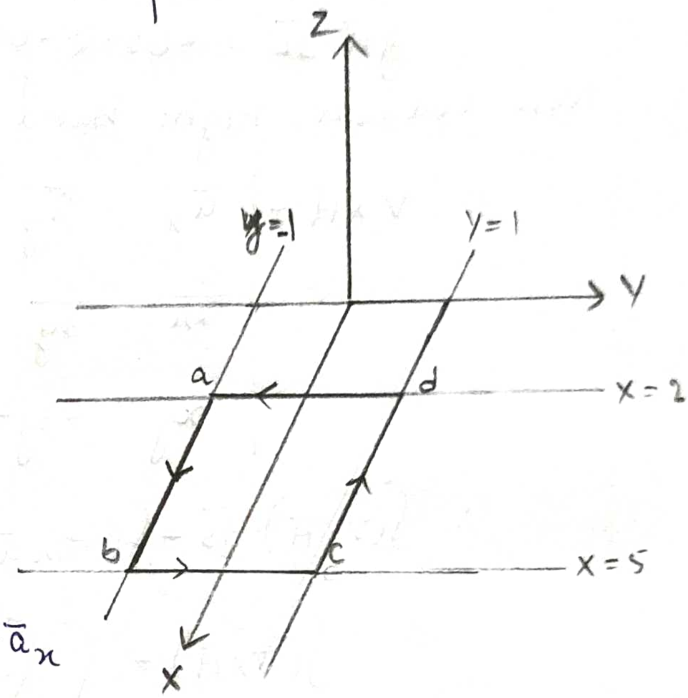
$$\therefore \nabla^2 V = \frac{1}{r}\cos\phi - \frac{\cos\phi}{r} = 0$$

So this field satisfies laplace's equation.

1) Evaluate both sides of the stokes theorem for the field $\bar{H} = 6xy\,\bar{a}_x - 3y^2\bar{a}_y$ A/m and the rectangular path around the region $2 < x < 5$, $-1 < y < 1$, $z = 0$. let the positive direction of ds be $\bar{a}_z$.

sol:- Given field

$$H = 6xy\bar{a}_x - 3y^2\bar{a}_y$$



According to stoke's theorem

$$\oint_L \bar{H}\cdot\bar{dL} = \int(\nabla\times\bar{H})ds$$

$$\therefore \oint \bar{H}\cdot\bar{dL} = \int_{ab} + \int_{bc} + \int_{cd} + \int_{da} \bar{H}\cdot\bar{dL}$$

$$\int_{ab} \bar{H}\cdot\bar{dL} = \int_{x=2}^{5} (6xy\bar{a}_x - 3y^2\bar{a}_y)dx\cdot\bar{a}_x$$

$$= \int_{x=2}^{5} 6xy\,dx = 6y\left[\frac{x^2}{2}\right]_2^5$$

$$= 3y(x^2)_2^5$$

$$= 3y(5^2 - 2^2) = 3y(25-4)$$

$$= 3y(21)$$

$$\int_{ab} \bar{H}\cdot\bar{dL} = 63y$$

$$\Rightarrow \int_{ab} \bar{H}\cdot\bar{dL} = 63y = 63(-1) = 63$$ (now $y=-1$ for path ab)

Similarly $\int_{bc} \bar{H}\cdot\bar{dL} = \int_{y=-1}^{1} -3y^2dy = -3\left[\frac{y^3}{3}\right]_{-1}^{1} = -1[y^3]_{-1}^{1} = -1[1+1] = -1[2]$

$$\int_{bc} \bar{H}\cdot\bar{dL} = -2$$

$$\int_{ad} \bar{H}\cdot\bar{dL} = \int_{x=5}^{2} 6xy\,dx = 6\left[\frac{x^2}{2}\right]_5^2 y = 3y[x^2]_5^2 = 3y[2^2 - 5^2]$$

$$\int_{ad} \bar{H}\cdot\bar{dL} = 3y[4-25] = 3y[-21] = -63y$$

But $y=1$ for path cd hence $\int_{cd} \bar{H}\cdot\bar{dL} = -63$.

$$\int_{da} \bar{H}\cdot\bar{dL} = \int_{y=1}^{-1} -3y^2 dy = -3\left[\frac{y^3}{3}\right]_1^{-1} = -1[-1\mp1] = -1[-2] = 2$$

$$\int_{da} \bar{H}\cdot\bar{dL} = 2$$

$$\therefore \oint \bar{H}\cdot\bar{dL} = -63-2-63+2 = -126A$$

Now evaluate right hand side

$$\nabla\times\bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & -3y^2 & 0 \end{vmatrix} = \bar{a}_x[0-0]+\bar{a}_y[0-0]+\bar{a}_z[0-6$$
$$= -6x\bar{a}_z.$$

$$\therefore \int_s (\nabla\times\bar{H})\cdot\bar{ds} = \int_s (-6x\bar{a}_z)(dxdy\bar{a}_z)$$ ($\bar{ds} = dxdy\bar{a}_z$ normal to direction $\bar{a}_n$)

$$\int_s (\nabla\times\bar{H}) = \int_{y=-1}^{1}\int_{x=2}^{5} -6x\,dx\,dy = -6\left[\frac{x^2}{2}\right]_2^5 [y]_{-1}^{1}$$
$$= -3[x^2]_2^5[1-(-1)] = -3[5^2-2^2][1+$$
$$= -3[25-4][2]$$
$$= -3[21][2]$$
$$\therefore \int_s (\nabla\times\bar{H})\cdot\bar{ds} = -126A$$

Thus both the sides are same, here stoke's theorem verified.

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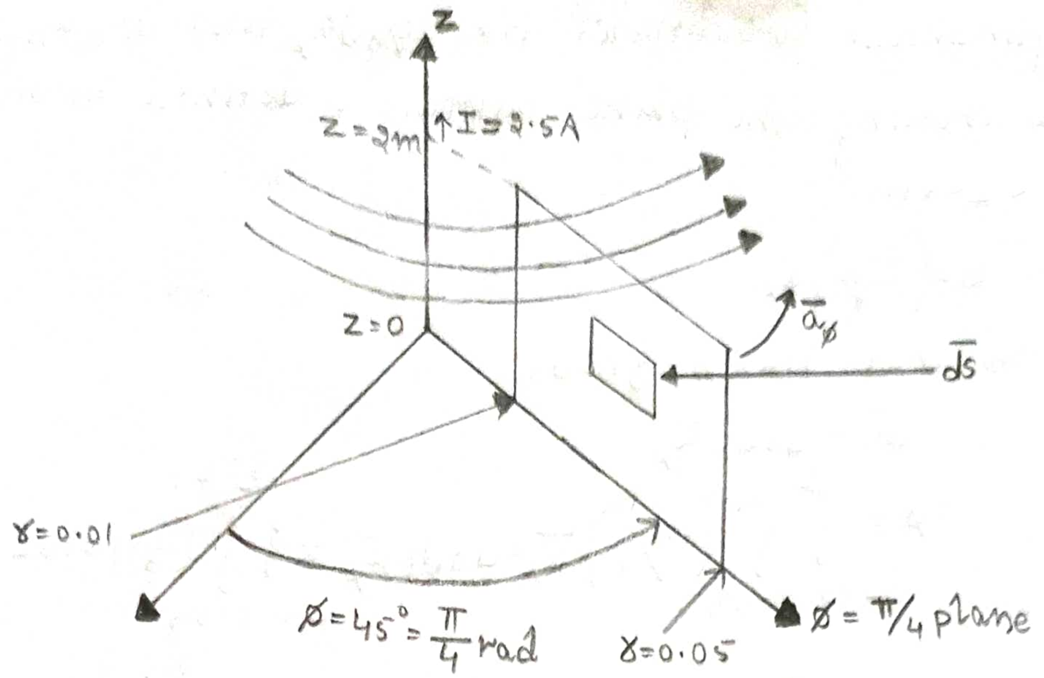
2) Find the flux passing the portion of the plane $\phi = \frac{\pi}{4}$ defi by $0.01<r<0.05m$ and $0<z<2m$. A current filament of 2.5A is along the z-axis in the a_z direction in free spac

sol:- Given,

flux (ϕ) passing through the portion of plane $\phi = \pi/4$ given by $0.01<r<0.05m$ & $0<z<2m$.

A current filament of 2.5A is along the z-axis in the a_z direction in free space. The arrangement is as shown in the figure



Due to current carrying Conductor in free space along z-axis, $\bar{H}$ is given by $\bar{H} = \frac{I}{2\pi r} \bar{a}_\phi$

$$= \frac{2.5}{2\pi r} \bar{a}_\phi$$

$$\bar{H} = \frac{0.3978}{r} \bar{a}_\phi \text{ A/m}$$

$$\bar{B} = \mu_0 \bar{H} = \frac{4\pi \times 10^{-7} \times 0.3978}{r} \bar{a}_\phi = \frac{5\times10^{-7}}{r} \bar{a}_\phi \text{ Wb/m}^2$$

$$= \frac{5\times10^{-7}}{r} \bar{a}_\phi \text{ Wb/m}^2$$

The flux Crossing the surface,

$$\phi = \int_s \bar{B} \cdot \bar{ds}$$

(normal to $\bar{a}_\phi$ direction)

Now, $\bar{ds} = dr\,dz\,\bar{a}_\phi$

$$\phi = \int_{z=0}^{2} \int_{r=0.01}^{r=0.05} \frac{5\times10^{-7}}{r} \bar{a}_\phi \cdot dr\,dz\,\bar{a}_\phi$$

$$\phi = \int_{z=0}^{2} \int_{r=0.01}^{r=0.05} \frac{5\times10^{-7}}{r} dr\,dz = 5\times10^{-7} [\ln r]_{0.01}^{0.05} [z]_0^2$$

$$\phi = 5\times10^{-7} \ln\left[\frac{0.05}{0.01}\right][2] = 1.6094\,\mu\text{Wb}$$

$\therefore \phi = 1.6094\,\mu$Wb

The flux (x) Crossing the surface be $\phi = 1.6094\,\mu$Wb.

3) In cylindrical Co-ordinates $B=(2.0/r)a_\phi$ tesla. Determine the
4(b) flux ϕ crossing the plane surface is defined by $0.5<r<$
and $0<z<2m$.

sol:- Given, $B=\left(\frac{2.0}{r}\right)\bar{a}_\phi$

We know that flux $\phi=\oint \bar{B}\cdot\bar{ds}$

$$\bar{ds}=dr\,dz\,\bar{a}_\phi$$

$$\therefore \phi=\int_{z=0}^{2}\int_{r=0.5}^{2.5}\left(2.0/r\right)\bar{a}_\phi\cdot dr\,dz\,\bar{a}_\phi=\int_{z=0}^{2}\int_{r=0.5}^{2.5}\left(\frac{2}{r}\right)dr\,dz$$

$$\phi=4\ln\left[\frac{2.5}{0.5}\right]=6.4377wb$$

$$\therefore \phi=6.4377wb.$$

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4) In cylindrical Co-ordinates $A=50r^2a_z\,wb/m$ is a vector magnetic
5) potential in a certain region of free space. find the H, B, J
and using J find the total current I crossing the surface
$0<r<1$, $0<\phi<2\pi$ and $z=0$.

sol:- Given; Vector magnetic potential, $A=50r^2a_z\,wb/m$

Now $\bar{B}=\nabla\times\bar{A}$

$$\bar{B}=\left[\frac{1}{r}\frac{\partial A_z}{\partial\phi}-\frac{\partial A_\phi}{\partial z}\right]\bar{a}_r+\left[\frac{\partial A_r}{\partial z}-\frac{\partial A_z}{\partial r}\right]\bar{a}_\phi+\frac{1}{r}\left[\frac{\partial(rA_\phi)}{\partial r}-\frac{\partial A_r}{\partial\phi}\right]\bar{a}_z$$

Now $A_r=0$, $A_\phi=0$, $A_z=50r^2$

$$\bar{B}=\left[\frac{1}{r}\frac{\partial(50r^2)}{\partial\phi}-0\right]\bar{a}_r+\left[0-\frac{\partial(50r^2)}{\partial r}\right]\bar{a}_\phi+\frac{1}{r}[0-0]\bar{a}_z$$

$$\bar{B}=-100r\,\bar{a}_\phi\ wb/m^2$$

$$\bar{H}=\frac{\bar{B}}{\mu_0}=\frac{-100}{\mu_0}r\,a_\phi\ A/m$$

Now $\bar{J}=\nabla\times\bar{H}$

$$H_r=0,\ H_\phi=\frac{-100r}{\mu_0},\ H_z=0$$

$$\nabla\times\bar{H}=\left[0-\frac{\partial\left(\frac{-100r}{\mu_0}\right)}{\partial z}\right]\bar{a}_r+[0-0]\bar{a}_\phi+\frac{1}{r}\left[\frac{\partial\left(\frac{-100r^2}{\mu_0}\right)}{\partial r}-0\right]\bar{a}_z$$

$\nabla \times \bar{H} = [0-0]\,\bar{a}_r + 0\,\bar{a}_\phi + \frac{1}{r}\left[\frac{-100}{\mu_0}\right][2r]\,\bar{a}_z = \frac{-200}{\mu_0}\,\bar{a}_z\ A/m^2$

$\therefore\ \bar{J} = \frac{-200}{\mu_0}\,\bar{a}_z\ A/m^2$

Now $I = \int_s \bar{J}\cdot d\bar{s}$ where $d\bar{s} = r\,dr\,d\phi\,\bar{a}_z$

$= \int_{\phi=0}^{2\pi}\int_{r=0}^{1} \frac{-200}{\mu_0}\,\bar{a}_z\cdot r\,dr\,d\phi\,\bar{a}_z = \int_{\phi=0}^{2\pi}\int_{r=0}^{1} \frac{-200}{\mu_0}\,r\,dr\,d\phi$

$I = \frac{-200}{\mu_0}\left[\frac{r^2}{2}\right]_0^1 [\phi]_0^{2\pi} = \frac{-200}{\mu_0}[1/2][2\pi]$ | where $\mu_0 = 1.256 \times 10^{-6}$

$I = -500 \times 10^{6} A$

so current is $500\mu A$ and negative sign indicates the direction of current.

A point charge of $Q = -1.2C$ has a velocity $V = (5\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z)\,m/s$. Find the magnitude of the force exerted on the charge if (i) $\bar{E} = -18\bar{a}_x + 5\bar{a}_y - 10\bar{a}_z$ V/m and (ii) $\bar{B} = -4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z\,T$.

Given,

Point charge $Q = -1.2C$

velocity $V = (5a_x + 2a_y - 3a_z)\,m/s$

(a) The electric force exerted by $\bar{E}$ on charge Q is given by

$\bar{F}_e = Q\bar{E}$

$\bar{F}_e = -1.2[-18\bar{a}_x + 5\bar{a}_y - 10\bar{a}_z]$

$\bar{F}_e = 21.6\bar{a}_x - 6\bar{a}_y + 12\bar{a}_z$

Thus the magnitude of the electric force is given by

$|\bar{F}_e| = \sqrt{(21.6)^2 + (-6)^2 + (12)^2}$

$\therefore\ |\bar{F}_e| = 25.4275 N$

(b) The magnitude force exerted by $\bar{B}$ on charge Q is given by

$\bar{F}_m = Q\bar{V} \times \bar{B}$

$= -1.2[(5\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z) \times (-4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z)]$

$= [(-6\bar{a}_x - 2.4\bar{a}_y + 3.6\bar{a}_z) \times (-4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z)]$

$$= \begin{vmatrix} a_x & a_y & a_z \\ -6 & -2.4 & 3.6 \\ -4 & 4 & 3 \end{vmatrix}$$

$$\bar{F}_m = [7.2-14.4]\bar{a}_x - [-18+14.4]\bar{a}_y + [-24-9.6]\bar{a}_z$$

$$\bar{F}_m = (-21.6\bar{a}_x + 3.6\bar{a}_y - 33.6\bar{a}_z)\,N$$

Thus, the magnitude of the magnetic force is given by

$$|\bar{F}_m| = \sqrt{(-21.6)^2 + (3.6)^2 + (-33.6)^2}$$

$$\therefore |\bar{F}_m| = 40.1058\,N$$

6a) A magnetic field $B = 3.5 * 10^{-2} a_z$ exerts a force on a 0.3m long
c(b) Conductor along x-axis. If a current of 5A flows in $-a_x$ direction, determine what force must be applied to hold Conductor in position.

sol:- Given,

A magnetic field $\bar{B} = 3.5 * 10^{-2} a_z$

length of the Conductors $= 0.3m\,(-\bar{a}_x)$ long

Current $I = 5A$

The force exerted on a straight Conductor is given by,

$$\bar{F} = I\bar{L} \times \bar{B}$$

$$\bar{F} = 5(-0.3\bar{a}_x) \times (3.5 \times 10^{-2}\bar{a}_z)$$

$$\bar{F} = -0.0525\,(-\bar{a}_y) \qquad \therefore \bar{a}_x \times \bar{a}_z = -\bar{a}_y$$

$$\bar{F} = 0.0525\bar{a}_y\,N$$

Hence the force applied to hold the Conductor in position must be

$$\bar{F} = 0.0525\,\bar{a}_y\,N.$$

6b) Determine the force per meter length between two long
d(c) parallel wires A & B separated by distances 5cm in air and carrying currents of 40A in the same direction.

sol:- Given $I_1 = I_2 = 40A$, d= seperation's distances (or) distances of separation i.e, $d = 5cm = 5\times10^{-2} m$

$\mu_r = 1$

∴ The force per meter length between the two long conductors is given by $\frac{F}{L} = \frac{\mu I_1 I_2}{2\pi d} = \frac{\mu_0 \mu_r I_1 I_2}{2\pi(5\times10^{-2})} \cancel{(\mu_0)} = 6.4\times10^{-3}\ N/m$

As the currents in both the parallel conductors ($\mu_0 = 1.2566\times10^{-6}$) flow in same direction, the force experienced will be force of attraction.

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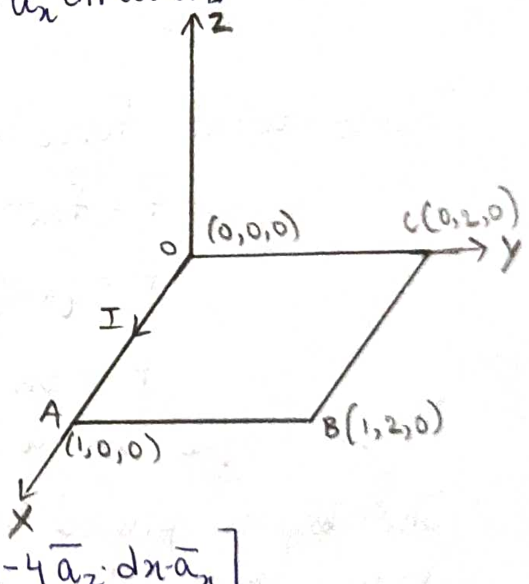
7) A rectangular loop in $z=0$ plane has corners at $(0,0,0)$ $(1,0,0)$ $(1,2,0)$ and $(0,2,0)$. The loop carries a current of 5A in a_x direction. find the total force and torque on the loop produced by magnetic field $B = 2a_x + 2a_y - 4a_z\ wb/m^2$.

A: ~~Given $I_1 = I_2 = 40A$, d= separation's distances (or) direction of separation. i.e, $d = 5cm = 5\times10^{-2} m$~~

A: Consider a rectangular loop as shown in the figure.

Given that,

the loop carries a current in a_x direction is $(I) = 5A$



Magnetic field $B = 2a_x + 2a_y - 4a_z\ wb/m^2$

force on side OA is given by,

$$\bar{F}_{OA} = -I\oint \bar{B}\times d\bar{L}$$

$$\bar{F}_{OA} = -5\int_{x=0}^{x=1}(2\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z)\times(dx\,\bar{a}_x)$$

$$\bar{F}_{OA} = -5\int_0^1\left[2\bar{a}_x\cdot dx\,\bar{a}_x + 2\bar{a}_y\cdot dx\,\bar{a}_x - 4\bar{a}_z\cdot dx\,\bar{a}_x\right]$$

$$\bar{F}_{OA} = -5\int_0^1\left[-2dx\,\bar{a}_z + 4dx\,\bar{a}_y\right]$$

$$\bar{F}_{OA} = 10\int_0^1 dx\,\bar{a}_z + 20\int_0^1 dx\,\bar{a}_y = 10\bar{a}_z + 20\bar{a}_y N$$

$$\bar{F}_{OA} = 20\bar{a}_y + 10\bar{a}_z N$$

Force on side AB is given by, $\bar{F}_{AB} = -I\oint\bar{B}\times d\bar{L} = -5\int_{y=0}^{y=2}(2\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z)\times(dy\,\bar{a}_y)$

$$\bar{F}_{AB} = -5\int_0^2 (2dy\,\bar{a}_z + 4dy\,\bar{a}_x) = -10\int_0^2 dy\,\bar{a}_z - 20\int_0^2 dy\,\bar{a}_x$$

$$\bar{F}_{AB} = -40\,\bar{a}_x - 20\,\bar{a}_z\,N.$$

Force on side BC is given by

$$\bar{F}_{BC} = -I\oint \bar{B}\times d\bar{L} = -5\int_{x=1}^{x=0} (2\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z)\times(dx\,\bar{a}_x)$$

$$\bar{F}_{BC} = -5\int_1^0 (-2\bar{a}_z\,dx + (-4\bar{a}_y\,dx))$$

$$\bar{F}_{BC} = 10\int_1^0 dx\,\bar{a}_z + 20\int_1^0 dx\,\bar{a}_y$$

$$\bar{F}_{BC} = -10\,\bar{a}_z - 20\,\bar{a}_y\,N$$

Force on side CO is given by,

$$\bar{F}_{CO} = -I\oint \bar{B}\times d\bar{L} = -5\int_{Y=2}^{Y=0} (2\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z)\times(dy.\bar{a}_y)$$

$$\bar{F}_{CO} = -5\int_2^0 (2dy\,\bar{a}_z + 4dy\,\bar{a}_x)$$

$$\bar{F}_{CO} = -10\int_2^0 dy\,\bar{a}_z - 20\int_2^0 dy\,\bar{a}_x$$

$$\bar{F}_{CO} = 20\,\bar{a}_z + 40\,\bar{a}_x$$

$$\bar{F}_{CO} = 40\,\bar{a}_x + 20\,\bar{a}_z$$

Hence total force on a rectangular loop is given by

$$\bar{F} = \bar{F}_{OA} + \bar{F}_{AB} + \bar{F}_{BC} + \bar{F}_{CO}$$

$$\bar{F} = [20\bar{a}_y + 10\bar{a}_z] + [-40\bar{a}_x - 20\bar{a}_z] + [-20\bar{a}_y - 10\bar{a}_z] + [40\bar{a}_x + 20\bar{a}$$

$$\bar{F} = 0N$$

Total torque on a loop is given by, $\bar{T} = I(d\bar{S})\times\bar{B}$.

The loop is placed in x-y plane i.e, z=0 plane with dimensions of the rectangular is as 1x2.

Hence $d\bar{S} = (1)(2)\,\bar{a}_z = 2\bar{a}_z$

$$\therefore \bar{T} = 5(2\bar{a}_z)\times(2a_x + 2a_y - 4a_z) = \cancel{20a_y + 20a_y + 40a0} = 20a_y - 20a_x + 0$$

$$\bar{T} = -2a_x + 2a_y$$

$$\bar{T} = 20(-\bar{a}_x + \bar{a}_y)\,N.m$$

Total torque on a loop is given by $\bar{T} = 20(-\bar{a}_x + \bar{a}_y)\,N.m.$

8) Calculate the inductances of a solenoid of 200 ~~times~~ turns wound tightly on a cylindrical tube of 6 cm diameter. The length of the tube is 60 cm and the solenoid is in air.

sol:- For a given solenoid in air, $\mu = \mu_0 = 4\pi \times 10^{-7}$ wb/Am, $N = 200$ turns

$l = 60cm = 60 \times 10^{-2} m$, $d = 6cm = 6 \times 10^{-2} m$ hence $r = 3 \times 10^{-2} m$.

The inductance of a solenoid is given by,

$$L = \frac{\mu N^2 A}{l} = \frac{\mu_0 N^2 (\pi r^2)}{l} = \frac{4\pi \times 10^{-7} \times (200)^2 \times \pi (3 \times 10^{-2})^2}{60 \times 10^{-2}}$$

$$\therefore L = 2.3687 \times 10^{-4} H = 0.2368 \ mH.$$

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9) Find the inductance per unit length of a co-axial cable of
(10) radius of inner and outer conductors are 1mm and 3mm respectively. Assume relative permeability unity.

sol:- For a co-axial cable, the inductances per unit length is given by,

$$\frac{L}{d} = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{b}{a}\right)$$

where from the given data the values of $\mu_r = 1$ $\quad b = 3 \times 10^{-3} m$

$\mu_0 = 1.2566 \times 10^{-6}$ $\quad a = 1 \times 10^{-3} m$

$$\therefore \frac{L}{d} = \frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{1.2566 \times 10^{-6} \times 1}{2\pi} \ln\left(\frac{3 \times 10^{-3}}{1 \times 10^{-3}}\right) = 0.2197 \times 10^{-7} H/m.$$

Hence, the inductances per unit length is given by

$$\frac{L}{d} = 2.197 \times 10^{-6} H/m$$

$$\therefore \frac{L}{d} = 2.197 \ \mu H/m.$$

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10) Calculate the inductances of a 10m length of coaxial cable filled with a material for which $\mu_r = 80$ and radii inner and outer conductors are 1mm and 4mm respectively.

sol:- Given, length of coaxial cable $(d) = 10m$

$\mu_r = 80$, $\mu_0 = 1.2566 \times 10^{-6}$

radii at inner $\quad a = 1mm = 1 \times 10^{-3} m$

radii at outer $\quad b = 4mm = 4 \times 10^{-3} m$

Inductance $\quad L = x$

The inductances of a Co-axial cable ~~of~~ is given by,

$$L = \frac{\mu d}{2\pi} \ln(b/a)$$

$$L = \frac{(\mu_0 \mu_r)d}{2\pi} \ln(b/a)$$

$$L = \frac{1.2566 \times 10^{-6} \times 10 \times 80}{2\pi} \ln\left(\frac{4\times10^{-3}}{1\times10^{-3}}\right)$$

$$L = 221.8 \times 10^{-6} H$$

$$\therefore L = 221.8 \mu H$$

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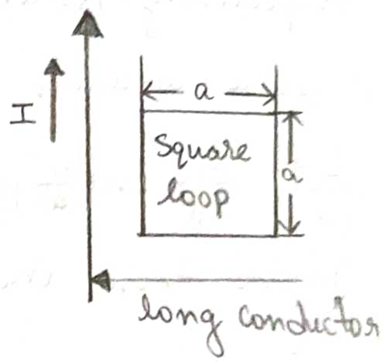
10 (b) 11) A straight long wire is situated parallel to one side of a square Coil has a length of 10cm. The distance between straight wire and the centre of coil is 20cm. find the mutual inductances of the system.



Sol: Given,

$a = 10cm = 10 \times 10^{-2} = 0.1m$, $d = 20cm$

$d = 20 \times 10^{-2}$

$d = 0.2m$

The mutual inductances between the long straight wire and is given by $M = \frac{\mu_0 a}{2\pi} \ln[1 + a/d]$

$$M = \frac{4\pi \times 10^{-7} \times 0.1}{2\pi} \ln\left[1 + \frac{0.1}{0.2}\right]$$

$$\therefore M = 8.1093 nH.$$

UNIT - 5

1) An area of $0.65\,m^2$ in the plane $z=0$ encloses a filamentary conductors. Find the induced voltage if $\vec{B} = 0.05\cos 10^3 t \left(\frac{a_y + a_z}{\sqrt{2}}\right)$ tesla.

10a

Sol:- Given,

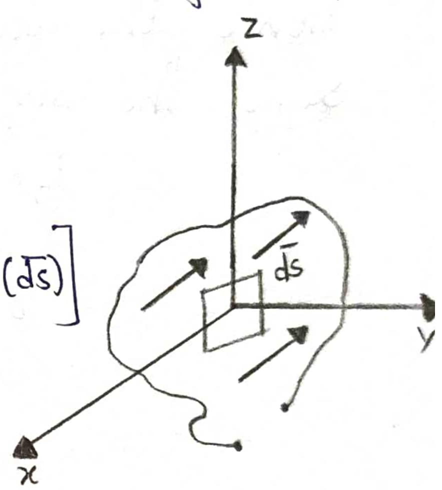
An area of $0.65\,m^2$ in the plane $z=0$ encloses a filamentary conductors.

Here the filamentary conductors is fixed and is placed in $z=0$ plane. It encloses area of $0.65\,m^2$.

$$\therefore \overline{ds} = ds\,\overline{a_z}$$

Induced e.m.f according to faraday's law is given by,

$$e = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot \overline{\partial s}$$



given, $\bar{B} = 0.05\cos 10^3 t \left(\frac{a_y + a_z}{\sqrt{2}}\right)$ Tesla

$$\therefore e = -\int_S \frac{\partial}{\partial t}\left[0.05\cos 10^3 t \left(\frac{a_y + a_z}{\sqrt{2}}\right) \text{Tesla} \cdot (\overline{ds})\right]$$

$$e = -\int_S \frac{\partial}{\partial t}\left[0.05\cos 10^3 t \left(\frac{a_y + a_z}{\sqrt{2}}\right)\right] \cdot (ds\,\overline{a_z})$$

$$e = -\int_S \frac{0.05(10^3)(-\sin(10^3)t)}{\sqrt{2}}\,ds$$

$\bar{a}_y \cdot \bar{a}_z = 0$

$\bar{a}_z \cdot \bar{a}_z = 1$

$$e = 35.355 \sin 10^3 t \left[\int_S ds\right]$$

But $\int_S ds$ is given as $0.65\,m^2$

$$\therefore e = 35.355 \sin 10^3 t\,[0.65] = 22.98 \sin 10^3 t\,V$$

Hence the induced voltage $e = 22.98 \sin 10^3 t\,V$.

2) A parallel plate capacitor with plate area of $50\,cm^2$ and plate separation of $3\,mm$ has a voltage of $50\sin 10^3 t$ volts applied to its plates. Calculate the displacement current assuming $\varepsilon = 2\varepsilon_0$.

9a

Sol:- Given,

A parallel plate capacitor with plate area of (A) $50\,cm^2$ and plate seperation (d) of $3\,mm$ has a voltage (V) of $50\sin 10^3 t$ volts applied to its plates, and to find the displacement current assuming $\varepsilon = 2\varepsilon_0$.

$$D = \varepsilon E = \varepsilon (V/d)$$

Hence the displacement current density is given by,

$$J_D = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}\left(\varepsilon \frac{V}{d}\right) = \frac{\varepsilon}{d}\frac{\partial V}{\partial t}.$$

Hence the displacement current is given by,

$$i_D = J_D \cdot Area = \left(\frac{\varepsilon}{d} \cdot \frac{\partial V}{\partial t}\right)(A) = \frac{\varepsilon A}{d} \cdot \frac{\partial V}{\partial t} = C\frac{dV}{dt}$$

This current is same as conduction current

$$i_c = \frac{dQ}{dt} = A \cdot \frac{dD}{dt} = \varepsilon A \frac{dE}{dt}.$$

Hence the conduction current and displacement current same. The displacement current is given by

$$i_D = \frac{\varepsilon A}{d} \cdot \frac{dV}{dt} = \frac{(2\varepsilon_0)A}{d} \cdot \frac{dV}{dt}$$

$$i_D = \frac{2 \times 8.854 \times 10^{-12} \times 5 \times 10^{-4}}{3 \times 10^{-3}} \frac{d}{dt}\left(50 \sin 10^3 t\right)$$

$$i_D = \frac{2 \times 8.854 \times 10^{-12} \times 5 \times 10^{-4} \times 50 \times 10^3 \, \cancel{\cos 50}}{3 \times 10^{-3}} \times \cos 10^3 t$$

$$i_D = \frac{2 \times 8.854 \times 10^{-12} \times 5 \times 10^{-4} \times 50 \times 10^3}{3 \times 10^{-3}} \cos 10^3 t$$

$$i_D = 0.1475 \cos 10^3 t \, \mu A.$$